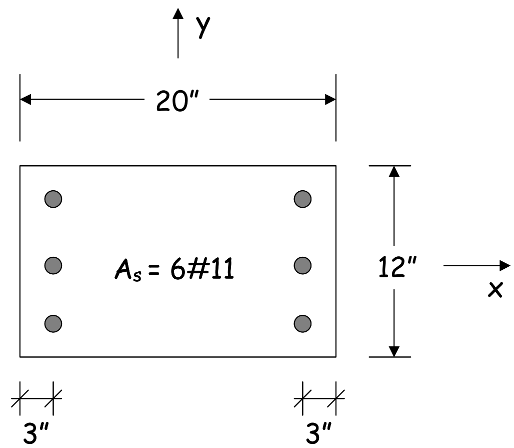
1.051 Structural Engineering Design

Problem Set 4 Solutions

Design of Short Columns

1. Problem 8.3 in the textbook.
Given: Bending about y-axis
$b = 12$ in
$h = 20$ in
$d = 17$ in
$A_{st} = 6\#11 = 6(1.56) = 9.36\ \text{in}^2$
$A_s = A_s' = 3(1.56) = 4.68\ \text{in}^2$
$f_y = 60$ ksi
$f'_c = 4$ ksi



(a) Nominal strength interaction curve

Axial compression ($c=\infty$, $e=0$)

$$P_0 = 0.85 f'_c A_c + A_{st} f_y = 0.85(4)(12)(20) + (9.36)(60) = 1378 \text{ kips}$$

Balanced condition

$$\frac{\varepsilon_u}{c_b} = \frac{\varepsilon_u + \varepsilon_y}{d} \Rightarrow c_b = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d = \frac{0.003}{0.003 + \frac{60}{29000}}(17) = 10.1 \text{ in}$$

$$a_b = \beta_1 c = (0.85)(10.1) = 8.6 \text{ in}$$

$$\frac{\varepsilon_u}{c} = \frac{\varepsilon'_s}{c - d'} \Rightarrow \varepsilon'_s = \varepsilon_u \frac{c - d'}{c}$$

$$f'_s = E\varepsilon'_s = E\varepsilon_u \frac{c - d'}{c} = (29000)(0.003)\frac{10.1 - 3}{10.1} = 61.2 \text{ kips} > f_y$$

$$f'_s = f_y = 60 \text{ ksi}$$

$$C = 0.85 f'_c a_b b = (0.85)(4)(8.6)(12) = 351 \text{ kips}$$

$$P_b = C + A'_s f'_s - A_s f_s = C + f_y(A'_s - A_s) = C = 351 \text{ kips}$$

$$M_b = C\left(\frac{h}{2} - \frac{a}{2}\right) + A'_s f'_s\left(\frac{h}{2} - d'\right) + A_s f_y\left(d - \frac{h}{2}\right)$$

$$= 351\left(10 - \frac{8.6}{2}\right) + (4.68)(60)(10 - 3) + (4.68)(60)(17 - 10)$$

$$= 5932 \text{ kips-in} = 494.3 \text{ kips-ft}$$

Pure bending ($e=\infty$, $P_n=0$)

$$\rho = \rho' = \frac{4.68}{(12)(17)} = 0.023$$

Ignoring compression steel (see section 3.7.a in textbook)

$$c = \frac{A_s f_y}{0.85\beta_1 f_c' b} = \frac{(4.68)(60)}{0.85^2(4)(12)} = 8.1 \text{ in}$$

$$M_n = A_s f_y \left(d - \frac{\beta_1 c}{2}\right) = \rho b d^2 f_y \left(1 - 0.59\rho \frac{f_y}{f_c'}\right)$$

$$= (0.023)(12)(17)^2(60)\left(1 - (0.59)(0.023)\frac{60}{4}\right)$$

$$= 3812 \text{ kips-in} = 318 \text{ kips-ft}$$

Find P_n and M_n for two more points, one for tension and one for compression failure.

For c=5 in

$$a = (0.85)(5) = 4.25 \text{ in}$$

$$f_s = f_y \qquad f_s' = E\varepsilon_u \frac{c - d'}{c} = (29000)(0.003)\frac{5-3}{5} = 34.8 \text{ ksi} < 60 \text{ ksi OK}$$

$$C = 0.85 f_c' ab = (0.85)(4)(4.25)(12) = 173.4 \text{ kips}$$

$$P_n = C + f_s A_s' - f_y A_s = 173.4 + (34.8)(4.68) - (60)(4.68) = 55.5 \text{ kips}$$

$$M_b = C\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_y \left(d - \frac{h}{2}\right)$$

$$= (173.4)\left(10 - \frac{4.25}{2}\right) + (4.68)(34.8)(10-3) + (4.68)(60)(17-10)$$

$$= 4471 \text{ kips-in} = 372.6 \text{ kips-ft}$$

For c=15 in

$$a = (0.85)(15) = 12.75 \text{ in}$$

$$f_s = 29000\varepsilon_u \frac{d-c}{c} = (29000)(0.003)\frac{17-15}{15} = 11.6 \text{ ksi}$$

$$f_s' = E\varepsilon_u \frac{c-d'}{c} = (29000)(0.003)\frac{15-3}{15} = 69.6 \text{ ksi} > 60 \text{ ksi} \Rightarrow f_s' = f_y$$

$$C = 0.85 f_c' ab = (0.85)(4)(12.75)(12) = 520.2 \text{ kips}$$

$$P_n = C + f_y A_s' - f_s A_s = 520.2 + (4.68)(60) - (4.68)(11.6) = 747 \text{ kips}$$

$$M_n = C(\frac{h}{2} - \frac{a}{2}) + A_s' f_s' (\frac{h}{2} - d') + A_s f_y (d - \frac{h}{2})$$
$$= (520.2)(10 - \frac{12.75}{2}) + (4.68)(60)(10 - 3) + (4.68)(11.6)(17 - 10)$$
$$= 4231 \text{ kips-in} = 353 \text{ kips-ft}$$

Summary of points on the nominal strength interaction curve

Loading case	c (in)	e (in)	P_n (kips)	M_n (kips-ft)
Pure axial	∞	0	1378	0
Comp. failure	15	5.7	747	353
Balanced	10.1	16.9	351	494
Tens. failure	5	80.6	56	373
Pure bending	8.1	∞	0	318

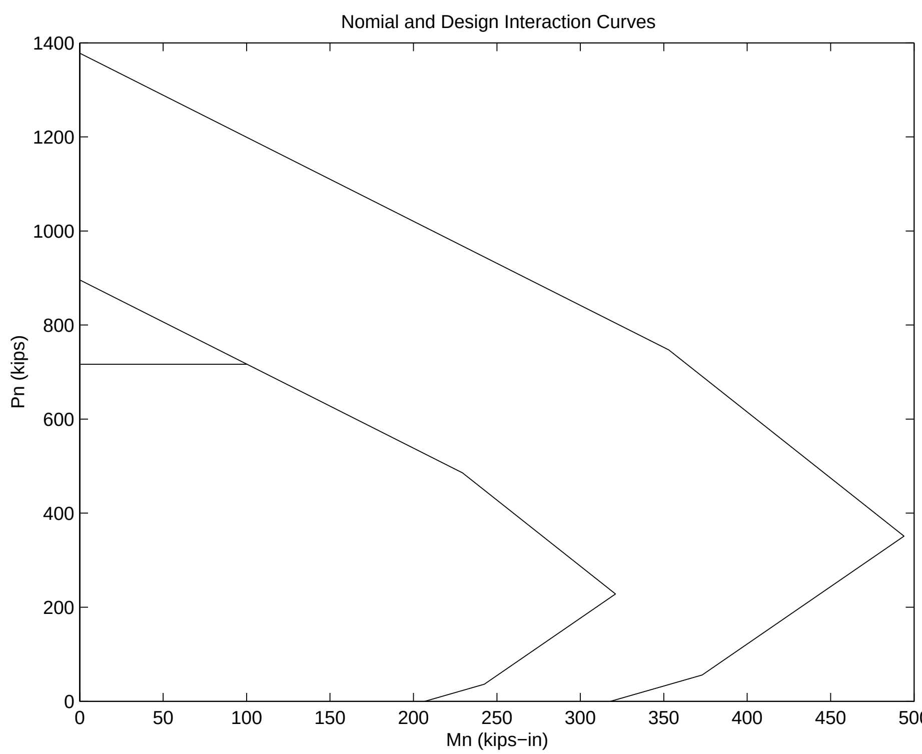


Figure 1. Nominal and Design strength interaction curves

(b)

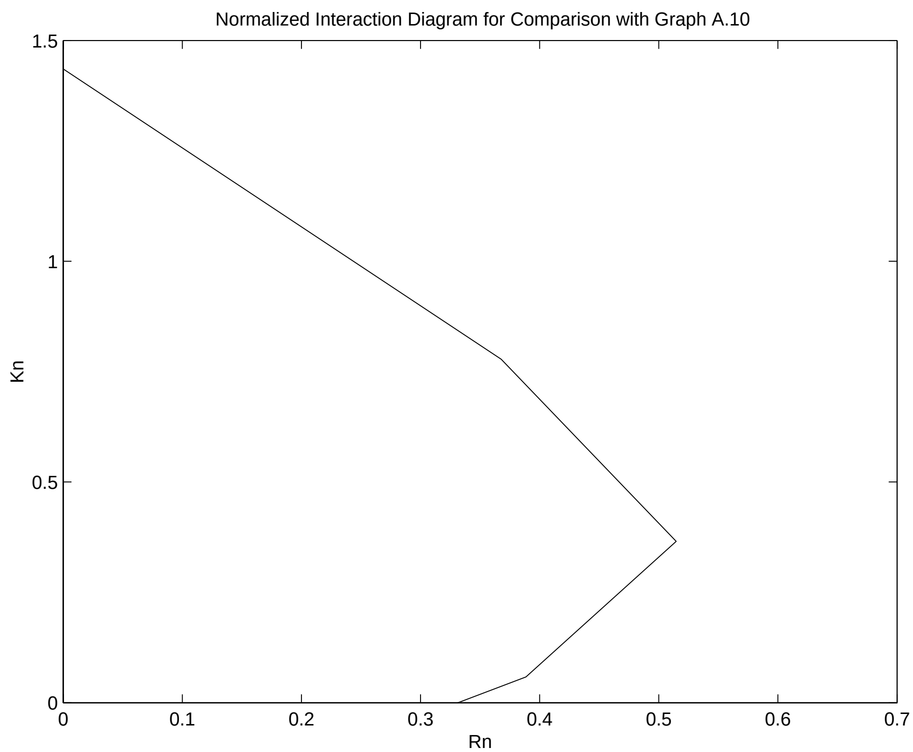


Figure 2. Normalized interaction curve for comparison with Graph A.10

The curve shown in Figure 2 is similar to that given for

$r_g = \frac{A_{st}}{bh} = \frac{9.36}{(12)(20)} = 0.039$ in Graph A.10

(c) The design strength curve is shown in Figure 1.

(d) Shear reinforcement

In accordance with Section 8.2 in the textbook, use #4 bars for lateral ties, at a spacing of:

$$s = \min(16d_b, 48d_v, b) = \min((16)(1.41), (48)(0.5), 12) = \min(22.6, 24, 12)$$

$$s = 12 \text{ in}$$

Design of Slabs

2. Problem 13.1 in the textbook

Given:

One way slab, masonry abutments (simply supported)

L = 16 ft

W = 6 ft

w_L = 100 psf = 100x6 = 600 lb/ft

P_L = 2000 lb

w_{D1} = 20x6+2x4x4x150/144 = 120+33=153 lb/ft

f_y = 60 ksi

f'_c = 4 ksi

Determine the slab thickness: For simply supported one way slabs, the minimum thickness according to ACI-389 is given by:

$$h_{min} = \frac{L}{20} = \frac{(16)(12)}{20} = 9.6 \text{ in} \Rightarrow \text{ take } h = 10 \text{ in}, \; d = 9 \text{ in}$$

Self-weight of the slab: $w_{D2} = (6)(10/12)(150) = 750$ lb/ft

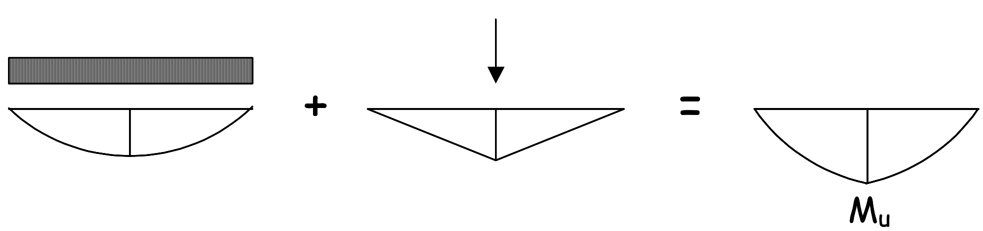
Total dead load: $w_D = w_{D1} + w_{D2} = 153 + 750 = 903$ lb/ft

Design Loads:

$w_u = 1.2w_D + 1.6w_L = (1.2)(903) + (1.6)(600) = 2044$ lb/ft = 2.04 kips/ft

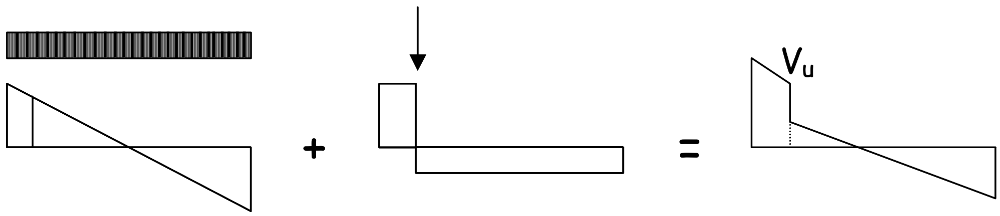
$P_u = 1.6P_L = (1.6)(2000) = 3200$ lb = 3.2 kips

Design moment (P_u placed at mid-span for maximum moment)



$$M_u = \frac{w_u L^2}{8} + \frac{P_u L}{4} = \frac{(2.04)(16)^2}{8} + \frac{(3.2)(16)}{4} = 78.1 \text{ kips-ft} = 937 \text{ kips-in}$$

Design shear (P_u placed at the critical shear section, i.e. d away from the support for maximum shear)



$$V_u = \frac{w_D L}{2}\left(\frac{\frac{L}{2} - d}{\frac{L}{2}}\right) + P_u\left(1 - \frac{d}{L}\right) = \frac{(2.04)(16)}{2}\left(\frac{8 - (9/12)}{8}\right) + 3.2\left(1 - \frac{9/12}{16}\right) = 17.8 \text{ kips}$$

Flexural reinforcement (One can either assume a and make iterations, or solve for ρ from the moment equation):

Assume a=1 in (similar to the example in the textbook)

$$M_u = \phi M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \Rightarrow \quad A_s = \frac{M_u}{\phi f_y (d - a/2)}$$

$$A_s = \frac{937}{(0.9)(60)(9 - 1/2)} = 2.04 \text{ in}^2 \quad \text{(Note that b=6 ft)}$$

Calculate a,

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(2.04)(60)}{(0.85)(4)(72)} = 0.5 \text{ in}$$

Recalculate As,

$$A_s = \frac{937}{(0.9)(60)(9 - 0.5/2)} = 1.98 \text{ in}^2 \approx 2 \text{ in}^2$$

Use #6 rebars, A_b=0.44 in^2

$$s = \frac{A_b}{A_s} b = \frac{0.44}{2}(6)(12) = 15.8 \text{ in} \quad \Rightarrow \quad s = 15 \text{ in}$$

Shear reinforcement:

$$\phi V_c = \phi 2\sqrt{f_c'}\, b_w d = (0.85)(2)\sqrt{4000}(72)(9) = 69.7 \text{ kips}$$

$$(V_u = 17.8 \text{ kips}) < \left(\frac{\phi V_c}{2} = 34.8 \text{ kips}\right) \quad \Rightarrow \quad \text{No shear reinforcement needed}$$

Temperature and shrinkage reinforcement in the transverse direction:

$\min \rho_{t\&s} = 0.0018$ (Table 13.2)

$A_s = 0.0018bd = 0.0018(12)(9) = 0.19$ in^2/ft

Use #5@15 in spacing, $A_s = 0.31/(15/12) = 0.25$ in^2/ft OK.