LECTURE #1

1.060 ENGINEERING MECHANICS II

1.050 EM I = Solid Mechanics
1.060 EM II = Fluid Mechanics

Common : Mechanics, i.e. based on Newton's Laws & Continuum Hypothesis
Difference : Material whose behavior we want to describe.

Definition of a FLUID:

A fluid is a substance that can not support a shearing force (shear stress) without being in motion.

or, in the vocabulary of 1.050,

A fluid is a Tresca Material with zero cohesion, i.e. unless $\sigma_I = \sigma_{II} = \sigma_{III}$ the fluid is in a state of failure

There are two types of fluids:
Liquids (exhibit a "free surface")
Gasses (need a "lid" to be contained)
but same principles (often) apply to both.

In <u>solid mechanics</u> we need to determine <u>stresses</u>, σ_{ij}, and <u>displacements</u>, ξ_i, from external forces, material properties and boundary conditions.

In <u>fluid mechanics</u> we need to determine <u>stresses</u>, σ_{ij}, and <u>rates of displacements</u>, $\dot{\xi}_i$ = <u>velocities</u> = u_i, from external forces, material properties and boundary conditions

Stress tensor in solid mechanics

is replaced σ_{ij} by

$$\sigma_{ij} = -p\,\delta_{ij} + \tau_{ij}$$

where

<u>p = fluid pressure (positive for compression)</u>

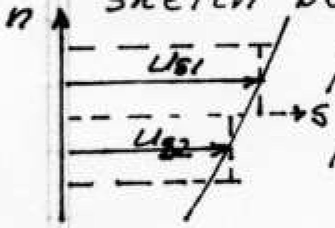
$$\delta_{ij} = \text{Kronecker Delta} = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Bmatrix}$$

τ_{ij} = <u>viscous</u> stress tensor (same sign conventions as in solid mech., i.e. $\tau_{xx}, \tau_{yy}, \tau_{zz} > 0$ for <u>tension</u>)

in fluid mechanics.

Fluid pressure is isotropic (same in all direction) and it follows from the definition of a fluid that $\tau_{ij} \equiv 0$ if the fluid is at rest or in motion as a solid body.

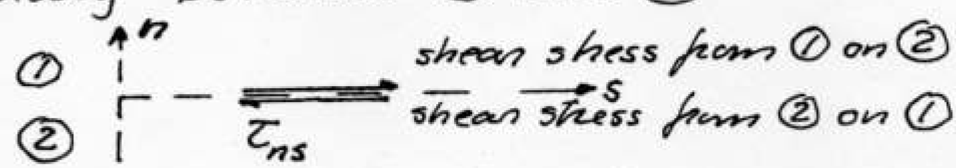
The nature of viscous (shear) stresses in a moving fluid is illustrated in the sketch below



layer ① moves at velocity $\sim u_{s1}$

layer ② moves at velocity $\sim u_{s2}$

Relative to layer ② the fluid in layer ① moves faster, i.e. it slides over ②, and tries to "pull" the fluid in layer ② along. Similarly, the fluid in layer ② tries to "hold back" the fluid in layer ① This interaction is equivalent to a shear stress acting at the common boundary between ① and ②



$$\tau_{ns} = \mu \frac{\partial u_s}{\partial n} = \rho \nu \frac{\partial u_s}{\partial n}$$

μ = dynamic viscosity

ρ = fluid density

ν = kinematic viscosity ($= \mu / \rho$)

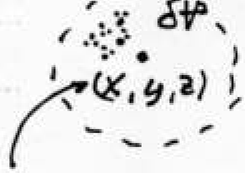
If there is no "shear" in the velocity field of a fluid, i.e. $\partial u_i / \partial x_j = 0$, then there can be no viscous stresses, i.e. $\tau_{ij} = 0$.

The use of differential calculus, e.g. the term $\partial u_s / \partial n$ in the expression for the viscous shear stress, in fluid (as well as in solid) mechanics is based on the

<u>Continuum Hypothesis</u>

As an example illustrating this, we take Fluid density at a point $= \rho(x, y, z)$

By definition



$$\rho(x,y,z) = \lim_{\delta \mathcal{V} \to 0} \left(\frac{\delta m}{\delta \mathcal{V}} = \frac{N m_m}{\delta \mathcal{V}} \right)$$

N molecules of mass m_m in $\delta \mathcal{V}$

If $\delta \mathcal{V} \to 0$ we may "hit" a molecule or we may not. Even if we "hit" a molecule, it may be gone in an instant.

With $\delta \mathcal{V} \to 0 =$ "absolute zero" ρ is undefined!

With $\delta \mathcal{V} \to$ "0" = a volume much smaller in scale than anything we aim to resolve, then ρ at a "point" is defined.

$N \cong 10^{11}$ (Liquid); 10^{8} (Gas) if $\delta \mathcal{V} = 1\ \mu m^3$,
a cube of sidelength $= 1\mu m = 10^{-3} mm = 10^{-6} m$

For $\delta \mathcal{V} =$ "0" $= O(1 \mu m^3)$ N may not be exactly constant, but even if it varied ± 100 that would only be 10^{-7} %, i.e., nothing!