

New Approaches to Add Robustness into Airline Schedules

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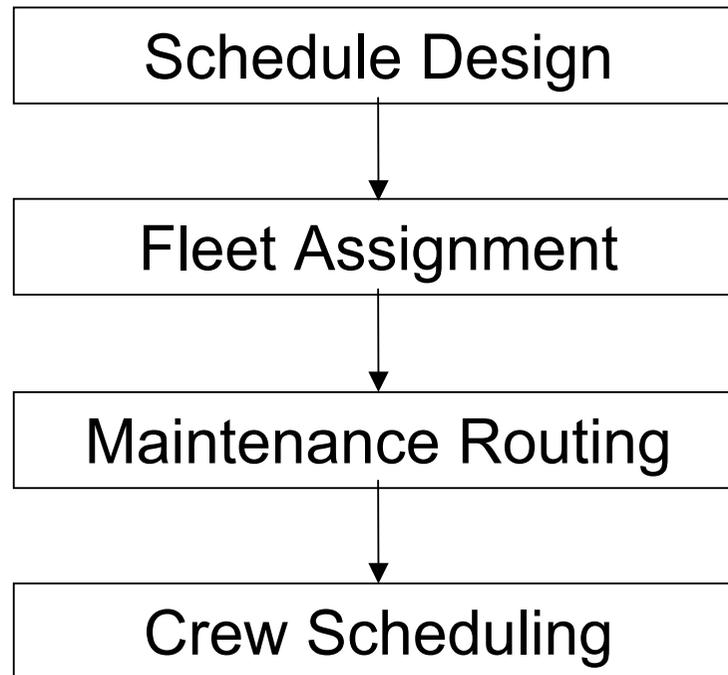
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Outline

- **Background, Motivation and Our Contributions**
- **Overview of Robust Airline Schedule Planning**
- **Robust Aircraft Maintenance Routing – reduce delay propagation**
- **Flight Schedule Retiming – reduce passenger missed connections**
- **Summary and Future Research Directions**

Airline Schedule Planning Process



- **Most existing planning models assume that aircraft, crew, and passengers will operate as planned**

Airline Operations

- **Many reasons can cause delays**
 - Severe weather conditions, unexpected aircraft and personnel failures, congested traffic, etc.
- **Delays may propagate through the network**
- **Long delays and cancellations cause schedule disruptions**
- **Airlines must reschedule aircraft/crew and re-accommodate passengers**
- **Huge revenue loss:**
 - Delays cost consumers and airlines about \$6.5 billion in 2000 (Air Transport Association)

Flight Delays & Cancellations

- **Trend (1995-1999) (Bratu and Barnhart, 2002)**
 - Significant increase (80%) in flights delayed more than 45 min
 - Significant increase (500%) in the number of cancelled flights

- **Year 2000 (Bratu and Barnhart, 2002)**
 - 30% of flights delayed
 - 3.5% of flights cancelled

- **Future:**
 - Air traffic in US is expected to double in the next 10-15 years (Schaefer et al. (2001))
 - Each 1% increase in air traffic → a 5% increase in delays (Schaefer et al. (2001))
 - Lead to more frequent and serious delay and schedule disruptions

Passenger Disruptions

- **Passengers are disrupted if their planned itineraries are infeasible because**
 - flights cancellation
 - Insufficient time to connect

- **4% of passengers disrupted in 2000 (Bratu and Barnhart, 2002)**
 - Half of them are connecting passengers

- **Very long delays for disrupted passengers**
 - Average delay for disrupted passengers is approx. 419 minutes (versus 14 min delay for non-disrupted passengers) (Bratu and Barnhart, 2002)

- **Significant revenue loss**

Our Contributions

- ❑ **Provide alternative definitions for robustness in the context of airline schedule planning**
- ❑ **Develop an optimization model and solution approach that can generate aircraft maintenance routes to minimize delay propagation**
- ❑ **Develop optimization models and solution approach to minimize the expected total number of passengers missing connection, and analyze the model properties**
- ❑ **Proof-of-concept results show that these approaches are promising**
- ❑ **Develop integrated models for more robustness**

Outline

- **Background, Motivation and Our Contributions**
- **Overview of Robust Airline Schedule Planning**
 - How to deal with schedule disruptions
 - Challenges of building robust airline schedules
 - Definitions of robustness
 - Robust airline schedule planning approaches
- **Robust Aircraft Maintenance Routing -- reduce delay propagation**
- **Flight Schedule Retiming – reduce passenger missed connections**
- **Summary and Future Research Directions**

How to Deal with Schedule Disruptions

- **Two ways to deal with schedule disruptions**
 - Re-optimize schedule after disruptions occur (operation stage)
 - Build robustness into the schedules (planning stage)
- **Existing planning systems do not have effective methods to manage disruptions**
- **A more robust plan can reduce the effect of disruptions on the operations → reduce operation costs and improve quality of service**
- **Robust airline schedule planning methods are needed**

Challenges of Building Robust Plans

- ❑ **Lack of a systematic way to define robustness in the context of airline schedule planning**
- ❑ **Aircraft, crew and passenger flows interact in the hub-and-spoke network**
- ❑ **Huge problem size → tractability issue**
- ❑ **Difficult to balance robustness and costs**

Definitions of Robustness

- ❑ **Minimize cost**
- ❑ **Minimize aircraft/passenger/crew delays and disruptions**
- ❑ **Easy to recover (aircraft, crew, passengers)**
- ❑ **Isolate disruptions and reduce the downstream impact**

Robust Airline Schedule Planning

	Min Cost	Min delays/ disruptions	Ease of recovery	Isolation of disruptions
Schedule Design		This Thesis		Kang & Clarke
Fleet Assignment				Rosenberger, <i>et al.</i> (2001)
Maintenance Routing		This Thesis	Ageeva & Clarke(2000)	Kang & Clarke This thesis
Crew Scheduling	Yen & Birge, Schaefer, <i>et al.</i> (2001)		Chebalov & Klabjan	

Where Should We Start?

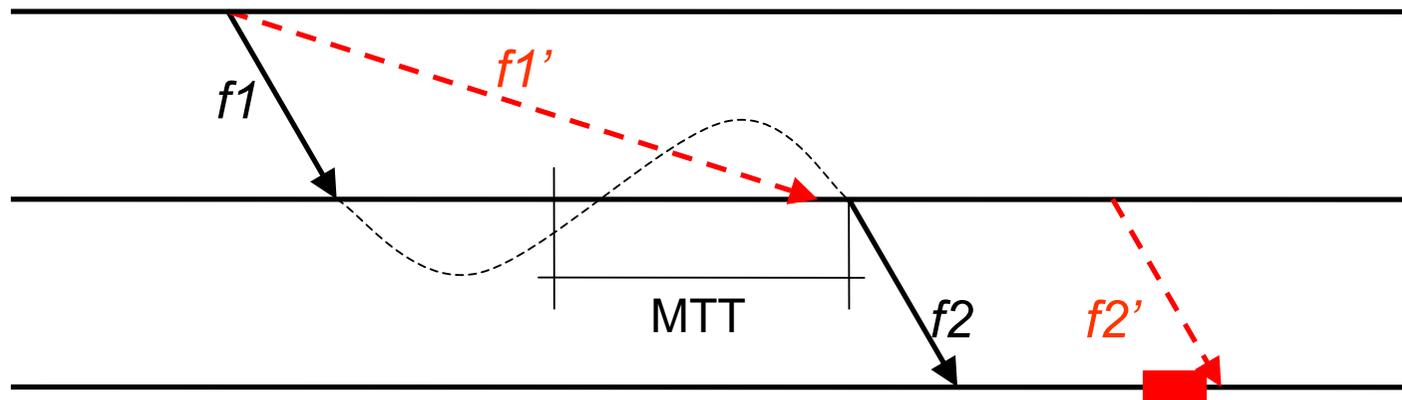
- **Difficult to balance cost that airlines are willing to pay for robustness versus cost of operation**
- **Looking for robust solution without significant added costs**
 - Aircraft maintenance routing problem: The financial impact is relatively small → It is more a feasibility problem
 - How to route aircraft has impacts on flight delays and cancellations, passengers, crews
 - Question:
 - What robustness can be achieved for the maintenance routing problem?

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- **Overview of Robust Airline Schedule Planning**
- **Robust Aircraft Maintenance Routing – reduce delay propagation**
 - Delay Propagation
 - Modeling Idea
 - String based formulation
 - Solution approach
 - Proof-of-concept results
- **Flight Schedule Retiming – reduce passenger missed connections**
- **Summary and Future Research Directions**

Delay Propagation

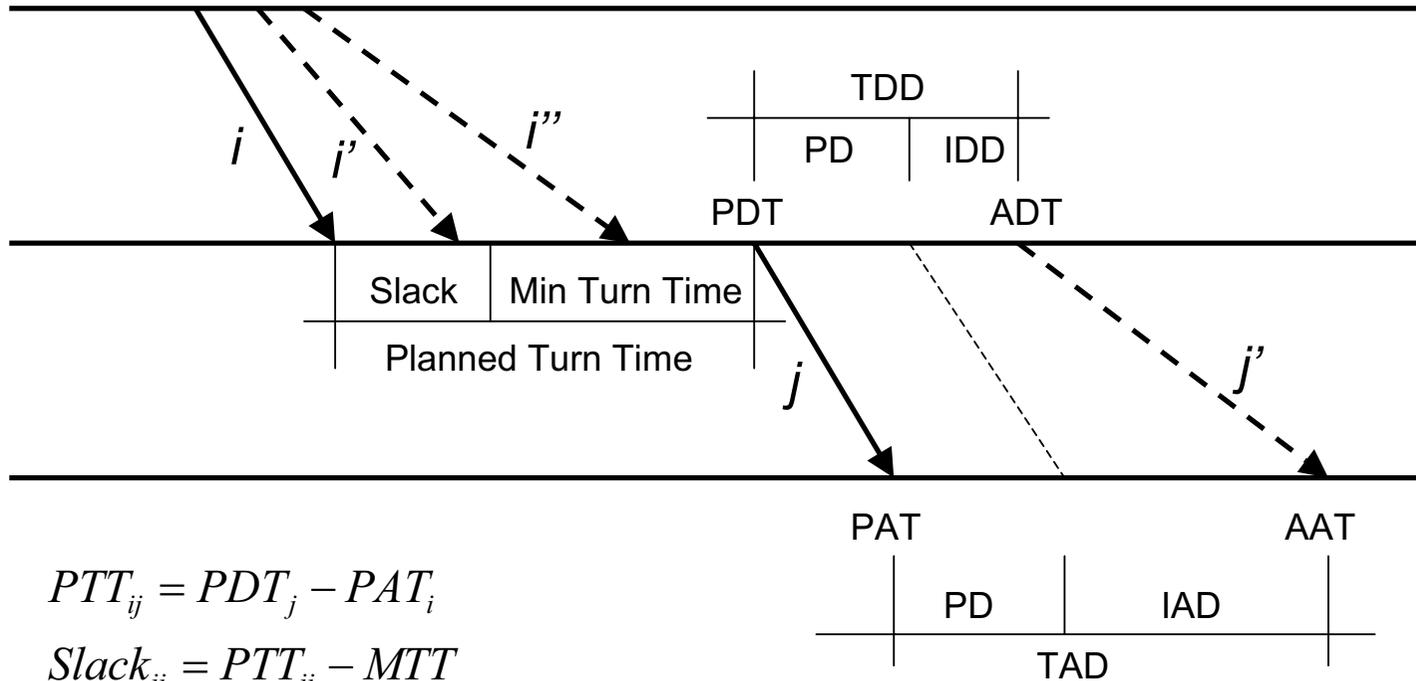
- Arrival delay may cause departure delay for the next flight that is using the same aircraft if there is not enough slack between these two flights
- Delay propagation may cause schedule, passenger and crew disruptions for downstream flights (especially at hubs)



Propagated Delay vs. Independent Delay

- **Flight delay may be divided into two categories:**
 - Propagated delay
 - Caused by inbound aircraft delay – function of routing
 - 20-30% of total delay (Continental Airlines)
 - Independent delay
 - Caused by other factors – not a function of routing

Definitions



$$PTT_{ij} = PDT_j - PAT_i$$

$$Slack_{ij} = PTT_{ij} - MTT$$

$$PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$$

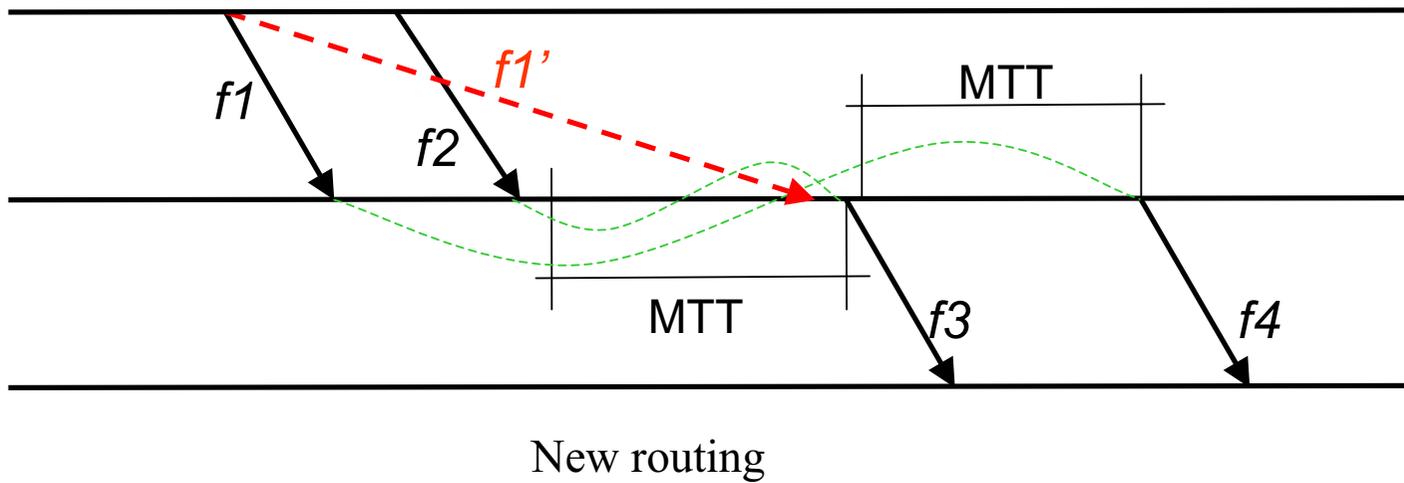
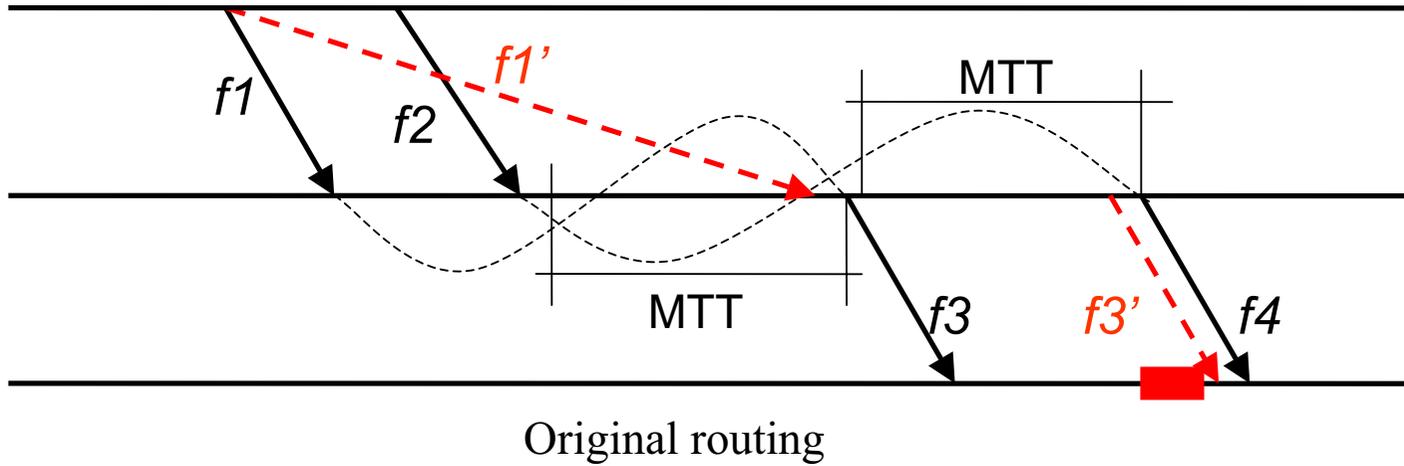
$$IDD_j = TDD_j - PD_{ij}$$

$$IAD_j = TAD_j - PD_{ij}$$

Modeling Idea

- **Delays propagate along aircraft routes**
- **Only limited slack can be added**
- **Appropriately located slack can prevent delay propagation**
- **Routing aircraft intelligently → better allocated slack**
- **Essentially add slack where advantageous, reducing slack where less needed**

Illustration of the Idea



Modeling Issues

- **Difficult to use leg-based models to track the delay propagation**
- **One variable (string) for each aircraft route between two maintenances (Barnhart, et al. 1998)**
 - A string: a sequence of connected flights that begins and ends at maintenance stations
 - Delay propagation for each route can be determined
- **Need to determine delays for each feasible route**
 - Most of the feasible routes haven't been realized yet
 - PD and TAD are a function of routing
 - PD and TAD for these routes can't be found in the historical data
 - IAD is not a function of routing and can be calculated by tracking the route of each individual aircraft in the historical data

Generating Flight Delays for Any Feasible Route

- **Step 1: Determine propagated delays from historical data:**
 - $PD_{ij} = \max(TAD_i - \text{slack}_{ij}, 0)$
- **Step 2: Determine Independent Arrival Delays (IAD) from historical data:**
 - $IAD_j = TAD_j - PD_{ij}$
- **Step 3: Determine TAD and PD for feasible routes:**
 - For the first flight on each string, $\text{New_TAD} = IAD$
 - $\text{New_PD}_{ij} = \max(\text{New_TAD}_i - \text{slack}_{ij}, 0)$
 - $\text{New_TAD}_j = IAD_j + \text{New_PD}_{ij}$

String Based Formulation

$$\min E \left(\sum_s \left(\sum_{(i,j) \in s} p d_{ij}^s \right) x_s \right)$$

s.t. :

$$\sum_{s \in S} a_{is} x_s = 1, \forall i \in F$$

$$\sum_{s \in S_i^+} x_s - y_{i,d}^- + y_{i,d}^+ = 0, \forall i \in F$$

$$- \sum_{s \in S_i^-} x_s - y_{i,a}^- + y_{i,a}^+ = 0, \forall i \in F$$

$$\sum_{s \in S} r_s x_s + \sum_{g \in G} p_g y_g \leq N$$

$$y_g \geq 0, \forall g \in G$$

$$x_s \in \{0,1\}, \forall s \in S$$

Objective Function Coefficient

- **Random variables (PD) can be replaced by their mean**

$$\min \sum_s x_s \times E\left[\sum_{(i,j) \in s} pd_{ij}^s\right] = \min \sum_s x_s \times E\left[\sum_{(i,j) \in s} pd_{ij}^s\right] = \min \sum_s x_s \times \left(\sum_{(i,j) \in s} E[pd_{ij}^s]\right)$$

- **Distribution of Total Arrival Delay**

- Possible distributions analyzed: Normal, Exponential, Gamma, Weibull, Lognormal, etc.
- Our statistical analysis shows that lognormal distribution is the best fit

- **A closed form of expected value function can be obtained**

$$E(pd) = \left(1 - \Phi\left(\frac{\ln(-\theta/m)}{\delta}\right)\right) \left(\theta + m e^{\frac{1}{2}\delta^2}\right)$$

Solution Approach

- **This formulation is a deterministic mixed-integer program with a huge number of 0-1 variables**
- **Branch-and-price**
 - Branch-and-Bound with a linear programming relaxation solved at each node of the branch-and-bound tree using column generation
- **IP solution**
 - A special branching strategy: branching on follow-ons (Ryan and Foster 1981, Barnhart et al. 1998)

Computational Results

□ Test Networks

Network	Num of flights	Num of strings
N1	20	7,909,144
N2	59	614,240
N3	97	6,354,384
N4	102	51,730,736

□ Data divided into two sets:

- First data set (Jul 2000) used to build model and generate routes
- Second data set (Aug 2000) used to test these new routes

Results - Delays

□ July 2000 data

Network	Old PD	New PD	PD reduced	% of PD reduced
N1	5723	4091	1632	29%
N2	3553	1388	2165	61%
N3	7128	3217	3911	55%
N4	9152	7108	4321	47%
Total	25556	15804	12029	47%

□ August 2000 data

Network	Old PD	New PD	PD reduced	% of PD reduced
N1	6749	4923	1826	27%
N2	4106	2548	1558	38%
N3	8919	4113	4806	54%
N4	14526	9921	6940	48%
Total	34300	21505	15130	44%

Results - Delay Distribution

□ Total delays for existing and new routings

	Total delay			on-time performance		
	> 15 min	> 60 min	> 120 min	15 min	60 min	120 min
Old	22.3%	7.9%	2.9%	77.7%	92.1%	97.1%
New	20.7%	6.9%	2.6%	79.3%	93.1%	97.4%

Results - Passenger Disruptions

□ Disruptions calculated at the flight level

- If a flight was cancelled, all passengers on that flight is disrupted
- If actual departure time of flight B – actual arrival time of flight A < minimum connecting time → all passengers connecting from A to B are disrupted

Network	Total num of D-pax	D-pax reduces	D-pax reduced (%)
N1	986	147	14.9%
N2	1070	79	7.4%
N3	1463	161	11.0%
N4	3323	355	10.7%
Total	6842	742	10.8%

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- **Robust Aircraft Maintenance Routing**
- **Flight Schedule Retiming – reduce passenger missed connections**
 - Passenger delays and disruptions
 - Modeling Idea
 - Formulations and their properties
 - Solution approach
 - Proof-of-concept results
- **Summary and Future Research Directions**

Passenger Delays and Disruptions

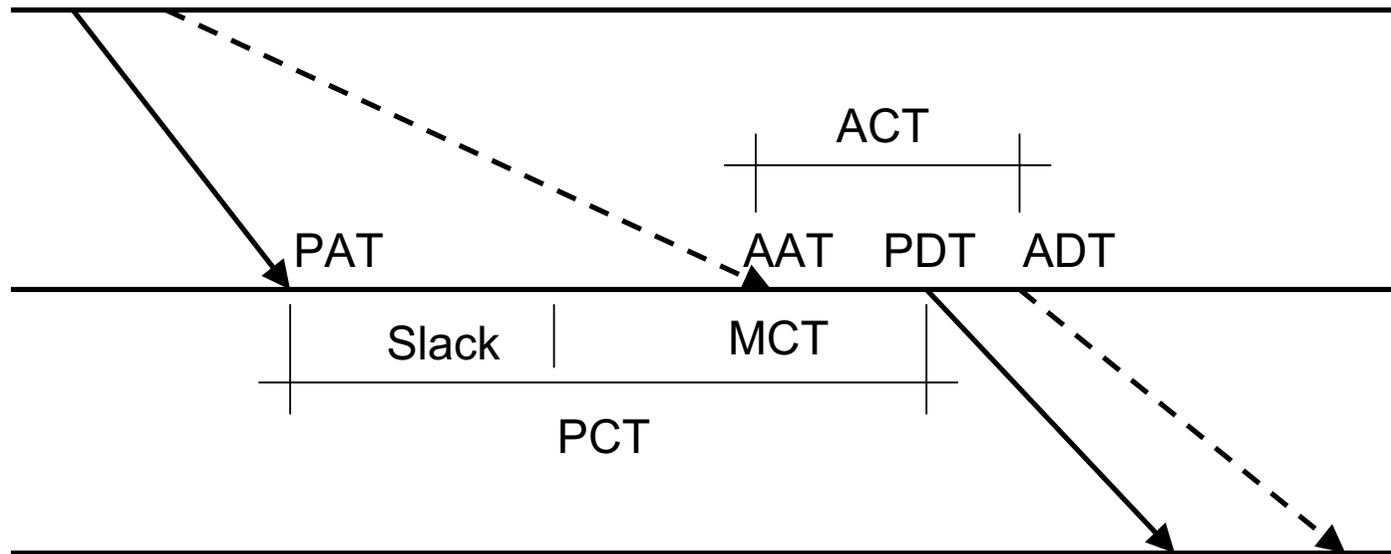
- **Flight delay and passenger delay (Bratu and Barnhart, 2002)**

	Ave. delay	% Pax	% Total pax delay
Disrupted pax	419 min	4%	51%
Nondisrupted pax	14 min	96%	49%
All pax	31 min		
Flights	16 min		

- **Passenger delay caused by disruptions is the most critical part**
- **Minimize number of disrupted passengers**
 - A good proxy for passenger delays

Definitions Related to Passenger Disruption

If $ACT - MCT < 0$, passengers are disrupted

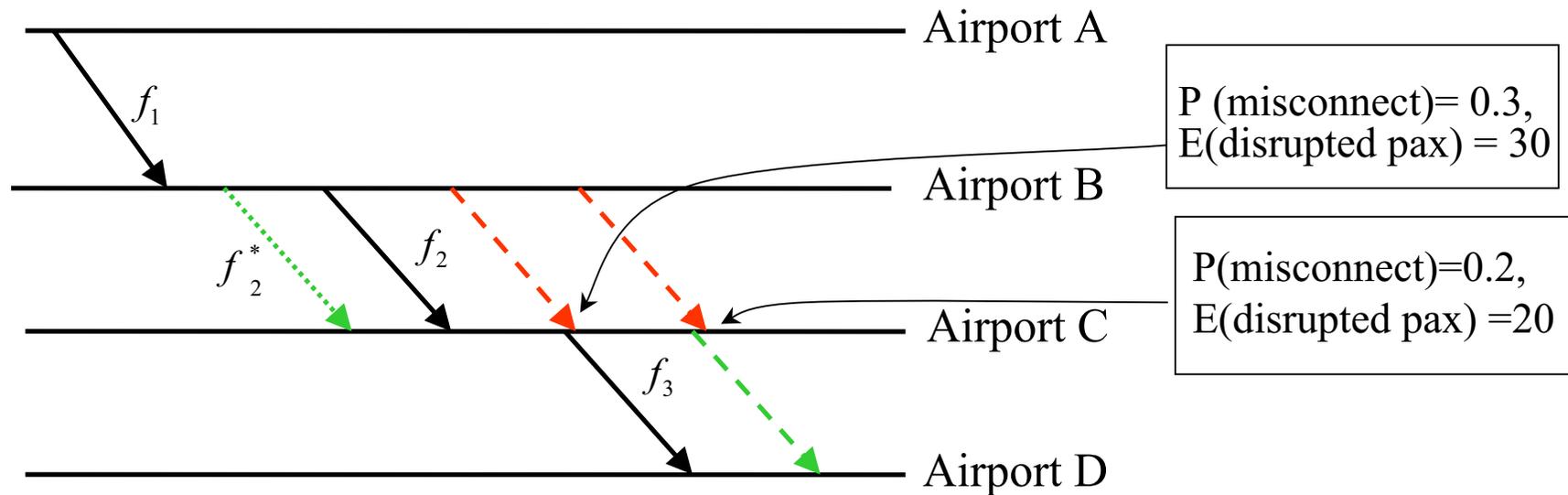


Minimize Passenger Missed Connections

- ❑ If the slack is “eaten” by flight delay, passengers are disrupted
- ❑ Adding more slack can be good for connecting passengers, but can result in reduced productivity
- ❑ Appropriately located slack can prevent passenger disruptions
- ❑ Moving flight departure times in a small time window can lead to better allocated slack

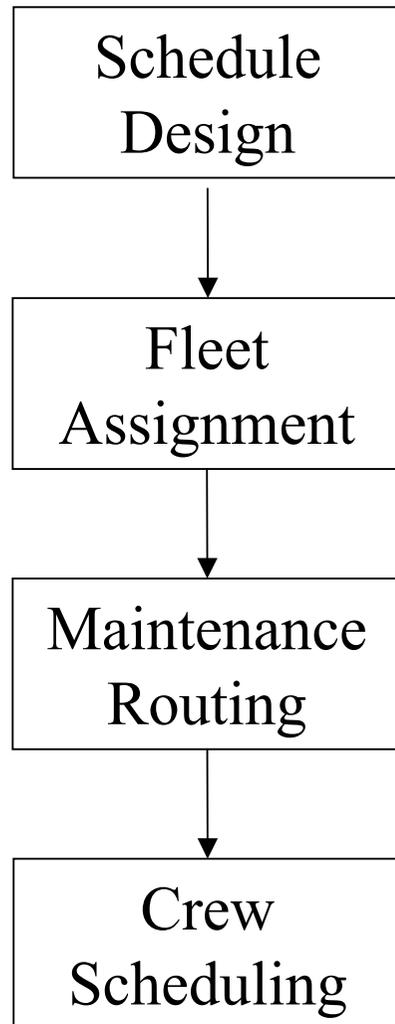
Illustration of the Idea

Suppose 100 passengers in flight f_2 will connect to f_3



→ Expected disrupted passengers reduced: 10

Where to Apply



- **Whether a passenger will be disrupted depends on flight delays, a function of fleet and routing**
- **Before solving maintenance routing**
 - Impact of the propagation of flight delays won't be considered
 - New fleet and routing solution may cause delay propagate in a different way → may eventually change the number of disrupted passengers
- **After solving fleet and routing problem**
 - Delay propagation has been considered
 - Need to maintain the current fleet and routing solution

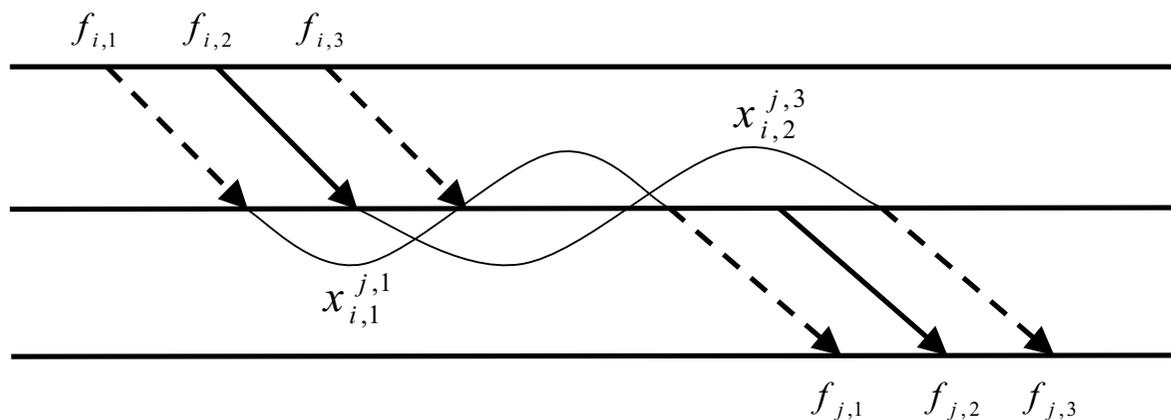
Connection-Based Formulation

□ Objective

- minimize the expected total number of passengers missing connection

□ Constraints:

- For each flight, exactly one copy will be selected.
- For each connection, exactly one copy will be selected and this selected copy must connect the selected flight-leg copies.
- The current fleeting and routing solution cannot be altered.



Connection-Based Formulation

$$\text{Min } E \left[\sum_{i,n,j,m} x_{i_n,j_m} \times DP_{i_n,j_m} \right]$$

s.t.

$$\sum_n f_{i,n} = 1, \forall i;$$

$$\sum_m \sum_n x_{i_n,j_m} = 1, \forall i, j;$$

$$\sum_m x_{i_n,j_m} = f_{i,n}, \forall i, n, j \in C^+(i);$$

$$\sum_n x_{i_n,j_m} = f_{j,m}, \forall j, m, i \in C^-(j);$$

$$x_{i_n,j_m} \in \{0,1\}; \forall i, n, j \in C^+(i), m;$$

$$f_{i,n} \in \{0,1\}; \forall i, n$$

□ Theorem 1:

- The second set of constraints are redundant and can be relaxed

□ Theorem 2:

- The integrality of the connection variables can be relaxed

Alternative Connection-based Formulations

• Formulation II

$$\text{Min } E \left[\sum_{i,n,j,m} x_{i_n,j_m} \times DP_{i_n,j_m} \right]$$

s.t.

$$\sum_n f_{i,n} = 1, \forall i;$$

$$\sum_n \sum_m x_{i_n,j_m} = 1, \forall i, j;$$

$$\sum_{j \in C^+(i)} \sum_m x_{i_n,j_m} = |C^+(i)| f_{i,n}, \forall i, n;$$

$$\sum_{i \in C^-(j)} \sum_n x_{i_n,j_m} = |C^-(j)| f_{j,m}, \forall j, m;$$

$$f_{i,n} \in \{0,1\}, \forall i, n;$$

$$x_{i_n,j_m} \in \{0,1\}, \forall i, n, j \in C^+(i), m.$$

• Formulation III

$$\text{Min } E \left[\sum_{i,n,j,m} x_{i_n,j_m} \times DP_{i_n,j_m} \right]$$

s.t.

$$\sum_n f_{i,n} = 1, \forall i;$$

$$\sum_m \sum_n x_{i_n,j_m} = 1, \forall i, j;$$

$$x_{i_n,j_m} \geq f_{i,n} + f_{j,m} - 1, \forall i, n, j \in C^+(i), m;$$

$$x_{i_n,j_m} \in \{0,1\}; \forall i, n, j \in C^+(i), m;$$

$$f_{i,n} \in \{0,1\}; \forall i, n$$

Model Properties

□ Theorems on constraints:

- The second set of constraints are redundant and can be relaxed in formulations two and three
- The integrality constraints of the connection variables can be relaxed in formulations two and three

□ Theorem on LP relaxations

- The LP relaxation of formulation one is at least as strong as those of formulations two and three

Problem Size

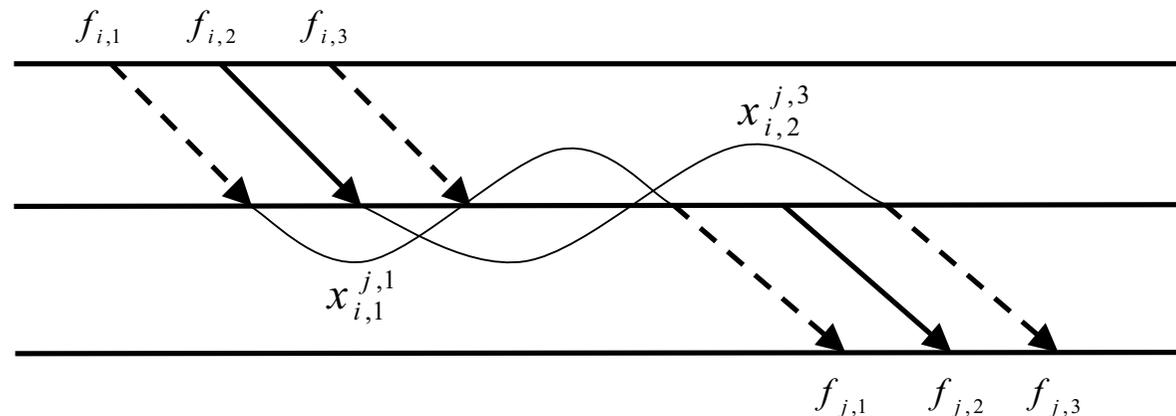
□ **A network from a major US airline used by Barnhart et al. (2001)**

- 2,044 flights and 76,641 itineraries.
- Suppose 7 copies will be generated for each flight (if 5 minutes interval is used, 7 copies correspond to a 30 minute time window)
- Assume on average every flight connects to 12 flights with connecting passengers.

	Number of Variable	Number of Integer Variables	Number of Rows
F1	1,216,180	14,308	345,436
F2	1,216,180	14,308	30,660
F3	1,216,180	14,308	1,203,916

How to Maintain Current Fleeting and Routing Solution

- For an aircraft maintenance route: the planned turn time \geq minimum turn time
- Force $x_{i,n}^{j,m} = 0$, if the time between the arrival of flight copy $f_{i,n}$ and the departure of flight copy $f_{j,m}$ is less than the minimum turn time.
- The upper bounds will be set to zero for these x variables



Solution Approach

□ Random variables can be replaced by their mean

➤ Deterministic Problem

$$E\left[\sum_{i,n,j,m} x_{i_n,j_m} \times DP_{i_n,j_m}\right] = \sum_{i,n,j,m} E[x_{i_n,j_m} \times DP_{i_n,j_m}] = \sum_{i,n,j,m} x_{i_n,j_m} \times E[DP_{i_n,j_m}]$$

➤ Distribution of DP_{i_n,j_m}

$$DP_{i_n,j_m} = \begin{cases} c_{i,j}, & \text{with prob } p \\ 0, & \text{with prob } 1-p \end{cases}$$

Probability p can be determined by considering

$$-- \text{prob} \left(ADT_{j_m} - AAT_{i_n} < MCT \right)$$

□ Branch-and-Price

Computational Results

□ Network

- We use the same four networks, but add all flights together and form one network with total 278 flights.

□ Data divided into two sets:

- First data set (Jul 2000) used to build model and generate schedule
- Second data set (Aug 2000) used to test the new schedule

□ Strength of the formulations

Models	Value at node 0	Optimal value	Num of nodes searched	Time to solve
RAFRS	10,437	10,899	≥43,590 (out of memory)	≥ 386,756 sec
RFRS	10,899	10,899	1	13 sec

Computational Results

□ Assume 30 minute minimum connecting time

➤ For July 2000 data

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	17,459	10,899	6,560	37.6%
±10min(5 copies)	17,459	12,070	5,389	30.9%
±5min(3 copies)	17,459	14,069	3,390	19.4%

➤ For August 2000 data

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	18,808	11,348	7,460	39.7%
±10min(5 copies)	18,808	12,732	6,076	32.3%
±5min(3 copies)	18,808	15,042	3,766	20.0%

Computational Results

□ August 2000 data

- Assume 25 minute minimum connecting time

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	15,102	10,144	4,958	32.8%
±10min(5 copies)	15,102	11,237	3,865	25.6%
±5min(3 copies)	15,102	12,753	2,349	15.6%

- Assume 20 minute minimum connecting time

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	12,724	9,275	3,449	27.1%
±10min(5 copies)	12,724	10,054	2,670	21.0%
±5min(3 copies)	12,724	11,107	1,617	12.7%

Computational Results

□ How many copies to generate

Time window	Num of Rows	Num of Cols	Num of Non-zeros	Increase
±15min(7 copies)	7,506	27,013	59,836	
±15min(31 copies)	32,514	507,253	1,040,236	17.4
±10min(5 copies)	5,422	14,085	32,320	
±10min(21 copies)	22,094	234,213	485,856	15.0
±5min(3 copies)	3,338	5,325	13,140	
±5min(11 copies)	11,674	65,373	139,876	10.6

Time window	Tot num of D-pax	Output	D-pax reduced	Improve (%)
±15min(7 copies)	17,459	10,899	6,560 (37.6%)	
±15min(31 copies)	17,459	10,865	6,594 (37.8%)	0.52%
±10min(5 copies)	17,459	12,070	5,389 (30.9%)	
±10min(21 copies)	17,459	12,056	5,403 (30.9%)	0.26%
±5min(3 copies)	17,459	14,069	3,390 (19.4%)	
±5min(11 copies)	17,459	14,058	3,401 (19.5%)	0.28%

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 - **Summary of Contributions**
 - **Future Research Directions**

Summary of Contributions

- ❑ **Provide alternative definitions for robustness in the context of airline schedule planning**
- ❑ **Develop an optimization model and solution approach that can generate aircraft maintenance routes to minimize delay propagation**
- ❑ **Develop optimization models and solution approach to minimize the expected total number of passengers missing connections, and analyze the model properties**
- ❑ **Proof-of-concept results show that these approaches are promising**
- ❑ **Develop integrated models for more robustness**

Future Research Directions

□ Integrated Models

- Integrated robust aircraft maintenance routing with fleet assignment
- Robust aircraft maintenance routing with time window
- Integrated flight schedule re-timing with FAMTW

□ Other approaches

- Fleet assignment with minimal expected cost
- Fleet assignment under demand uncertainty
- Aircraft routes with swap opportunities
- Aircraft routes with short cycles



Computational Results

□ July 2000 data

- Assume 25 minute minimum connecting time

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	14,199	9,866	4,333	30.5%
±10min(5 copies)	14,199	10,778	3,421	24.1%
±5min(3 copies)	14,199	12,026	2,173	15.3%

- Assume 20 minute minimum connecting time

Time window	Tot num of D-pax	Output	D-pax reduced	D-pax reduced (%)
±15min(7 copies)	12,090	9,148	2,942	24.3%
±10min(5 copies)	12,090	9,812	2,278	18.8%
±5min(3 copies)	12,090	10,767	1,323	10.9%

Impact on Passengers

❑ **Disruptions calculated at the flight level**

- If a flight was cancelled, all passengers on that flight is disrupted
- If actual departure time of flight B – actual arrival time of flight A < minimum connecting time → all passengers connecting from A to B are disrupted

❑ **Number of disrupted passengers only calculated for connections between flights that both have ASQP records**

- ASQP has records only for domestic flights flown by jet airplanes and major airlines
- Actual departure and arrival times for flights without ASQP records are unknown → Assume no disruptions for these flights

❑ **Passengers only counted as disrupted once**

- If passenger is disrupted on any flight leg of itinerary, passenger not counted as disrupted on the following flight legs

Passenger Delays and Disruptions

□ Passenger delays

- the difference between scheduled and actual arrival time at passengers' destination

□ Passengers are disrupted if their planned itineraries are infeasible

□ Flight delay and passenger delay (Bratu and Barnhart, 2002)

	Ave. delay	% Pax	% Total pax delay
Disrupted pax	419 min	4%	51%
Nondisrupted pax	14 min	96%	49%
All pax	31 min		
Flights	16 min		

Passenger Disruption

□ **Disrupted passengers**

- Significant numbers: 4% → 20-30 million in U.S.
- Experience very long delay
- Contribute to more than half of the total passenger delay
- Cause huge revenue loss
- Destroy airlines' image

□ **Reduce disrupted passengers**

- Passenger delay caused by disruption is the most critical part
- Hard to determine the delays for each disrupted passengers
- ➔ Minimize number of disrupted passengers

LP Solution

□ Algorithm for LP relaxation

- Step 0: Create initial feasible solution
- Step 1: Solve the restricted master problem (RMP)
 - Find optimal solution to RMP with a subset of all strings
- Step 2: Solve the pricing problem
 - Generate strings with negative reduced cost
 - If no string is generated, stop: the LP is solved
- Step 3: Construct a new restricted master problem
 - Add the strings generated
 - Go to step 1

Notation

- S : set of feasible strings
- F : set of flights
- G : set of ground variables
- $S_i^- (S_i^+)$: set of strings ending (starting) with flight i
- x_s : binary decision variable for each feasible string s
- y : integer variable to count number of aircraft on the ground at maintenance stations
- $y_{i,d}^- (y_{i,d}^+)$: number of aircraft on the ground before (after) flight i departs at the maintenance station from which flight i departs
- $y_{i,a}^- (y_{i,a}^+)$: number of aircraft on the ground before (after) flight i arrives at the maintenance station from which flight i arrives

Notation (Cont.)

- pd_{ij}^s : propagated delay from flight i to flight j if flight i and flight j are in string s
- a_{is} : indicator variable, equals 1 if flight i is in string s , and equals 0 otherwise
- r_s : number of times string s crosses the count time, a single point time at which to count aircraft
- p_g : number of times ground arc g crosses the count time
- N : number of planes available.

Data

- **Airline Service Quality Performance (ASQP) provides good source of delay information**
- **ASQP provides flight operation information:**
 - For all domestic flights served by jet aircraft by major airlines in U.S.
 - Planned departure time and arrival time, actual departure time and arrival time (including wheels-off and wheels-on time, taxi-out and taxi-in time, airborne time)
 - Aircraft tail number for each flight
 - Cancelled flights (reasons for cancellation, and aircraft tail number are not available)

Effect of Cancellations

- **For cancelled flights in the historical data**
 - we don't know which aircraft supposed to fly them
 - We don't have the delay information
 - We assume the propagated delays for these flights are zero
- **Lower cancellation rates**
 - Less passengers disrupted because of cancellation
 - More passengers disrupted because of flight delays
- **7 days in Aug 2000 with very few cancellations (cancellation rate = 0.19%)**
 - For Aug 2000, 65% of disrupted passengers are disrupted because of flight delays
 - For 7 selected days in Aug 2000, 92% of disrupted passengers are disrupted because of flight delays

Results - Low Cancellation Days

- **Passenger disruptions for 7 selected days in Aug 2000 with very few cancellations**

Network	D-pax Reduced	Total Num D-pax	D-pax Reduced (%)
N1	8	51	13,6%
N2	45	209	17,7%
N3	6	197	3,0%
N4	100	455	18,0%
Total	159	912	14,8%

- **Reduction in number of disrupted passengers per non-cancelled flights is same as that for entire month**

Extensions

- **Combine with scheduling**

- More slacks may be added → further reduce delay propagation

- **Combine with fleet assignment**

- Need to determine cost for propagated delay
- More feasible strings → better solution
- Minimum turn time is a function of fleet type

- **Integrate with fleet assignment and schedule generation**