1.051 Structural Engineering Design

Problem Set 3 Solutions

Shear in beams

1. Given

w_D = 1200 lb/ft (including own weight)
w_L = 900 lb/ft.
b_w = 12 in.
d = 17 in.
h = 20 in.
$A_s = 6.0\ \text{in}^2$
f'_c = 4000 psi
f_y = 60,000 psi

Maximum factored vertical shear V_u at the critical section

$$w_u = 1.2w_D + 1.6w_L = 1.2(1200) + 1.6(900) = 2880 \text{ lb/ft}$$

$$V_{max} = \frac{w_u L}{2} = \frac{2880(22)}{2} = 31680 \text{ lb} = 31.7 \text{ kips}$$

$$V_u = V_{max}\left(\frac{L-d}{L}\right) = 31.7\left(\frac{22-17/12}{22}\right) = 29.7 \text{ kips} \quad (d \text{ away from the supports})$$

$$M_u = \frac{w_u L^2}{8} = \frac{2.8(22)^2}{8} = 169.4 \text{ kips-ft}$$

Design the size and spacing of the shear reinforcement.

(a) V_c given by Eq. 4.12b in the textbook

$$V_c = 2\sqrt{f'_c}\,b_w d = 2\sqrt{4000}(12)(17) = 25804 \text{ lb} = 25.8 \text{ kips}$$

$$\phi V_c = 0.85(25.8) = 21.9 \text{ kips} < V_u \quad \text{Shear reinforcement needed}$$

$$V_u \le \phi(V_c + V_s) \Rightarrow V_s = \frac{V_u - \phi V_c}{\phi} = \frac{29.7-21.9}{0.85} = 9.2 \text{ kips}$$

Use #3 reinforcement, $A_v = 2(0.11) = 0.22\ \text{in}^2$

$$V_s = \frac{A_v f_y d}{s} \Rightarrow s = \frac{A_v f_y d}{V_s} = \frac{(0.22)(60)(17)}{9.2} = 24.4 \text{ in}$$

Check s_{max}

$$
\begin{aligned}
s_{max} &= \frac{A_v f_y}{0.75\sqrt{f_c'}\, b_w} \le \frac{A_v f_y}{50 b_w} \\
&= \frac{(0.22)(60000)}{0.75\sqrt{4000}(12)} \le \frac{(0.22)(60000)}{(50)(12)} \\
&= 23.2 \text{ in} \le 22 \text{ in} \\
&= 22 \text{ in}
\end{aligned}
$$

$$s_{max} = \frac{d}{2} = \frac{17}{2} = 8.5 \text{ in}$$

$$s_{max} = \min\{22, 8.5, 24\} = 8.5 \text{ in}$$

$$s = 24.4 \text{ in} < s_{max} = 8.5 \text{ in}$$

$$\Rightarrow \; s = 8.5 \text{ in}$$

(b) Using the more accurate value of V_c given by Eq. 4.12a in the textbook

Parameters needed to use this equation:

$$M_u = \frac{d}{L/2} M_{max} = \frac{17}{(11)(12)} 169.4 = 21.8 \text{ kips-ft}$$

$$\rho_w = \rho = \frac{A_s}{b_w d} = \frac{6}{(12)(17)} = 0.029$$

$$\frac{V_u d}{M_u} = \frac{(29.7)(17)}{(21.8)(12)} = 1.93 > 1 \;\Rightarrow\; \frac{V_u d}{M_u} = 1$$

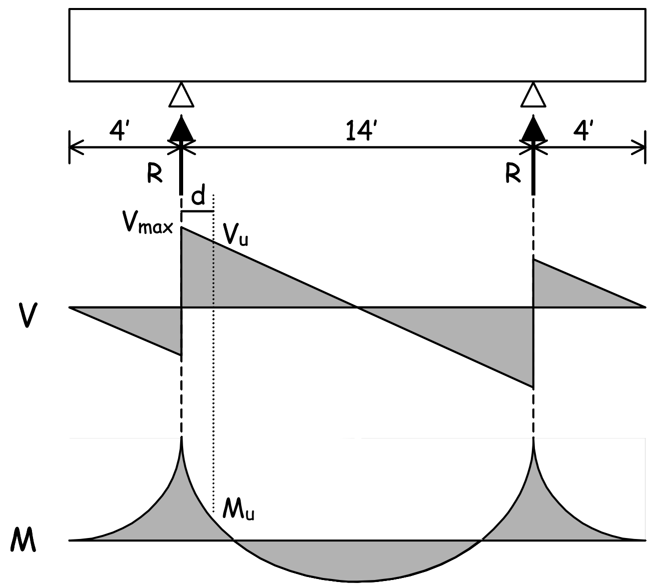
$$
\begin{aligned}
V_c &= \left(1.9\sqrt{f_c'} + 2500\frac{\rho_w V_u d}{M_u}\right) b_w d \le 3.5\sqrt{f_c'}\, b_w d \\
&= \left((1.9)(\sqrt{4000} + (2500)(0.0294)(1)\right)(12)(17) \le 3.5\sqrt{4000}(12)(17) \\
&= 39.5 \text{ kips} \le 45.2 \text{ kips} \\
&= 39.5 \text{ kips}
\end{aligned}
$$

$$\phi V_c = 0.85(39.5) = 33.6 \text{ kips} \qquad \phi V_c / 2 = 16.8 \text{ kips}$$

$$\phi V_c / 2 < V_u = 29.7 \text{ kips} < \phi V_c \;\Rightarrow\; \text{need minimum shear reinforcement}$$

The minimum shear reinforcement size is #3, and obviously $s_{max} = d/2 = 8.5$ in will control. Thus, the shear reinforcement configuration is same as part (a), i.e. #3@8.5 in. However, note the difference in the calculated shear capacity V_c.

2. Modified support conditions,



Support reactions: $R_1 = R_2 = \dfrac{w_u L}{2} = \dfrac{(2.88)(22)}{2} = 31.7$ kips

$V_{max} = R - 4w_u = 31.7 - (4)(2.88) = 20.2$ kips

The critical shear section is @d distance to the supports towards the center

$V_u = 20.2\dfrac{7 - (17/12)}{7} = 16.1$ kips

$M_u = (31.7)(17/12) - \dfrac{(2.88)(4 + 17/12)^2}{2} = 2.7$ kips-ft

Obviously, $\dfrac{V_u d}{M_u} = 1$

Calculate ϕV_c using the approximate expression

$\phi V_c = \phi 2\sqrt{f_c'}\, b_w d = 21.9$ kips (calculated in problem 1)

$\dfrac{\phi V_c}{2} = 11$ kips $< V_u = 16.1$ kips $< \phi V_c = 21.9$ kips

Use minimum reinforcement as calculated in Problem 1, i.e. #3@8.5 in.

Calculate ϕV_c using the accurate expression

$$\phi V_c = \phi\left(1.9\sqrt{f_c'} + 2500\frac{\rho_w V_u d}{M_u}\right) b_w d \le \phi 3.5\sqrt{f_c'}\, b_w d$$

$= 33.6$ kips (calculated in Problem 1)

(Note the assumption that $A_s' = A_s$, otherwise, V_c would be different.)

$(V_u = 16.1 \text{ kips}) < (\frac{\phi V_c}{2} = 16.8 \text{ kips}) \Rightarrow$ No shear reinforcement needed!

Even so, this is a beam, take accidental loads into consideration and provide minimum shear reinforcement!

Bond and Anchorage

3. *Given:* w_D = 2 kips/ft (including own weight)
w_L = 3 kips/ft.
b_w = 11 in.
d = 21 in.
h = 24 in.
A_s = 2#11 (A_s = 2(1.56) = 3.12 in^2)
Av = #3 (1.5" cover, spacing from column face: 4", 3@8", 5@10.5")

(a) The critical section for bond development is at the column face
f'_c = 3000 psi
f_y = 60,000 psi

Using the simplified equation:
$\alpha = 1.3$ (more than 12" of fresh concrete is cast below the reinforcement)
$\beta = \lambda = 1$ (uncoated reinforcement and normal density concrete)

$$l_d = \left(\frac{f_y \alpha \beta \lambda}{20\sqrt{f'_c}}\right) d_b = \left(\frac{(60000)(1.3)(1)(1)}{20\sqrt{3000}}\right)(1.41) = 100.4 \text{ in} > 96 \text{ in}$$

The available development length is insufficient

(b) Use the basic equation for development length

$\gamma = 1.0$ (#7 and larger bars)

$$c = \min\left\{3, 1.5 + \frac{0.11}{2} + \frac{1.41}{2}\right\} = \min\{3, 2.26\} = 2.26 \text{ in}$$

$$K_{tr} = \frac{A_{tr} f_{yt}}{1500 s n} = \frac{(0.11)(1+3+5)(60000)}{(1500)(10.5)(2)} = 1.89$$

$$\frac{c + K_{tr}}{d_b} = \frac{2.26 + 1.89}{1.41} = 2.94 > 2.5 \Rightarrow \frac{c + K_{tr}}{d_b} = 2.5 \text{ (upper limit)}$$

$$l_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f_c'}} \frac{\alpha\beta\gamma\lambda}{\left(\frac{c + K_{tr}}{d_b} \right)} \right) d_b = \left(\frac{3}{40} \frac{60000}{\sqrt{3000}} \frac{(1.3)(1)(1)(1)}{2.5} \right)(1.41) = 60.2 \text{ in}$$

Note that the ratio of the calculated values development length in (a) and (b) is $\frac{100.4}{60.2} = \frac{2.5}{1.5}$ since the simplified equations use $\frac{(c + K_{tr})}{d_b} = 1.5$.

(c) Column material properties: f'_c = 3000 psi , f_y = 60,000 psi.
There is no doubt that hooks will be necessary, so use expressions for hooked bars

$$l_{dh} = \left(\frac{0.02\beta\lambda f_y}{\sqrt{f_c'}} \right) d_b = \left(\frac{(0.02)(1)(1)(60000)}{\sqrt{5000}} \right)(1.41) = 23.9 \text{ in}$$

Modification factors:

For #11 and smaller bar hooks with side cover not less than 2½ in, and for 90° hooks with cover on bar extension beyond hook not less than 2 in: 0.7 (satisfied)

$l_{dh} = (0.7)(23.9) = 16.7$ in $\Rightarrow$ Adequate embedment length can be provided.

Detailed dimensions (See Fig. 5.10 in textbook):

The embedment length:	17″
Diameter of bend:	$r = 8d_b = (8)(1.41) = 11.3$ in
Length after bend:	$12d_b = (12)(1.41) = 16.9$ in