## TEPE: Transport Economics and Project Evaluation

"Transport Demand and Pricing" Module

## Question 1

## Pricing a Bus Company in SmallTown (12 p)

Consider the beloved city of SmallTown where a single bus company operates in the single OD pair of the city. The morning peak demand (Y [People/Morning Peak]) for bus trips can be described as a function of the bus fare ( $\mathrm{P}[\$ /$ Trip $]$ ) by the following demand function:

$$
Y=30-P
$$

Assume now that the morning peak total cost (C $\$ \$ /$ Morning Peak]) of the bus firm can be described as a function of the patronage, with the following function.

$$
C=28+0.05 Y^{2.8}
$$

Considering the morning peak isolated, answer the following questions:
a. (1p) What is the service fare that maximizes the social welfare? Explain and plot.
b. (1p) What is the amount of the subsidy required to run this service at the social optimum fare? Explain and plot.
c. (1p) What is the fare that maximizes social welfare subject to no losses for the bus company? Explain and plot.
d. (1p) Considering that the company is privately managed and deregulated, what would be the fare of the service? Explain and plot.
e. (1p) What is the amount of social welfare loss due to private deregulated operation compared to the social optimum situation? Explain and plot.

Assume now that, due to a technological change (a change in the size of the doors and the room available to carry people in the buses), the cost function of the bus company changes to the following

$$
C=28+0.0005 Y^{2.8}
$$

f. (1p) Which kind of technological change (an increment or a decrement in the size of the doors and the room available for carrying people in the buses) may produce such a change in the cost function? Explain.
g. (5p) Answer questions a, b, c, d and e, but now for this new cost function. Explain the potentially different results obtained for this new cost function, compared to the previous one.
h. (1p) Consider that SmallTown has to decide whether or not to perform this revolutionary technological change. Constraining the analysis to the morning peak, what would be the
maximum cost $X[\$ /$ Morning Peak] that the city should be willing to pay to perform such a hitech upgrade?

## Question 2

## Congestion Pricing (12p)

Consider a single OD pair connected by three routes with the following average user's travel time [min] as a function of their respective flow $q_{i}$ [veh/hour].


Answer the following questions:
a. (1p) Determine the user equilibrium considering that the total demand is inelastic and equal to 40 [veh/hour]. Hint: in a user equilibrium, the travel cost of all used routes should be equal.
b. (1p) Assuming that the value of time is $1[\$ / \mathrm{min}]$ and that there is only one driver per vehicle, what are the total costs for all users using route $i$, (where $i=1,2,3$ respectively)? What are the marginal costs for these three routes, respectively? Comment.
c. (1p) Determine the equilibrium flows on route $i,(i=1,2,3)$ under the system optimal condition. Hint: In a system optimal situation, the marginal costs for all used routes should be equal.
d. (1p) Calculate the difference in social welfare between the system optimum and the user's equilibrium.
e. (1p) Assuming the toll collection costs are zero, what toll should be charged to the users so that the flows on the routes are system optimal?
f. (1p) Compute the change in welfare experienced by the drivers using each route, by toll collection agency, and by the whole society, due to the application of the tolls. Comment.
g. (1p) Assume now that toll collection is not free. For each dollar collected, 50 cents should be spent in collection devices and administration. Would you recommend implementing congestion pricing in such case? What is the limit cost, as a percentage of collected tolls, which would make it socially profitable to implement congestion pricing in this example? Comment.
h. (2p) Would it be possible to attain the system or social optimum equilibrium by paying instead of tolling some drivers? If this is possible, calculate such payments and analyze the welfare
change compared to the congestion pricing case for each type of driver and the toll (in this case) paying agency. Comment.
i. (3p) Consider a social movement of DFS (Drivers For Sustainability) which is quickly gaining members. This group, instead of selfishly driving on routes that minimize their travel time, takes into consideration in their decision the social marginal cost or externality of their actions. In other words, they are willing to drive, for the sake of social welfare, on slower routes. Assuming perfect information on route costs, how many of the 40 [veh/hour] should belong to the DFS group in order to attain the system optimum? Analyze the social welfare change of the DFS drivers, the non DFS drivers and the tolling agency compared to the case of original user's equilibrium and congestion pricing.

## Question 3

## Revenue Forecasting (12p)

The New York City Department of Transportation is considering leasing the Brooklyn Bridge to a private toll-road operator to raise revenues and offload the responsibilities for maintaining and operating the bridge. The consultants on the project, Modeling Bridges Associated (MBA), have used their fancy micro-simulation models to make a prediction for the yearly revenue that the private operator can expect to collect. You are an infrastructure analyst at Fruman Sachs, Inc, the bank providing the financing for the deal. To know what interest rate to charge for your capital, you need to assign some level of risk to this revenue forecast. You have decided to use the method we have learned about in class (Bowman 2003); much of the preparatory work has already been done, you just need to put on the finishing touches. Please download the spreadsheet associated with this project from the class Stellar site (also shown in Table 1.1 on the next page). It shows the following information:

- The revenue forecast produced by the simulation model
- The variables we will use in our risk assessment, and the values used in MBA's final forecast
- The results of a number of runs with slightly changed values for each variable (one at a time)
- Simplified probability distributions for each variable
a) (2p) Estimate the elasticity of the revenue forecast with respect to each variable independently
b) Estimating the probability distribution function of the revenue forecast requires enumerating every possible scenario. To keep this assignment reasonable, we will limit ourselves to thinking about the best and worst case scenarios:
i) (2.5 p) Estimate the revenue forecast for the best and worst case scenarios
ii) (2.5 p) Estimate the probability of the best and worst case scenarios (these will be very small numbers, so please answer in scientific notation)
c) We would like to understand how sensitive our risk assessment is to the inclusion of certain kinds of variables. Estimate new best and worst case scenario revenue forecasts under the following circumstances, and compare your results to the results from part (b):
i) (2.5 p) If we ignore variability in the non-toll costs of driving (i.e. Fuel Efficiency, Gas Taxes, and Parking Costs)
ii) ( 2.5 p) If we ignore variability in the travel-time variables (i.e. Travel Time Improvement and Value of Time)

| Variable | Variable Name | Forecast Value | Value for Elasticity | Revenue Forecast |
| :---: | :---: | :---: | :---: | :---: |
| EG | Economic Growth Index | 1 | 1.05 | \$107,108,502.19 |
| FE | Fuel Efficiency (miles/gallon) | 30 | 31.5 | \$106,844,036.75 |
| GT | Gas Taxes (\$/Gallon) | \$0.32 | \$0.34 | \$104,569,633.99 |
| PC | Avg Parking Cost (\$/month) | \$450 | \$473 | \$102,083,658.88 |
| TS | Transit Service (Yearly Train-Miles + Bus-Miles) | 470,000,000 | 493,500,000 | \$104,781,206.34 |
| TL | Parallel Tolls | \$5 | \$5 | \$101,554,728.00 |
| TTI | Travel Time Improvement (minutes) | 20 | 21 | \$110,229,194.35 |
| VOT | Value of Time (\$/hour) | \$30 | \$32 | \$109,858,942.74 |
|  |  |  |  |  |
| R | Revenue | \$105,786,175 |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
| Forecast Revenue |  |  |  |  |
| 2007 Traffic | 67,595 |  |  |  |
| Toll | \$5 |  |  |  |
| Weekday Revenue | \$337,975 |  |  |  |
| Weekend Conversion | 0.50 |  |  |  |
| Weekend Day Revenue | \$168,988 |  |  |  |
| Weekend Days | 104.00 |  |  |  |
| Weekday Days | 261.00 |  |  |  |
| Total Revenue | \$105,786,175 |  |  |  |
|  |  |  |  |  |
| Factor Distributions |  |  |  |  |
|  |  | Value | P (Value) |  |
| EG | Economic Growth Index | 0.75 | 0.2 |  |
|  |  | 1 | 0.3 |  |
|  |  | 1.25 | 0.5 |  |
|  |  |  |  |  |
| FE | Fuel Efficiency (miles/gallon) | 30 | 0.33 |  |
|  |  | 40 | 0.33 |  |
|  |  | 50 | 0.33 |  |
|  |  |  |  |  |
| GT | Gas Taxes (\$/Gallon) | 0.32 | 0.25 |  |
|  |  | 0.5 | 0.5 |  |
|  |  | 1 | 0.25 |  |
|  |  |  |  |  |
| PC | Avg Parking Cost (\$/month) | 400 | 0.1 |  |
|  |  | 450 | 0.4 |  |
|  |  | 500 | 0.2 |  |
|  |  | 550 | 0.2 |  |
|  |  | 600 | 0.1 |  |
|  |  |  |  |  |
| TS | Transit Service (Yearly Train-Miles + Bus-Miles) | 430,000,000 | 0.1 |  |
|  |  | 470,000,000 | 0.7 |  |
|  |  | 510,000,000 | 0.2 |  |
|  |  |  |  |  |
| TL | Parallel Tolls | 5 | 0.33 |  |
|  |  | 6 | 0.33 |  |
|  |  | 7 | 0.33 |  |
|  |  |  |  |  |
| TTI | Travel Time Improvement (minutes) | 15 | 0.1 |  |
|  |  | 18 | 0.25 |  |
|  |  | 20 | 0.3 |  |
|  |  | 22 | 0.25 |  |
|  |  | 25 | 0.1 |  |
|  |  |  |  |  |
| VOT | Value of Time (\$/hour) | 25 | 0.1 |  |
|  |  | 30 | 0.8 |  |
|  |  | 35 | 0.1 |  |

Table 1.1: Inputs for Problem 3

MIT OpenCourseWare
http://ocw.mit.edu
1.201J / 11.545J / ESD.210J Transportation Systems Analysis: Demand and Economics Fall 2008

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