

LECTURE #2

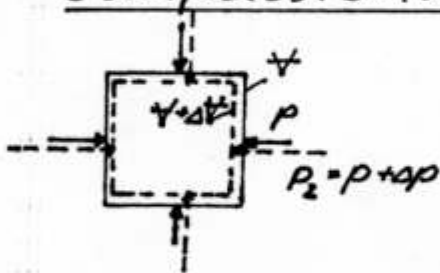
1.060 ENGINEERING MECHANICS II

Continuum Hypothesis

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta u}{\Delta x} = \lim_{\Delta x \rightarrow "0"} \frac{\Delta u}{\Delta x}$$

where "0" is a scale much smaller than any in which we are interested.

Compressibility of Fluids



$$E \frac{\Delta \nabla}{\nabla} = -\nabla p$$

$$E \frac{\Delta \rho}{\rho} = \nabla p$$

$E_v = \text{bulk modulus}$

For water:

$$E_v = 2.15 \cdot 10^9 \frac{\text{N}}{\text{m}^2}; \quad \nabla p = 10^8 \frac{\text{N}}{\text{m}^2} (\sim 10 \text{ km depth})$$

$$\left| \frac{\Delta \nabla}{\nabla} \right| = \left| \frac{\Delta \rho}{\rho} \right| \approx 5\% \quad \text{NOT MUCH}$$

For air

$$E_v = 1.4 \cdot 10^5 \frac{\text{N}}{\text{m}^2}; \quad \nabla p = 10^4 \frac{\text{N}}{\text{m}^2} (\sim 800 \text{ m height})$$

$$\left| \frac{\Delta \nabla}{\nabla} \right| = \left| \frac{\Delta \rho}{\rho} \right| \approx 7\% \quad \text{NOT MUCH}$$

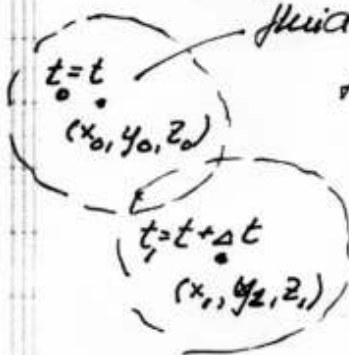
$$c = \text{speed of sound} = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\frac{\nabla p}{\Delta \rho}} = \sqrt{\frac{E}{\rho}}$$

{ 1,500 m/s (water)
335 m/s (air)

If $V = \text{fluid velocity} \ll C = \text{speed of sound}$ in fluid, the fluid can be considered ~ incompressible when pressure variations are not excessive.

Fluid Velocity

"fluid particle" = a small volume $\delta V = "0"$ that consists of "the same" molecules.

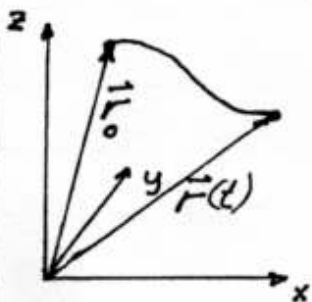


$$\vec{q} = \vec{q}(x_0, y_0, z_0, t) = \frac{(x_1 - x_0, y_1 - y_0, z_1 - z_0)}{t_1 - t_0}$$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x, \Delta y, \Delta z}{\Delta t} = (u_0, v_0, w_0) =$$

velocity (vector) at point (x_0, y_0, z_0) at time t_0 .

Choice of Coordinate System



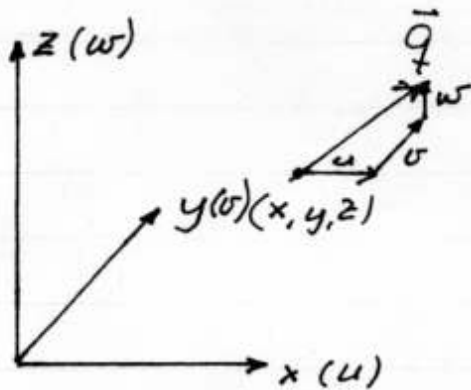
Lagrangian Coordinates

Identify a fluid particle by its position \vec{r}_0 at $t=0$ and determine its position $\vec{r}(t)$ at any subsequent time

$$\vec{r}(\vec{r}_0, t) \Rightarrow \vec{q} = \frac{d\vec{r}}{dt} \Rightarrow \frac{d\vec{q}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

position velocity acceleration

Eulerian Coordinates



Determine the velocity vector, \vec{q} , at a fixed point, (x, y, z) , as a function of time:

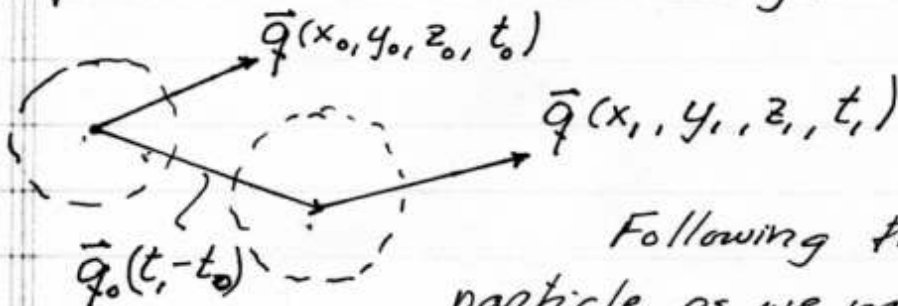
$$\vec{q}(x, y, z, t) = (u, v, w)$$

This coordinate system - Eulerian - is favored over Lagrange's in Fluid Mechanics.

If $\vec{q}(x, y, z, t)$ is not a function of time, i.e.

$$\frac{\partial \vec{q}}{\partial t} = \left(\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t} \right) = (0, 0, 0) = \underline{\underline{0}}$$

The flow is referred to as STEADY FLOW, but $\partial \vec{q} / \partial t = \underline{\underline{0}}$ does not imply that the fluid is not accelerating.



Following the same fluid particle as we must according to Newton's Law, we have

$$\vec{a} = \frac{d\vec{q}}{dt} = \frac{D\vec{q}}{Dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{q}(x_1, y_1, z_1, t_1) - \vec{q}(x_0, y_0, z_0, t_0)}{t_1 - t_0}$$

or, with $x_1 - x_0 = u_0(t_1 - t_0)$ and analogous we have

$$\frac{d\vec{q}}{dt} = \frac{D\vec{q}}{Dt} = \lim_{\Delta t \rightarrow 0} \frac{\frac{\partial \vec{q}}{\partial x} u_0 \Delta t + \frac{\partial \vec{q}}{\partial y} v_0 \Delta t + \frac{\partial \vec{q}}{\partial z} w_0 \Delta t + \frac{\partial \vec{q}}{\partial t} \Delta t}{\Delta t} =$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \text{grad}) \vec{q} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q}$$

where

$$\nabla = \text{"del" operator} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

In words:

$\frac{D\vec{q}}{Dt}$ Total derivative or Material Derivative = rate of change (in this case of velocity \vec{q}) following a fluid particle

"

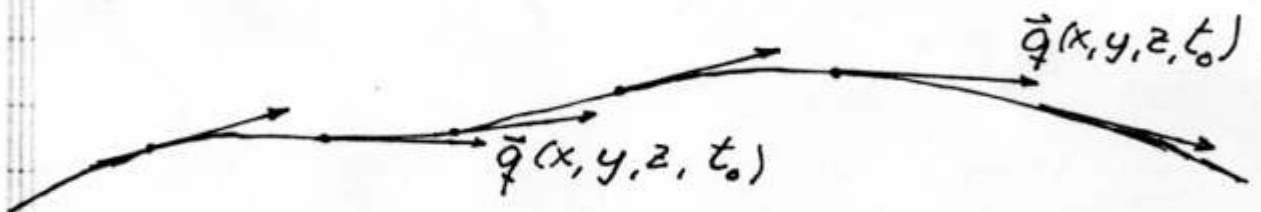
$\frac{\partial \vec{q}}{\partial t}$ Local rate of change, i.e. the rate of change taking place at the fixed location (x, y, z) . Note this would be zero if flow is steady

+

$(\vec{q} \cdot \nabla) \vec{q}$ The convective rate of change, i.e. the rate of change associated with the particle moving to a new location where conditions (here the velocity) has changed relative to the original location. Note this term would be zero if \vec{q} independent of location. UNIFORM FLOW

Streamline

Definition: A streamline is a line that at a given instant of time has the local velocity vector as its tangent at any point along the line.



By definition, it therefore follows that

$$d\vec{s} = \text{infinitesimal element along streamline} = (dx_s, dy_s, dz_s) \propto \vec{Q}(s, t_0) = (u_{s_0}, v_{s_0}, w_{s_0})$$

or

$$\frac{dx_s}{u_{s_0}} = \frac{dy_s}{v_{s_0}} = \frac{dz_s}{w_{s_0}}$$

If the flow is STEADY the velocity vector at any point does not vary with time, $\partial \vec{q} / \partial t = 0$. Hence the streamline is independent of time, and since a particle on a streamline always moves tangential to the line, it will follow a path (the PATHLINE) equal to the streamline.