

1.022 Introduction to Network Models

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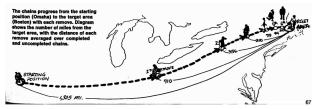
Lecture 11

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Small-world phenomenon



► Stanley Milgram's experiment ⇒ six degrees of separation



Milgram, Stanley. "The Small-World Today." Psychology Today 1 (1967): 61–67.© Sussex Publishers. All rights reserved. This content is excluded from our Creative Commons license. For more information, see https://ocw.mit.edu/help/faq-fair-use/.

▶ Get letter from 'starter' to target by forwarding to acquaintances ⇒ Letters arrive with a median of six steps

Two rather surprising facts

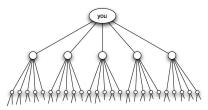
- \Rightarrow 1) Short paths between two nodes exist in abundance
- \Rightarrow 2) People without global knowledge can find these paths

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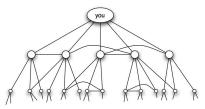
How can this happen?



Pure exponential growth produces a small world



Triadic closure reduces the growth rate



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Can a model exhibit both many closed triads and very short paths?

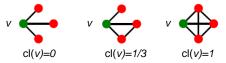
Clustering coefficient



- ▶ Q: What fraction of *v*'s neighbors are themselves connected?
- The clustering coefficient cl(v) of $v \in V$ is

$$\mathsf{cl}(v) = \frac{2|E_v|}{d_v(d_v-1)} \in [0,1]$$

 $\Rightarrow |E_v|$ is the number of edges among v's neighbors



- An indication of the extent to which edges 'cluster'
- The global (average) clustering coefficient is

$$\mathsf{cl}(G) = \frac{1}{|V|} \sum_{v \in V} \mathsf{cl}(v)$$

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Network	size	av. shortest path	Shortest path in fitted random graph	Clustering (averaged over vertices)	Clustering in random graph
Film actors	225,226	3.65	2.99	0.79	0.00027
MEDLINE co- authorship	1,520,251	4.6	4.91	0.56	1.8 x 10 ⁻⁴
E.Coli substrate graph	282	2.9	3.04	0.32	0.026
C.Elegans	282	2.65	2.25	0.28	0.05

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- Reconciling short paths and high clustering coefficients
- Desired number of nodes n, average degree K and probability p
 - \Rightarrow 1) Construct a circle of *n* nodes
 - \Rightarrow 2) Connect each node to its K closest neighbors
 - \Rightarrow 3) With probability *p* rewire each edge uniformly



Watts, Duncan J., and Steven H. Strogatz. "Collective dynamics of 'small-world' networks." Nature 393 (1998): 441–42. © Springer Nature. All rights reserved. This content is excluded from our Creative Commons license. For more information, see https://co.wnit.edu/he/fi/afa_fair-use/.

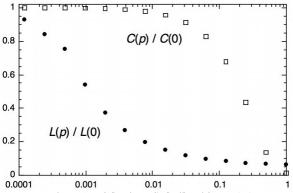
Small p: regular lattice. Large p: close to ER graph

 \Rightarrow There is a sweet spot in between

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- Plot clustering coefficient and average shortest path
 - \Rightarrow As a function of the rewiring probability p
- What happens for $p \approx 0.01$?



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