1.206J/16.77J/ESD.215J
Airline Schedule Planning

Cynthia Barnhart

Spring 2003

1.206J/16.77J/ESD.215J
The Passenger Mix Problem

Outline

- Definitions
- Formulations
- Column and Row Generation
- Solution Approach
- Results
- Applications and Extensions

Some Basic Definitions

- Market
  - An origin-destination airport pair, between which passengers wish to fly one-way
    - BOS-ORD and ORD-BOS are different
- Itinerary
  - A specific sequence of flight legs on which a passenger travels from their ultimate origin to their ultimate destination
- Fare Classes
  - Different prices for the same travel service, usually distinguished from one another by the set of restrictions imposed by the airlines

Some More Definitions

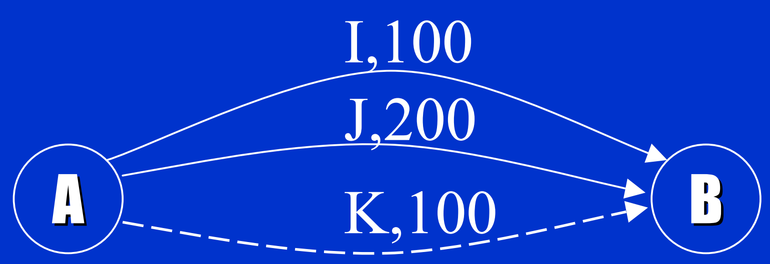
- Spill
  - Passengers that are denied booking due to capacity restrictions
- Recapture
  - Passengers that are recaptured back to the airline after being spilled from another flight leg

Problem Description

- Given
  - An airline's flight schedule
  - The unconstrained demand for all itineraries over the airline's flight schedule
- Objective
  - Maximize revenues by intelligently spilling passengers that are either low fare or will most likely fly another itinerary (recapture)
    - Equivalent to minimize the total spill costs

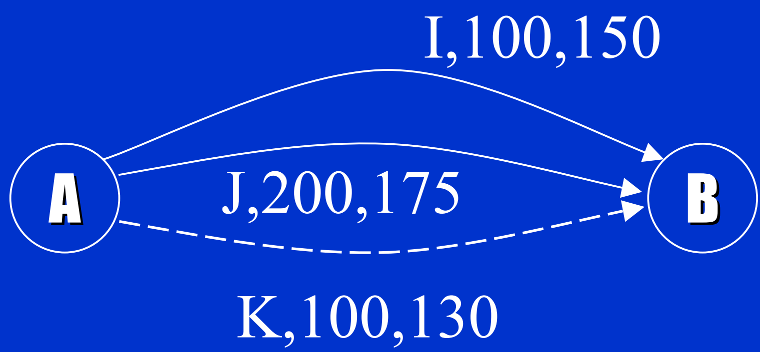
Example

- One market, 3 itineraries
- Unconstrained demand per itinerary
  - Total demand for an itinerary when the number of seats is unlimited



Example with Capacity Constraints

- One market, 3 itineraries
  - Capacity on itinerary I = 150
  - Capacity on itinerary J = 175
  - Capacity on itinerary K = 130
- Optimal solution:
  - Spill __0__ from I
  - Spill __25__ from J
  - Spill __0__ from K



Revenue Management: A Quick Look

- One flight leg
  - Flight 105, LGA-ORD, 287 seats available
- Two fare classes:
  - Y: High fare, no restrictions
  - M: Low fare, many restrictions
- Demand for Flight 105
  - Y class: 95 with an average fare of $400
  - M class: 225 with an average fare of $100
  - Optimal Spill Solution (0 Y and 33 M passengers)
    - Revenue: $ 95*400 + 192*100
    - Spill: $ 33*100

Network Revenue Management

- Two Flights
  - Flight 105, LGA-ORD, 287 seats
  - Flight 201, ORD-SFO, 287 seats
- Demand (one fare class)
  - LGA-ORD, 225 passengers $100
  - ORD-SFO, 150 passengers $150
  - LGA-SFO, 150 passengers, $225
- Optimal Solution: $ 150*100+150*150+137*225
  - LGA-ORD, 150 passengers
  - ORD-SFO, 150 passengers
  - LGA-SFO, 137 passengers

Quantitative Share Index or Quality of Service Index (QSI): Definition

- Quantitative Share Index or Quality of Service Index (QSI)
  - There is a QSI for each itinerary i in each market m for each airline a, denoted $QSI_{i(m)}^{a}$
  - The sum of $QSI_{i(m)}^{a}$ over all itineraries i in a market m over all airlines a is equal to 1, for all markets m

Market Share

- The market share of airline a in market m is the sum of $QSI_{i(m)}^{a}$ over all itineraries i in market m
- The market share of the competitors of airline a in market m is $1 -$ (the sum of $QSI_{i(m)}^{a}$ over all itineraries i in market m)
  - Denote this as msc_{m}^{a}

Recapture

- Consider a passenger who desires itinerary *I* but is redirected (spilled) to itinerary *J*
  - The passenger has the choice of accepting *J* or not (going to a competitor)
  - Probability that passenger will accept *J* (given an uniform distribution) is the ratio of $QSI_{J(m)}{}^a$ to $(QSI_{J(m)}{}^a + msc_m{}^a)$
- Probability that passenger will *NOT* accept *J* (given an uniform distribution) is the ratio of $msc_m{}^a$ to $(QSI_{J(m)}{}^a + msc_m{}^a)$
  - The ratios sum to 1
  - If *a* is a monopoly, recapture rate will equal 1.0

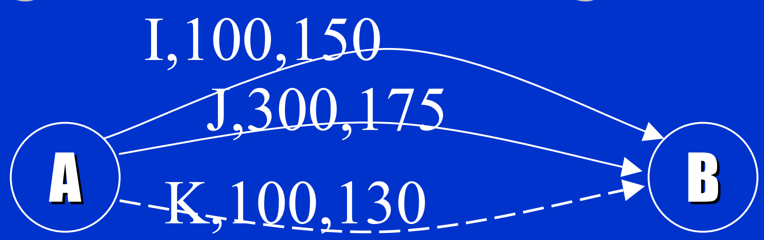
Recapture Calculation

- Recapture rates for airline *a*:
  - b_I^J: probability that a passenger spilled from I will accept a seat on J, <u>if one exists</u>
  - QSI mechanism for computing recapture rates

$$b_r^p = \frac{QSI_p^{\ a}}{msc_m^{\ a} + QSI_p^{\ a}}$$

Example with Recapture

- Recapture rates:
  - $b_I^J = 0.4$, $b_I^k = 0.1$
  - $b_J^I = 0.5$, $b_J^K = 0.1$
  - $b_K^I = 0.5$, $b_K^J = 0.4$
- Assume all itineraries have a single fare class, and their fares are all equal
- Optimal solution:
  - Spill ___0___ from I to J, Spill ___0___ from I to K
  - Spill ___100___ from J to I, Spill ___25___ from J to K
  - Spill ___0___ from K to I, Spill ___0___ from K to J



Mathematical Model: Notation

- Decision variables
  - x_p^r - the number of passengers that desire travel on itinerary *p* and then travel on itinerary *r*
- Parameters and Data
  - $fare_p$ - the average fare for itinerary *p*
  - D_p - the daily unconstrained demand for itinerary *p*
  - CAP_i - the capacity on flight leg i
  - b_p^r - the recapture rate of a passenger desiring itinerary *p* who is offered itinerary *r*

$$\delta_i^p = \begin{cases} 1 & \text{if flight leg } i \text{ is on itinerary } p \\ 0 & \text{otherwise} \end{cases}$$

Basic Formulation

$$\text{Maximize} \quad \sum_{p \in P} \sum_{r \in P} fare_r \, x_p^r$$

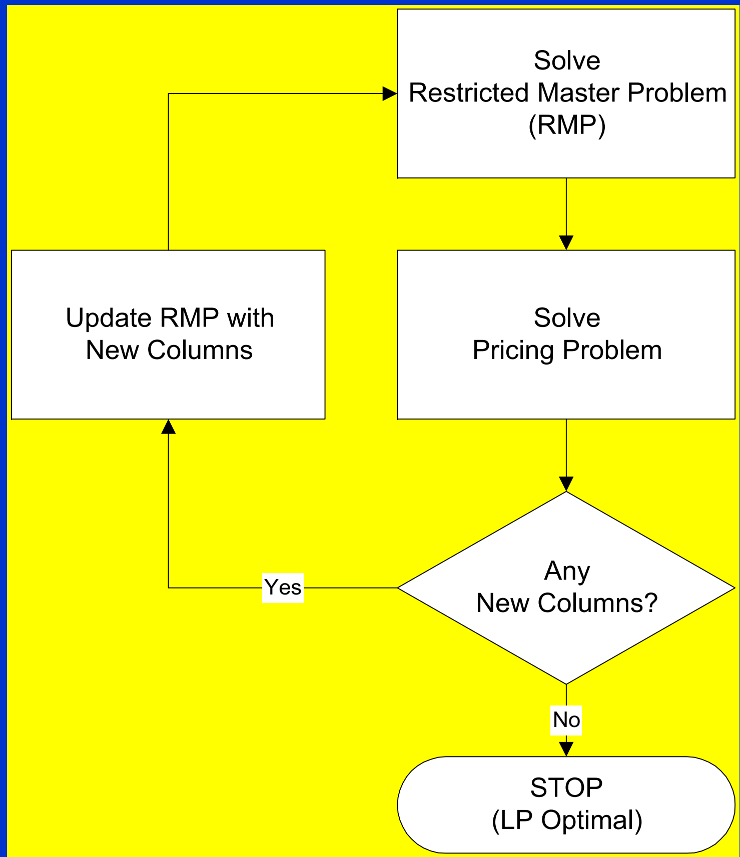
$$\text{subject to:}$$

$$\sum_{p \in P} \sum_{r \in P} \delta_i^r x_p^r \leq CAP_i \qquad \forall i \in L$$

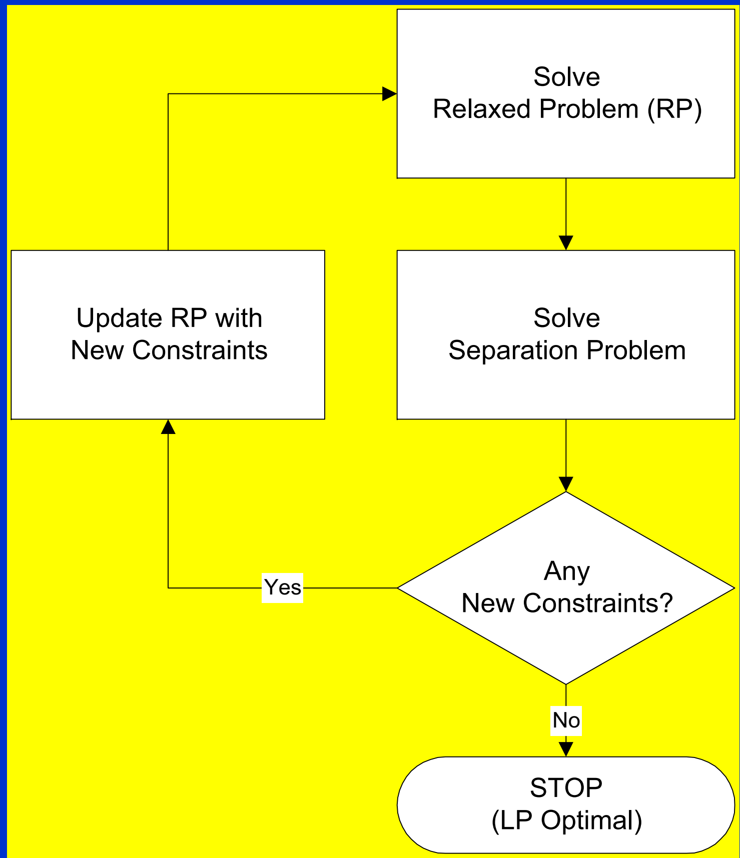
$$\sum_{r \in P} x_p^r / b_p^r \leq D_p \qquad \forall p \in P$$

$$x_p^r \geq 0 \qquad \forall p, r \in P$$

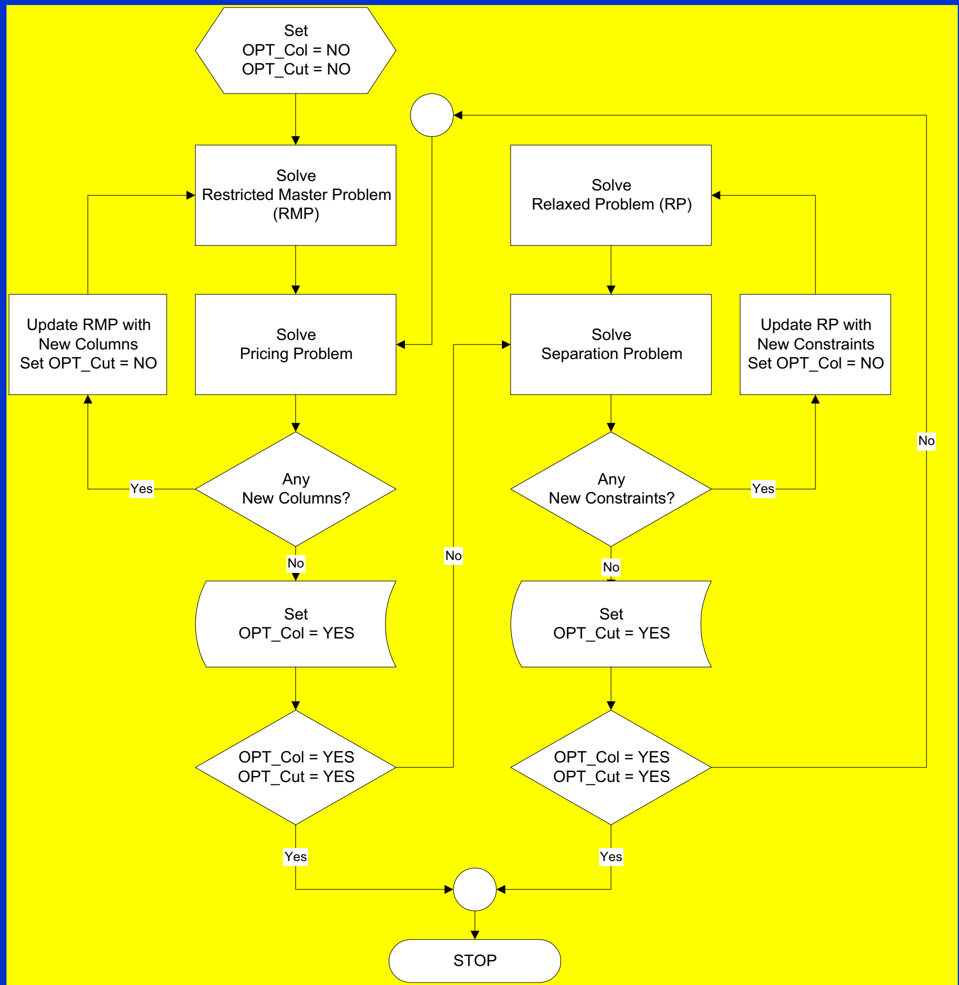
Column Generation



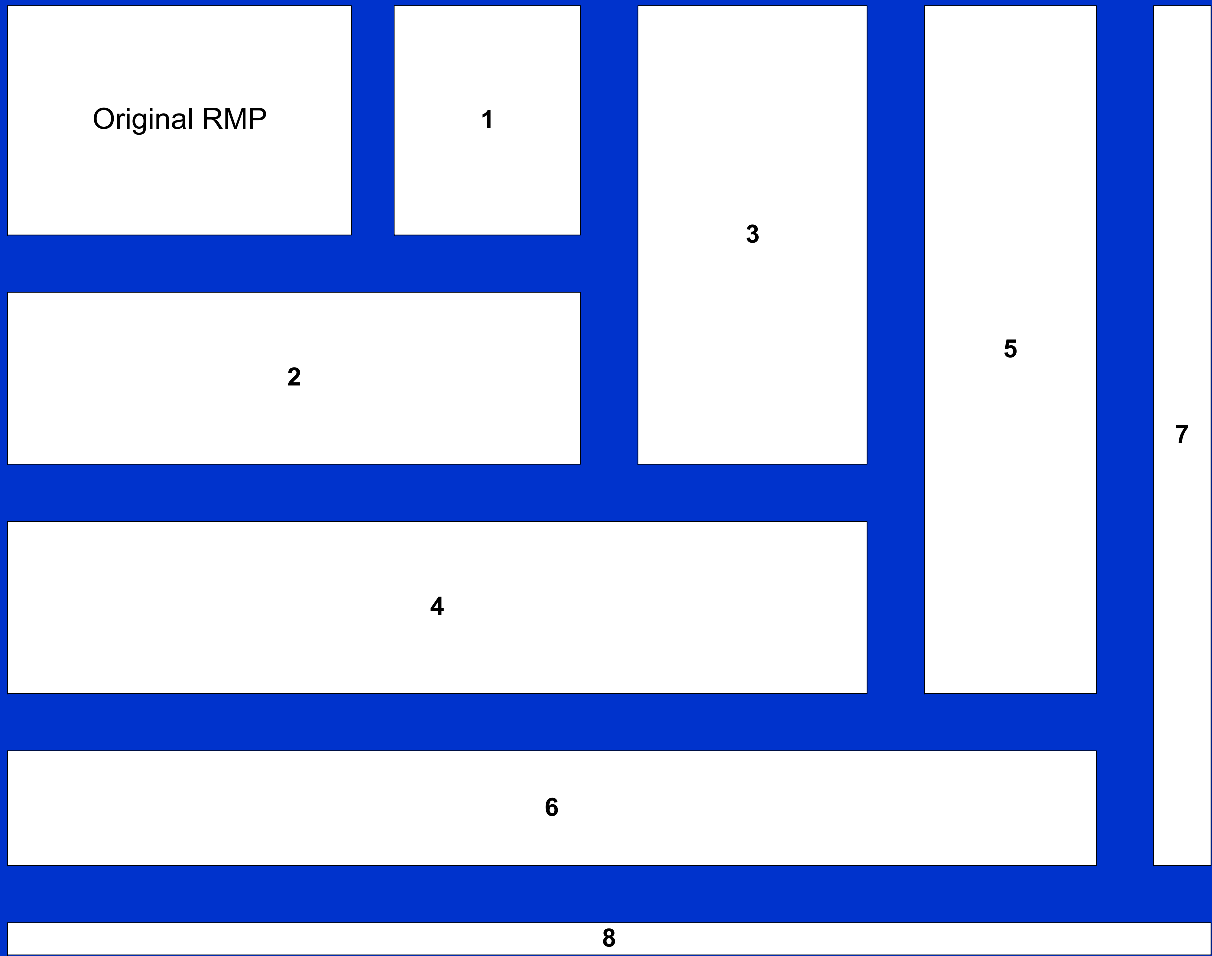
Row Generation



Column and Row Generation



Column and Row Generation: Constraint Matrix



The Keypath Concept

- Assume most passengers flow along their desired itinerary
- Focus on which passengers the airline would like to redirect on other itineraries
- New decision variable

t_p^r
  - The number of passengers who desire travel on itinerary *p*, but the airline attempts to redirect onto the itinerary *r*
- New Data

Q_i
  - The unconstrained demand on flight leg *i*

$$Q_i = \sum_{p \in P} \delta_i^p D_p$$

The Keypath Formulation

$$\text{Min} \quad \sum_{p \in P} \sum_{r \in P} \left(fare_p - b_p^r \, fare_r \right) t_p^r$$

s.t. :

$$\sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_r^p t_r^p \geq Q_i - CAP_i \qquad \forall i \in L$$

$$\sum_{r \in P} t_p^r \leq D_p \qquad \forall p \in P$$

$$t_p^r \geq 0$$

Change of variable Relationship

$$\boldsymbol{x_p^p = D_p - \sum_{r \neq p} t_p^r}$$

$$\boldsymbol{x_p^r = b_p^r t_p^r}$$

The Benefit of the Keypath Concept

- We are now minimizing the objective function and most of the objective coefficients are __positive__. Therefore, this will guide the decision variables to values of __0__.
- How does this help?

Solution Procedure

- Use Both Column Generation and Row Generation
- Actual flow of problem
  - Step 1- Define RMP for Iteration 1: Set k =1. Denote an initial subset of columns (A_1) which is to be used.
  - Step 2- Solve RMP for Iteration k: Solve a problem with the subset of columns A_k.
  - Step 3- Generate Rows: Determine if any constraints are violated and express them explicitly in the constraint matrix.
  - Step 4- Generate Columns: Price some of the remaining columns, and add a group (A*) that have a reduced cost less than zero, i.e., $A_{k+1}=[A_k \mid A^*]$
  - Step 5- Test Optimality: If no columns or rows are added, terminate. Otherwise, k =k+1, go to Step 2

Column Generation

- There are a large number of variables
  - n_m is the number of itineraries in market m
  - Most of them aren't going to be considered
- Generate columns by explicit enumeration and "pricing out" of variables

Computing Reduced Costs

- The reduced cost of a column is

$$\bar{c}_p^r = fare_p - \sum_{i \in p} \pi_i - b_p^r \left(fare_r - \sum_{j \in r} \pi_j \right) + \sigma_p$$

where π_i is the non-negative dual cost associated with flight leg *i* and σ_p is the non-negative dual cost associated with itinerary *p*

Solving the Pricing Problem (Column Generation)

- Can the column generation step be accomplished by solving shortest paths on a network with "modified" arc costs, or some other polynomial time algorithm?
  - Hint: Think about fare structure
  - What are the implications of the answer to this question?

Computational Experience

- Current United Data
  - Number of Markets -15,678
  - Number of Itineraries - 60,215
  - Maximum number of legs in an itinerary- 3
  - Maximum number of itineraries in a market- 66
  - Flight network (# of flights)- 2,037
- Using CPLEX, we solved the above problem in roughly 100 seconds, generating just over 100,000 columns and 4,100+ rows.

Applications: Irregular Operations

- When flights are cancelled or delayed
  - Passenger itineraries are cancelled
  - Passenger reassignments to alternative itineraries necessary
    - Flight schedule and fleet assignments (capacity) are known
    - Objective might be to minimize total delays or to minimize the maximum delay beyond schedule
- How are recapture rates affected by this scenario?
- How would the passenger-mix model have to be altered for this scenario?

Extensions: Yield Management

- Can the passenger mix problem be used as a tool for yield management?
- What are the issues?
  - Deterministic vs. stochastic
  - Sequence of requests
  - Small demands (i.e., quality of data)
- Advantages
  - Shows the expected makeup of seat allocation
  - Takes into account the probability of recapturing spilled passengers
  - Gives ideas of itineraries that should be blocked
  - Dual prices might give us ideas for contributions, or displacement costs

Extensions: Fleet Assignment

- To be explained in the next lectures…