

1.054/1.541 Mechanics and Design of Concrete Structures (3-0-9)

Outline 8

Biaxial Bending

- R/C columns under biaxial bending
 - The problem of columns under biaxial bending is nonlinear and the number of unknowns is large. For any defined column the problem may be expressed as:

$$(P, e_x, e_y) = f(c, \theta, \varepsilon_c)$$

where f = a nonlinear function of the variables that can be derived from the equilibrium equations and geometry of any given column section and the stress-strain curves of the materials,

P = axial load,

e_x, e_y = eccentricities measured parallel to x and y axes, respectively,

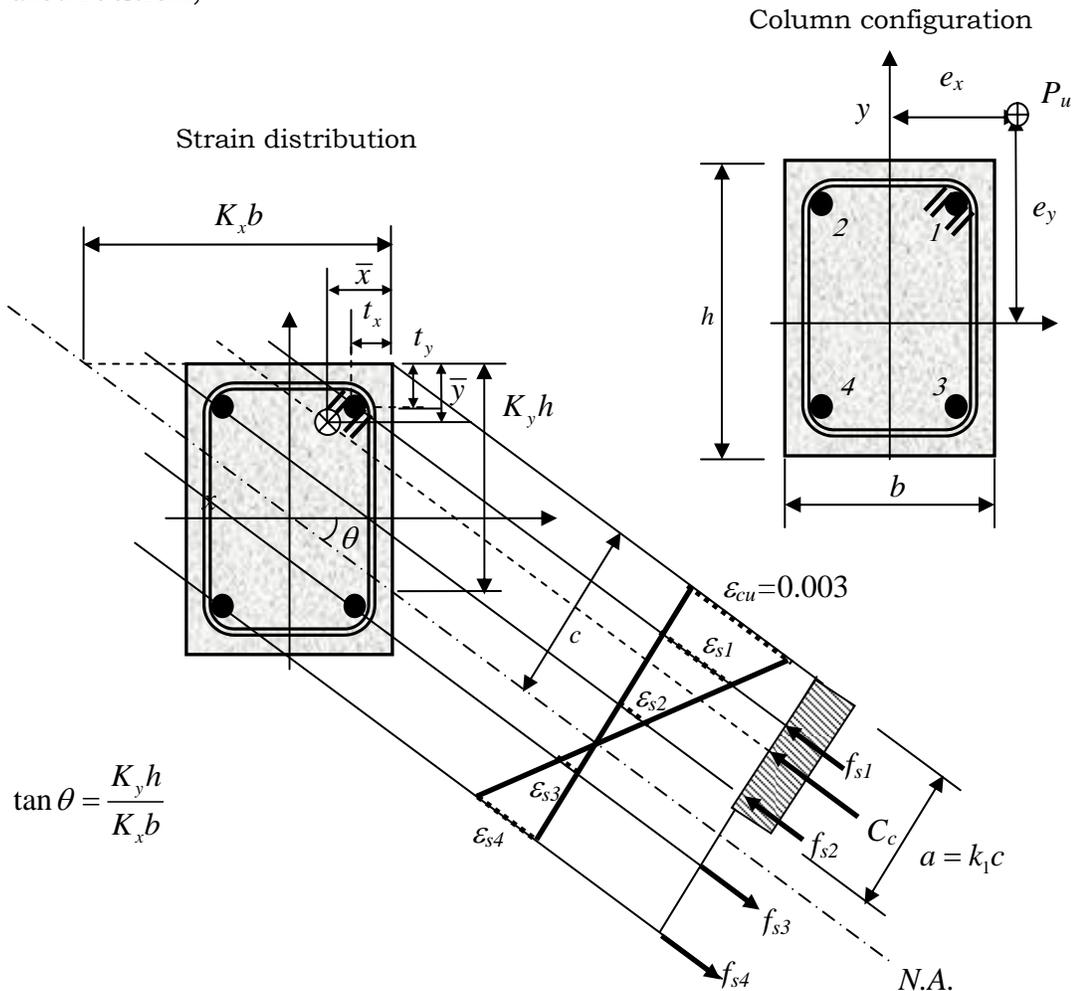
θ = inclination of neutral axis,

c = depth of neutral axis measured from extreme compressive fiber, and

ε_c = failure strain of concrete in compression.

- A number of approximate methods which are based on simplifying assumptions have been developed. However, for certain situations, the simplifying assumptions may lead to inaccurate results, and the use of presently available design charts is often limited.
- The use of computers to solve this problem with improved accuracy has been based upon iterative analysis of trial sections until a satisfactory result is achieved.

- Apart from these considerations, the design process should not be limited to a purely numerical evaluation of the loads, stresses, and strains involved; basic design issues, such as seismic requirements, architectural preferences, availability of material types and sizes, economy, and constructability must be taken into account.
- Basic concept of analysis
- The analysis is based on the strain compatibility and equilibrium equations for the column section.
 - For a given neutral axis position, the strains, stresses, and forces in the steel can be found. The resultant force in the concrete depends on the shape of the stress block.
 - Consider the following configuration of column section and its strain distribution,



Strains in steel bars:

$$\varepsilon_{s1} = 0.003 \left(1 - \frac{t_x}{K_x b} - \frac{t_y}{K_y h} \right)$$

$$\varepsilon_{s2} = 0.003 \left(1 - \frac{b-t_x}{K_x b} - \frac{t_y}{K_y h} \right)$$

$$\varepsilon_{s3} = \frac{0.003}{c} \left\{ [h(1-K_y) - t_y] \cos \theta + t_y \sin \theta \right\}$$

$$\varepsilon_{s4} = \frac{0.003}{c} \left\{ \frac{K_y h}{K_x b} [b(1-K_x) - t_x] + h - t_y \right\} \cos \left[\tan^{-1} \left(\frac{K_y h}{K_x b} \right) \right]$$

- o Determination of the stress of steel bars

For $\varepsilon_{si} < \varepsilon_y$, $f_{si} = E_s \varepsilon_{si}$.

For $\varepsilon_{si} \geq \varepsilon_y$, $f_{si} = f_y$.

- o Equilibrium conditions

Forces in steel bars:

$$S_1 = f_{s1} A_{s1} = (\varepsilon_{s1} E_s) A_{s1}$$

$$S_2 = f_{s2} A_{s2} = (\varepsilon_{s2} E_s) A_{s2}$$

$$S_3 = f_{s3} A_{s3} = (\varepsilon_{s3} E_s) A_{s3}$$

$$S_4 = f_{s4} A_{s4} = (\varepsilon_{s4} E_s) A_{s4}$$

Force equilibrium:

$$\sum F = 0; C_c + S_1 + S_2 - S_3 - S_4 = 0$$

Conditions of moment equilibrium are expressed in x and y directions.

$$M_{ux} = P_u e_y = C_c \left(\frac{h}{2} - \bar{y} \right) + (S_1 + S_2) \left(\frac{h}{2} - t_y \right) + (S_3 + S_4) \left(\frac{h}{2} - t_y \right)$$

$$M_{uy} = P_u e_x = C_c \left(\frac{b}{2} - \bar{x} \right) + S_1 \left(\frac{b}{2} - t_x \right) - S_2 \left(\frac{b}{2} - t_x \right) - S_3 \left(\frac{b}{2} - t_x \right) + S_4 \left(\frac{b}{2} - t_x \right)$$

Analysis and design requires trials and iterations to find the inclination and depth of the neutral axis satisfying the equilibrium equations.

□ Failure interaction curve

- The relationship expressed in

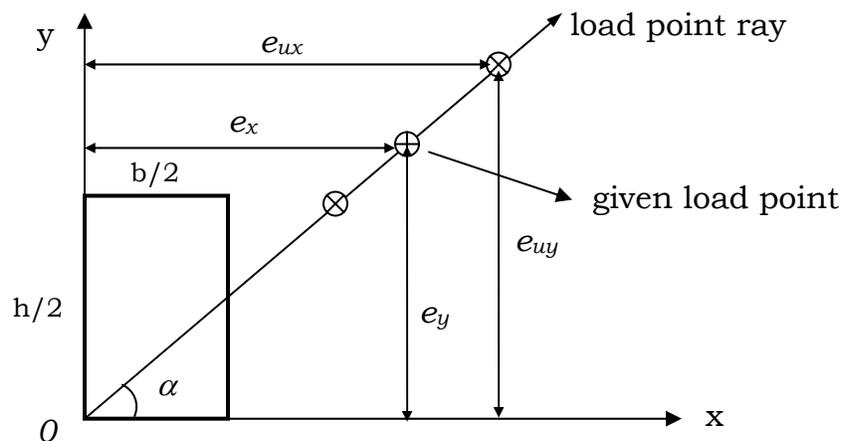
$$(P, e_x, e_y) = f(c, \theta, \varepsilon_c)$$

describes the three dimensional failure surface (failure interaction curve) if the concrete strain is taken to be ε_u (usually 0.003). Any combination of neutral axis depth c and inclination θ will give a unique triplet of P_u , e_{ux} , and e_{uy} corresponding to a point on this failure surface.

- Evaluation of column adequacy using a numerical scheme:

In order to declare the adequacy of the column section to resist a given combination of P , e_x , and e_y only one point on the failure surface need to be computed. Such point satisfies the following condition

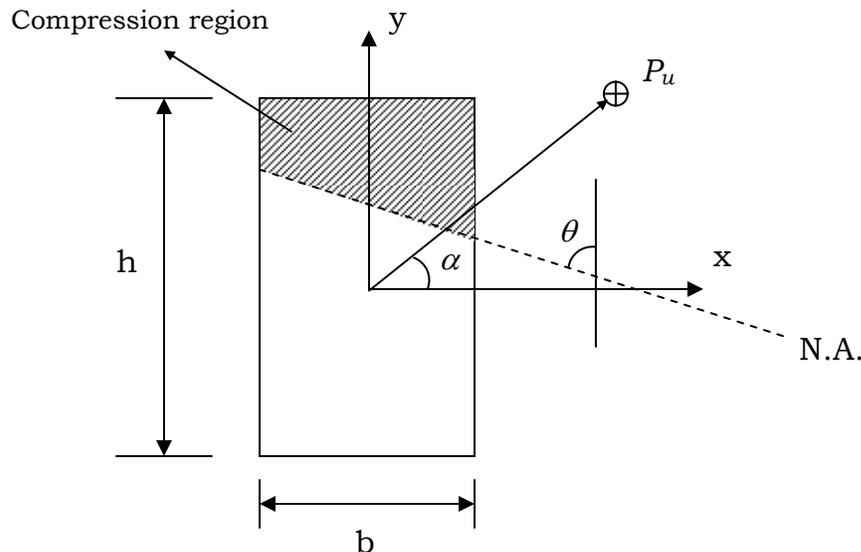
$$P_u = P \text{ and } \frac{e_{uy}}{e_{ux}} = \frac{e_y}{e_x}$$



- The procedure is summarized as follows:

- (a) Find the neutral axis inclination satisfying $\frac{e_{uy}}{e_{ux}} = \frac{e_y}{e_x}$.
 - (b) Set the neutral axis depth c equal to the neutral axis depth computed from balanced failure condition for the section.
 - (c) Compute the value of P_u and update c using a modified secant numerical method until $P_u = P$.
 - (d) Compute e_{ux} and e_{uy} and compare with e_x and e_y to decide whether the section is adequate or not.
- Approximate method for the determination of the neutral axis inclination θ

For rectangular column sections shown:



Approximately,

$$\theta = 90 - y + c - \frac{z}{\sqrt{2}} = \text{inclination of N.A. to } y \text{ axis}$$

$$\text{where } y = \frac{c}{2} + \sqrt{\frac{c^2}{4} - x^2 + cx}$$

$$c = \frac{127}{\frac{h}{b} - 1} = \frac{127b}{h - b} \quad (\text{if } \frac{h}{b} = 1, \theta = \alpha)$$

$$x = \alpha + c - \frac{z}{\sqrt{2}}$$

$$z = -13 \left(\frac{h}{b} - 1 \right)^2 + 39.4 \left(\frac{h}{b} - 1 \right) + 63.6$$

□ Approximate design methods

- Reciprocal load method:

$$\frac{1}{P_u} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} - \frac{1}{P_0}$$

where P_u = ultimate load under biaxial loading,

P_{ux} = ultimate load when only e_x is present,

P_{uy} = ultimate load when only e_y is present, and

P_0 = ultimate load when $e_x = e_y = 0$.

- Load contour method: ($M_{ux} - M_{uy}$ interaction curve in 2D)

For various loads of constant P_u ,

$$\left(\frac{M_{ux}}{M_{ux0}} \right)^m + \left(\frac{M_{uy}}{M_{uy0}} \right)^n = 1$$

where $M_{ux} = P_u e_y$, $M_{uy} = P_u e_x$, and M_{ux0}, M_{uy0} are uniaxial flexural strengths about x and y axes for the constant load level considered.

→ Experiments suggest that $m = n = \alpha$ depends on column geometry.

Typically $1.15 < \alpha < 1.55$ for most rectangular columns with uniform reinforcement.

→ There is no single value that can be assigned to the exponent to represent the true shape of the load contour in all cases.