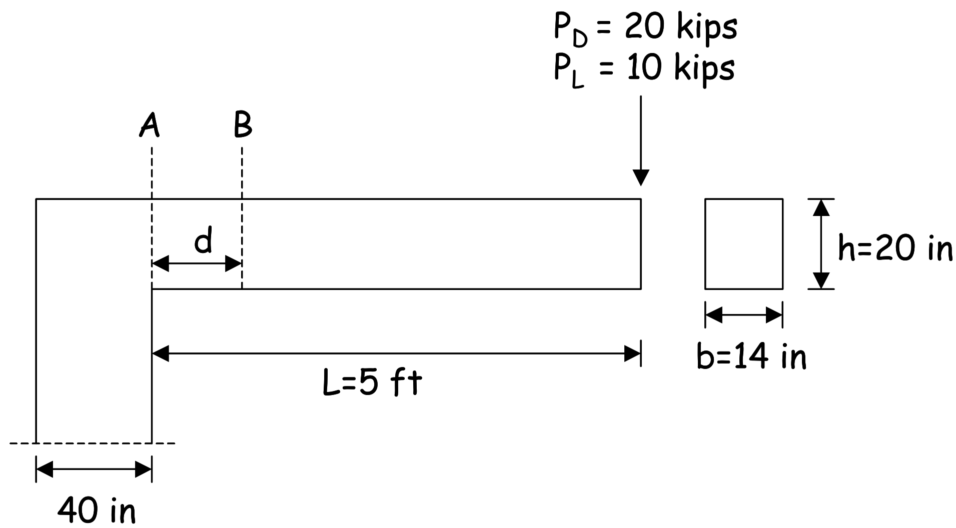
1.051 Structural Engineering Design

QUIZ 1 SOLUTIONS

QUESTION 1. – FLEXURAL DESIGN (40%)



d=17.5 in.
$f_c' = 4$ ksi, $f_y = 60$ ksi

(a) Distance to cracks from free end under service loading, P_s

$$P_S = P_D + P_L = 30 \text{ kips}$$

Cracking occurs when the maximum tensile stress in concrete reaches the modulus of rupture f_r of concrete

$$f_{ct}^{max} = f_r \qquad \text{(tensile stress calculated from elastic theory)}$$

ignoring the steel reinforcement

$$f_{ct} = \frac{My}{I}, \quad f_{ct}^{max} = \frac{M\frac{h}{2}}{I} = \frac{M}{S}, \quad S = \frac{bh^2}{6}$$

$$f_{ct}^{max} = \frac{6M}{bh^2} \quad \text{and} \quad f_r = 7.5\sqrt{f_c'}$$

$$\frac{6M}{bh^2} = 7.5\sqrt{f_c'} \text{ ® } M_{cr} = \frac{7.5}{6}bh^2\sqrt{f_c'}$$

$$M_{cr} = \frac{7.5}{6}(14)(20^2)\sqrt{4000} = 442{,}718 \text{ lb-in} = 36.9 \text{ kips-ft}$$

Moment in the beam at distance x from the free end is given by:

$$M = 30x$$

Distance to the beginning of cracks is given by:

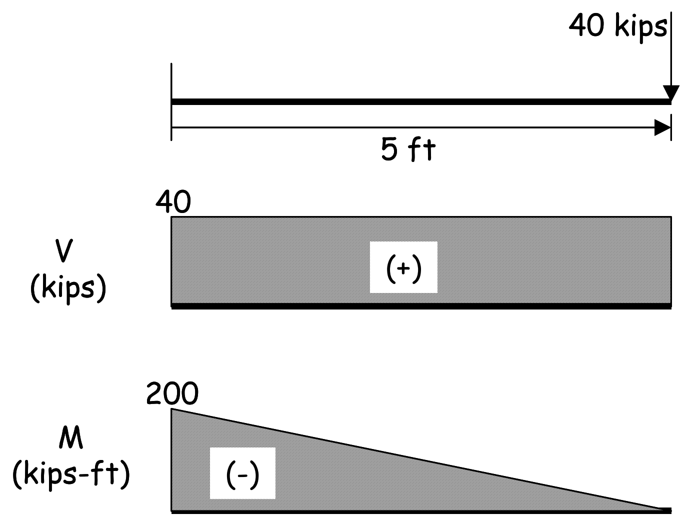
$$M = M_{cr}$$

$$30x = 36.9$$

$$\Rightarrow \; x = 1.23 \text{ ft} = 14.8 \text{ in}$$

(b) Design load, shear and moment diagrams

$$P_u = 1.2P_D + 1.6P_L = 1.2(20) + 1.6(10) = 40 \text{ kips}$$
$$V_u(x) = 40 \text{ kips}$$
$$M_u(x) = -40x$$



(c) Minimum and maximum reinforcement ratios

minimum reinforcement ratio:

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$
$$= \frac{3\sqrt{4000}}{60000} \geq \frac{200}{60000}$$
$$= 0.0032 \geq 0.0033$$

$$\boxed{\rho_{min} = 0.0033}$$

maximum reinforcement ratio

$$r_{max} = 0.85 b_1 \frac{f_c^{'}}{f_y} \frac{e_u}{e_u + 0.004} = (0.85^2)\frac{4}{60}\frac{0.003}{0.007}$$

$$\boxed{r_{max} = 0.0206}$$

alternatively

$$r_b = 0.85 b_1 \frac{f_c^{'}}{f_y} \frac{e_u}{e_u + e_y} = (0.85^2)\frac{4}{60}\frac{0.003}{0.003 + \frac{60}{29000}} = 0.0285$$

$$r_{max} = 0.75 r_b = 0.75(0.0285) = 0.0213 \text{ (close)}$$

Limits on the reinforcement ratio is imposed to ensure ductile behavior and failure of the beam

$$r_{min} \text{ £ } r \text{ £ } r_{max}$$

(d) Reinforcement ratio, r, and area, A_s, ($f = 0.9$). Check for underreinforced beam.

Expression for nominal moment capacity:

$$M_n = \frac{M_u}{f} = r b d^2 f_y (1 - 0.59 r \frac{f_y}{f_c^{'}})$$

$$= \frac{200(12 \text{ in/ft})}{0.9} = r(14)(17.5)^2(60)(1 - 0.59 r \frac{60}{4})$$

Þ $8.85 r^2 - r + 0.01037 = 0$

Þ $\boxed{r = 0.0115}$

$r_{min} < r < r_{max}$ OK

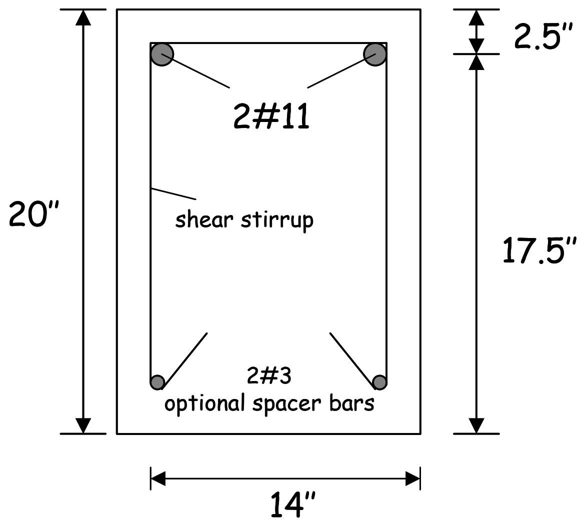
Reinforcement area:

$$A_s = r b d = 0.0115(14)(17.5) = 2.82 \text{ in}^2$$

(e) Reinforcement design using #11 rebars and design sketch.

$A_s = 2.82 \text{ in}^2$

Use 2#11, $A_s = 2(1.56) = 3.12 \text{ in}^2 > 2.82$ OK



(f) Net tensile strain, e_t

$A_s = 3.12 \text{ in}^2$ Þ $r = 0.0127$

Net tensile strain:

$$r = 0.85 b_1 \frac{f_c^{'}}{f_y} \frac{e_u}{e_u + e_t}$$

$$e_t = 0.85 b_1 \frac{f_c^{'}}{f_y} \frac{e_u}{r} - e_u$$

$$= 0.85^2 \frac{4}{60} \frac{0.003}{0.0127} - 0.003$$

Þ $\boxed{e_t = 0.0084}$ $\underline{e_t > 0.005 \text{ Þ } f = 0.9}$

or alternatively

$$c = \frac{A_s f_y}{0.85 b_1 f_c^{'} b}, \qquad e_t = e_u \frac{d - c}{c}$$

$$c = \frac{(3.12)(60)}{0.85^2(4)(14)} = 4.63 \text{ in}, \quad e_t = 0.003 \frac{17.5 - 4.63}{4.63} = 0.0083$$

QUESTION 2 – SHEAR DESIGN (20%)

(a) *Vc* using the simple expression, significance of critical shear section location.

$$
\begin{aligned}
V_c &= 2\sqrt{f_c'}\,bd \\
&= 2\sqrt{4000}(14)(17.5) \\
&= 30{,}990 \text{ lbs} = 30.1 \text{ kips}
\end{aligned}
$$

Since both V_u and V_c are constant along the beam span, location of the critical section bears no significance. Note that if we had used the more accurate equation for V_c, location of the shear section would bear some significance since V_c would vary along the beam section.

(b) *Vs* using #3 stirrups.

$$A_v = 2(0.11) = 0.22 \text{ in}^2$$

$$V_n = \frac{V_u}{\phi} = V_c + V_s \quad \Rightarrow \quad V_s = \frac{V_u}{\phi} - V_c = \frac{40}{0.85} - 30.1 = 17 \text{ kips}$$

$$V_s = A_v f_y \frac{d}{s} \quad \Rightarrow \quad s = \frac{A_v f_y d}{V_s} = \frac{0.22(60)(17.5)}{17} = 13.6 \text{ in}$$

$$
\begin{aligned}
s_{max} &= \frac{A_v f_y}{0.75\sqrt{f_c'}\,b_w} \le \frac{A_v f_y}{50 b_w} \\
&= \frac{0.22(60000)}{0.75(\sqrt{4000})(14)} \le \frac{0.22(60000)}{50(14)} \\
&= 19.9 \text{ in} \le 18.9 \text{ in}
\end{aligned}
$$

$$\underline{s_{max} = 18.9 \text{ in}}$$

$$\underline{s_{max} = \frac{d}{2} = \frac{17.5}{2} = 8.75 \text{ in}}$$

$$\underline{s_{max} = 24 \text{ in}}$$

$$s_{max} = 8.75 \text{ in controls}$$

Use #3 stirrups at 8 in spacing

QUESTION 3 – BOND AND ANCHORAGE (25%)

(a) Location of the critical section for bond and anchorage, necessary development length

Critical section for bond & anchorage is Section A

$$l_d = \left(\frac{f_y \alpha \beta \lambda}{20\sqrt{f_c'}} \right) d_b$$

$\alpha = 1.3$ top reinforcement at 17.5″ distance from bottom

$\beta = \lambda = 1$ uncoated reinforcement in normal weight concrete

$$l_d = \frac{60000(1.3)}{20\sqrt{4000}} 1.41 = 61.7 \text{ in}$$

$$\frac{A_{sr}}{A_{sp}} = \frac{2.82}{3.12} = 0.9$$

$$l_d = 0.9(61.7) = 55.5 \text{ in}$$

The necessary development length can be provided within the beam span but not in the column.

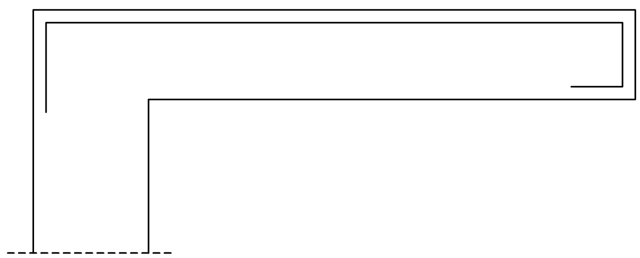
(b) Possible design modifications to meet the bond and anchorage requirements.

The necessary development length can be reduced through following design modifications:

- Providing hooks at the end of bars
- Using smaller diameter bars
- Increasing concrete strength
- Providing additional confinement through increased transverse reinforcement
- Increasing concrete cover thickness

(c) Considerations regarding bond and development length in case of an additional concentrated moment applied at the free end.

In case of a concentrated moment applied at the free end of the beam, proper anchorage of the reinforcement at section must also be ensured. This can be done by providing hooks at the beam-ends as shown below. Also, providing additional transverse reinforcement in this region promotes better confinement and improved anchorage.



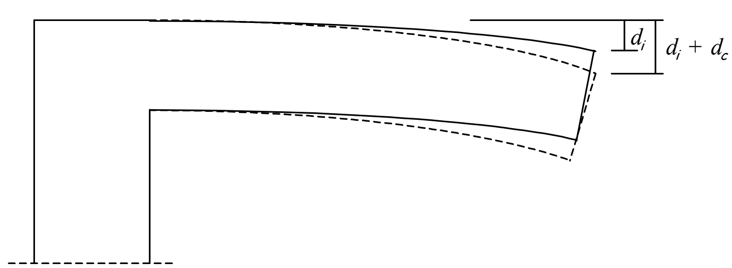
QUESTION 4 – SERVICEABILITY AND STRENGTH DESIGN (15%)

(a) Design for serviceability?

Structures must be designed for *controlled deflections* under service loading. Excessive deflections under service loading or long-term deflections due to sustained loads may cause *damage*. Besides being *visually unpleasant*, wide tension cracks in flexural members may adversely affect the *durability* of the member. Additional considerations such as *vibration* or *fatigue* also require serviceability design for controlled deflections.

(b) Effects of creep in concrete.

Due to creeping of the concrete, the initial deflection in the beam is likely to increase in time as shown in the figure below



Creeping of concrete is likely to increase strains in concrete, decreasing the neutral axis depth and increasing the steel stress. The strain profile at section A is shown in the figure below, solid lines showing the initial strains and the dashed lines showing the strains after the creep of concrete.