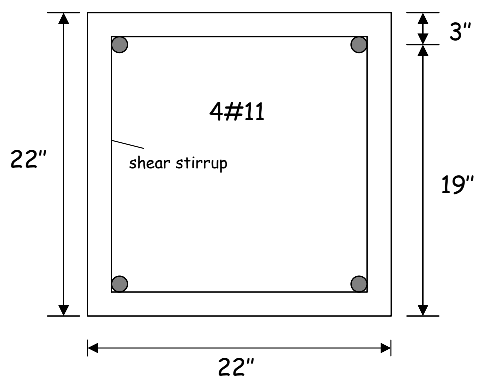
1.051 Structural Engineering Design

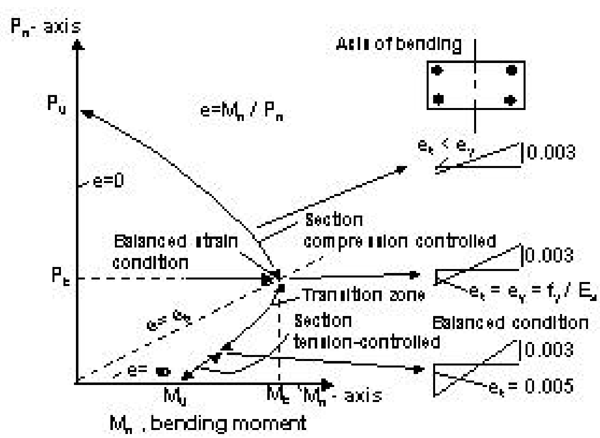
QUIZ 2 SOLUTIONS

QUESTION 1. – REINFORCED CONCRETE SHORT COLUMNS (30%)

$f_c^{'} = 4$ ksi, $f_y = 60$ ksi, $E_s = 29000$ ksi



(a) Interaction diagram. (8%)



(b) Calculate P_o (5%)

$$\begin{aligned} P_0 &= 0.85 f_c^{'} A_c + A_{st} f_y \\ &= 0.85(4)(22^2) + 4(1.56)(60) \\ &= 2020 \text{ kips-in} = 168.3 \text{ kips-ft} \end{aligned}$$

(c) Calculate P_b and Mb, and e_b. (12%)

$$\frac{e_u}{c} = \frac{e_u + e_y}{d} \Rightarrow c = d\frac{e_u}{e_u + e_y} = 19\frac{0.003}{0.003 + \frac{60}{29000}} = 11.2 \text{ in}$$

$$f_s^{'} = Ee_s^{'} = 29000e_u\frac{c - d^{'}}{c} = 29000(0.003)\frac{11.2 - 3}{11.2} = 63.7 > 60$$

$$f_s^{'} = f_y = 60 \text{ ksi}$$

$$P_b = C + T' - T = 0.85f_c^{'}ab + A_s^{'}f_y - A_sf_y$$

$$P_b = C = 0.85^2(4)(11.2)(22) = 712.1 \text{ kips}$$

$$M_b = C(\frac{h}{2} - \frac{a}{2}) + T^{'}(\frac{h}{2} - d^{'}) + T(\frac{h}{2} - d^{'})$$

$$= 712.1(11 - \frac{0.85(11.2)}{2}) + 4(1.56)(60)(11 - 3)$$

$$= 7439 \text{ kips-in} = 620 \text{ kips-ft}$$

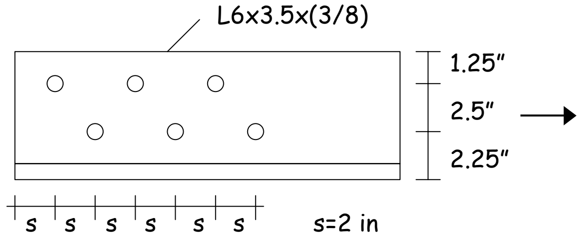
$$e_b = \frac{M_b}{P_b} = \frac{7439}{712.1} = 10.4 \text{ in}$$

(d) Calculate Mn for P_n=(P_o+P_b)/2. (5%)

Assume linear variation between P_b and P_o

$$M_n = \frac{M_b}{2} = \frac{4493}{2} = 2246 \text{ kips-in} = 187.2 \text{ kips-ft}$$

QUESTION 2 – TENSION MEMBERS (20%)



A36 steel (fy=36 ksi, fu=58 ksi)
Bolts are 1 in diameter
Assume U=0.9

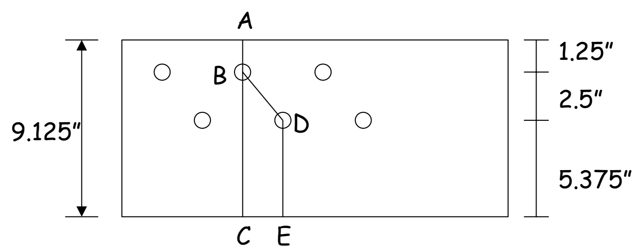
(a) Compute the design strength $\phi_t P_n$, neglecting block shear.

Yielding of the gross-section:

For L6x3.5x3/8, $A_g = 3.42\text{ in}^2$

$\phi_t P_n = 0.9 F_y A_g = 0.9(36)(3.42) = 110.8\text{ kips}$

Fracture of the net-section



$$d_e = 1 + \frac{1}{16} + \frac{1}{16} = 1.125\text{ in}$$

$$ABC = 9.125 - 1.125 = 8\text{ in}$$

$$ABDE = 9.125 - 2(1.125) + \frac{2^2}{4(2.5)} = 7.275\text{ in (controls)}$$

$$A_n = 7.275(3/8) = 2.73\text{ in}^2$$

$$A_e = UA_n = 0.9(2.73) = 2.46\text{ in}^2$$

$$\phi_t P_n = 0.75 A_e F_u = 0.75(2.46)(58) = 107\text{ kips}$$

Fracture of the net section controls : $\phi_t P_n = 107\text{ kips}$

BONUS (10%)

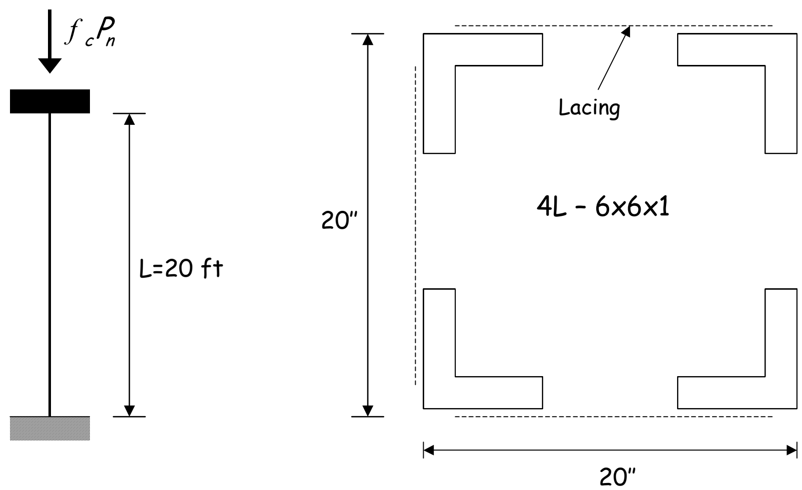
(b) Calculate s to maximize the fracture design strength of the connection.

$$ABC = ABDE \Rightarrow 8 = 9.125 - 2(1.125) + \frac{s^2}{4(2.5)}$$

$$\Rightarrow s = 3.4\text{ in}$$

QUESTION 3 – COMPRESSION MEMBERS (25%)

Determine the design strength, $\phi_c P_n$, for the compression member.
Fy=50 ksi



For L6x6x1: $A = 11\ \text{in}^2$, $\bar{x} = \bar{y} = 1.86\ \text{in}$, $I_x = I_y = I = 35.5\ \text{in}^4$, $r_x = r_y = 1.8\ \text{in}$

For the built-up section

$$A^b = 4(11) = 44\ \text{in}^2$$

$$I_x^b = I_y^b = I^b = 4\left[I + A\left(\frac{20}{2} - \bar{x}\right)^2\right] = 4\left[35.5 + 11(10 - 1.86)^2\right] = 3057\ \text{in}^4$$

$$r_x^b = r_y^b = r^b = \sqrt{\frac{I^b}{A^b}} = \sqrt{\frac{3057}{44}} = 8.34\ \text{in}$$

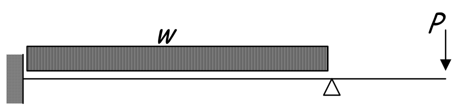
$K = 1.2$ (Ideally, $K = 1$)

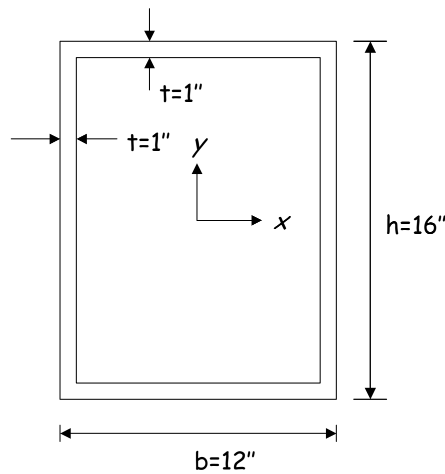
$$\lambda_c = \frac{KL}{r^b \pi}\sqrt{\frac{F_y}{E}} = \frac{1.2(20 \times 12)}{8.34\pi}\sqrt{\frac{50}{29000}} = 0.46 < 1.5$$

$$F_{cr} = (0.658^{\lambda^2})F_y = (0.658^{0.46^2})(50) = 45.8\ \text{ksi}$$

$$\phi_c P_n = \phi_c A^b F_{cr} = 0.85(44)(45.8) = 1713\ \text{kips}$$

QUESTION 4 – STEEL BEAMS (25%)





(a) Calculate the section modulus, S, the plastic modulus, Z, and the shape factor, X. (20%)

$$I = \frac{1}{12}\left(bh^3 - (b-2t)(h-2t)^3\right) = \frac{1}{12}\left((12)(16)^3 - (10)(14)^3\right) = 1809 \text{ in}^4$$

$$S = \frac{I}{h/2} = \frac{1809}{8} = 226 \text{ in}^3$$

$$Z = 2\left(b\frac{h}{2}\frac{h}{4} - (b-2t)(\frac{h}{2}-t)(\frac{h}{4}-\frac{t}{2})\right) = 2\left((12)(8)(4) - (10)(7)(3.5)\right) = 278 \text{ in}^3$$

$$X = \frac{Z}{S} = \frac{278}{226} = 1.23$$

(b) For Fy=50 ksi, calculate the yield moment, M_y, and plastic moment, M_p (5%)

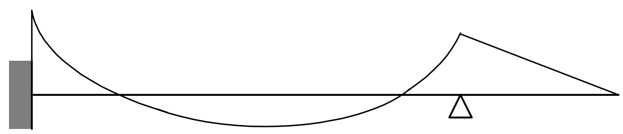
$$M_y = SF_y = 226(50) = 11300 \text{ kips-in} = 942 \text{ kips-ft}$$

$$M_p = ZF_y = 278(50) = 13900 \text{ kips-in} = 1158 \text{ kips-ft}$$

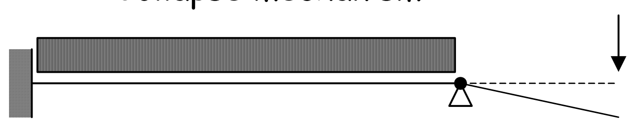
BONUS (10%)

(c) Sketch two alternative collapse mechanisms for the beam, showing the locations of the plastic hinges.

Typical moment diagram



Collapse mechanism 1



Collapse mechanism 2