

1.206J/16.77J/ESD.215J
Airline Schedule Planning

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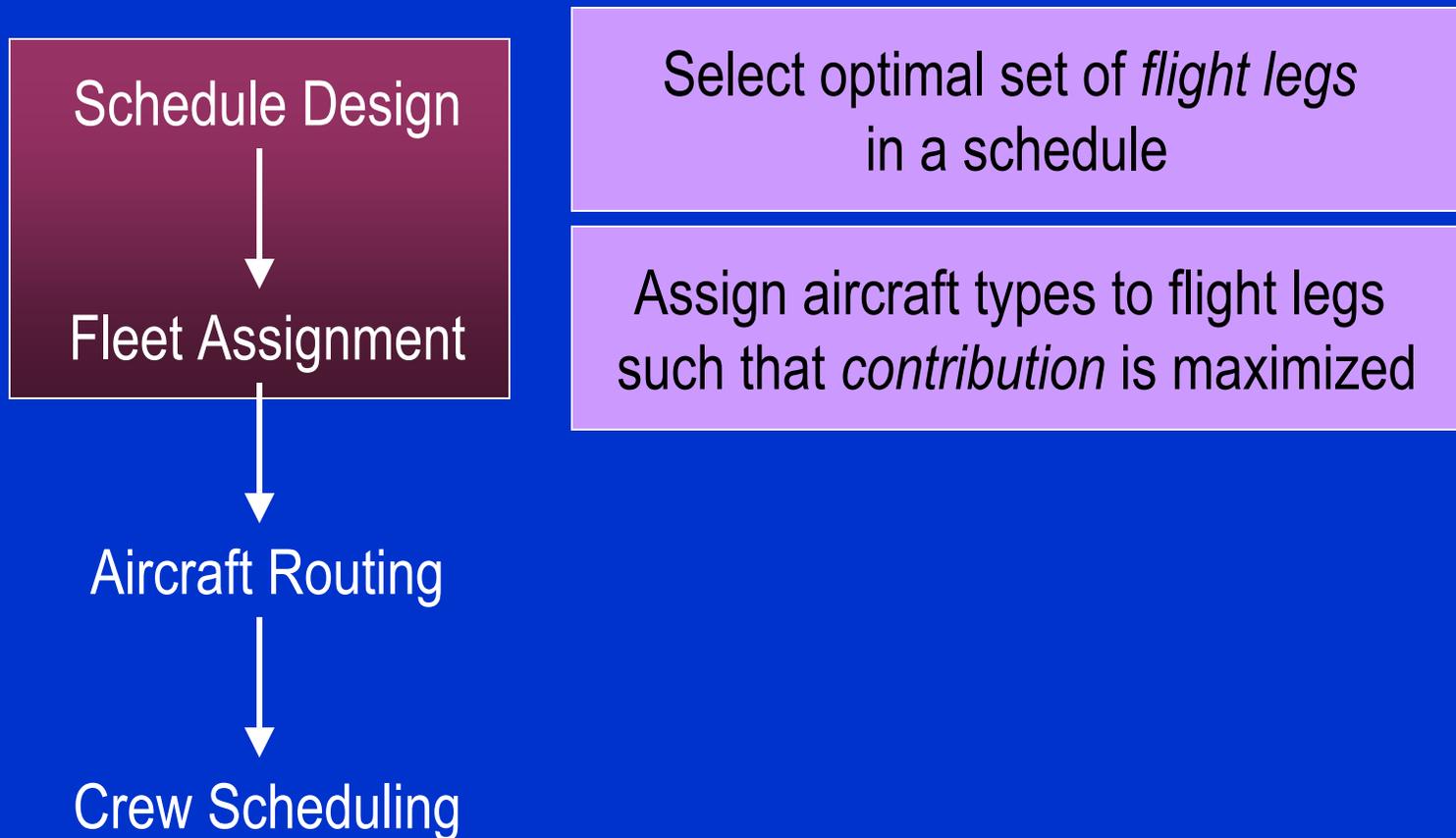
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The Schedule Design Problem

- Outline
 - Problem Definition and Objective
 - Schedule Design with Constant Market Share
 - Schedule Design with Variable Market Share
 - Schedule Design Solution Algorithm
 - Results
 - Next Steps
 - A Look to the Future in Airline Schedule Optimization

Airline Schedule Planning



Objectives

- Given origin-destination demands and fares, fleet composition and size, fleet operating characteristics and costs
- Find the revenue maximizing flight schedule

Schedule Design: Fixed Flight Network, Flexible Schedule Approach

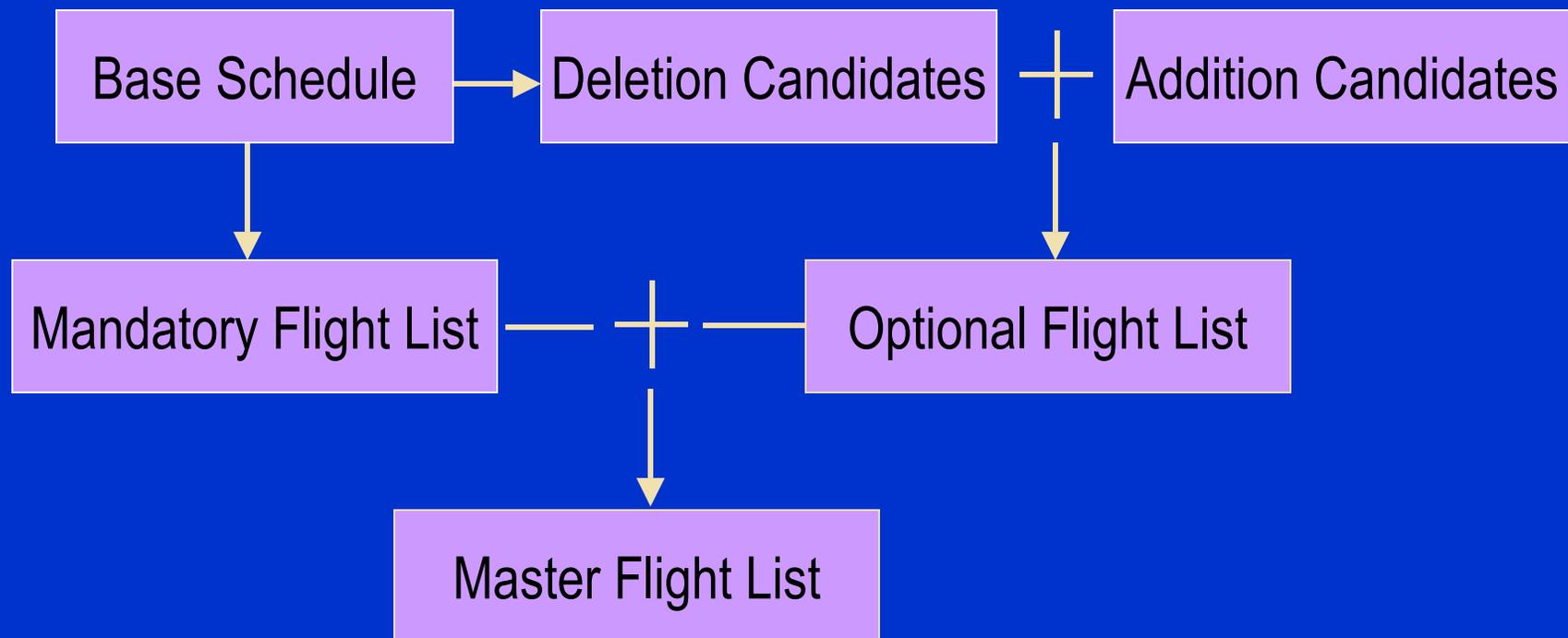
- Fleet assignment model with time windows
 - Allows flights to be re-timed slightly (plus/minus 10 minutes) to allow for improved utilization of aircraft and improved capacity assignments
- Initial step in integrating flight schedule design and fleet assignment decisions

Schedule Design: Optional Flights, Flexible Schedule Approach

- Fleet assignment with “optional” flight legs
 - Additional flight legs representing varying flight departure times
 - Additional flight legs representing new flights
 - Option to eliminate existing flights from future flight network

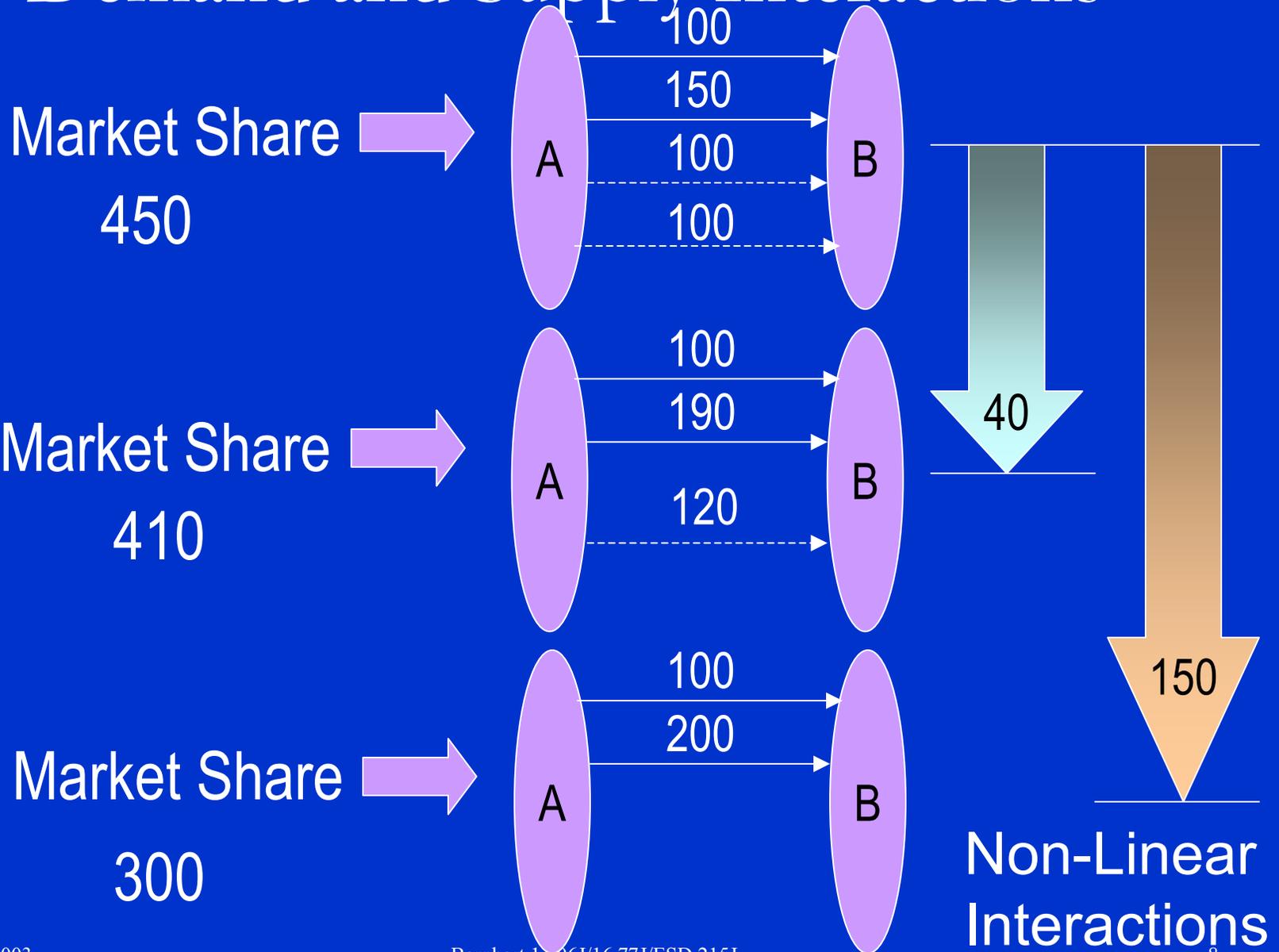
➤ Incremental Schedule Design

Integrated, Incremental Schedule Design and Fleet Assignment Models



Select optimal set of flight legs from master flight list
Assign fleet types to flight legs

Demand and Supply Interactions

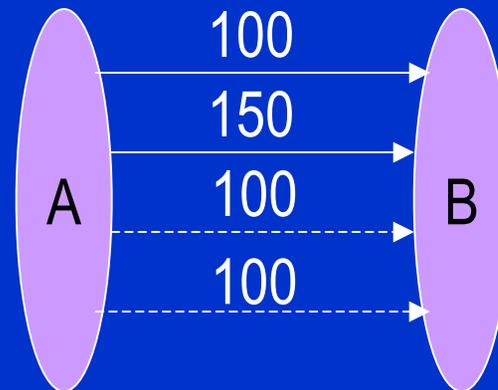
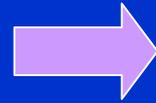


Schedule Design: Constant Market Share Model

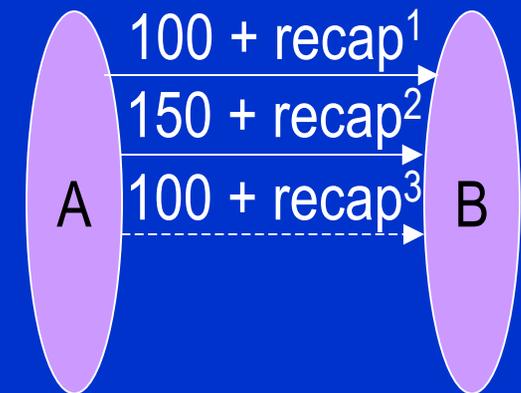
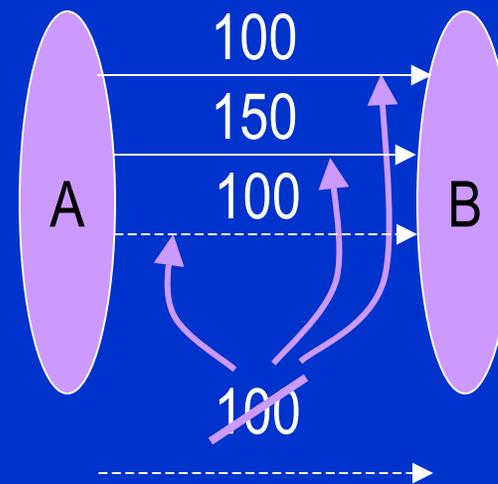
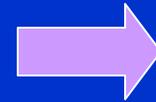
- Constant market share model
 - Integrated Schedule Design and Fleet Assignment Model (ISD-FAM)
 - Utilize recapture mechanism to adjust demand approximately

ISD-FAM: Example

Market Share
450



Market Share
450



ISD-FAM Formulation

$$\text{Min } \sum_{k \in K} \sum_{i \in L} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\text{fare}_p - b_p^r \text{fare}_r) t_p^r$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L^F$$

$$\sum_{k \in K} f_{k,i} \leq 1 \quad \forall i \in L^O$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k, o, t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \quad \forall k \in K$$

$$\sum_k f_{k,i} \text{SEATS}_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \geq Q_i \quad \forall i \in L$$

$$\sum_{r \in P} t_p^r \leq D_p \quad \forall p \in P$$

$$t_p^r \geq 0 \quad f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0$$

ISD-FAM Formulation

+

Flight Selection

FAM

PMM

$$t_p^r \geq 0 \quad f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0$$

ISD-FAM Formulation

Schedule Design

Fleet Assignment

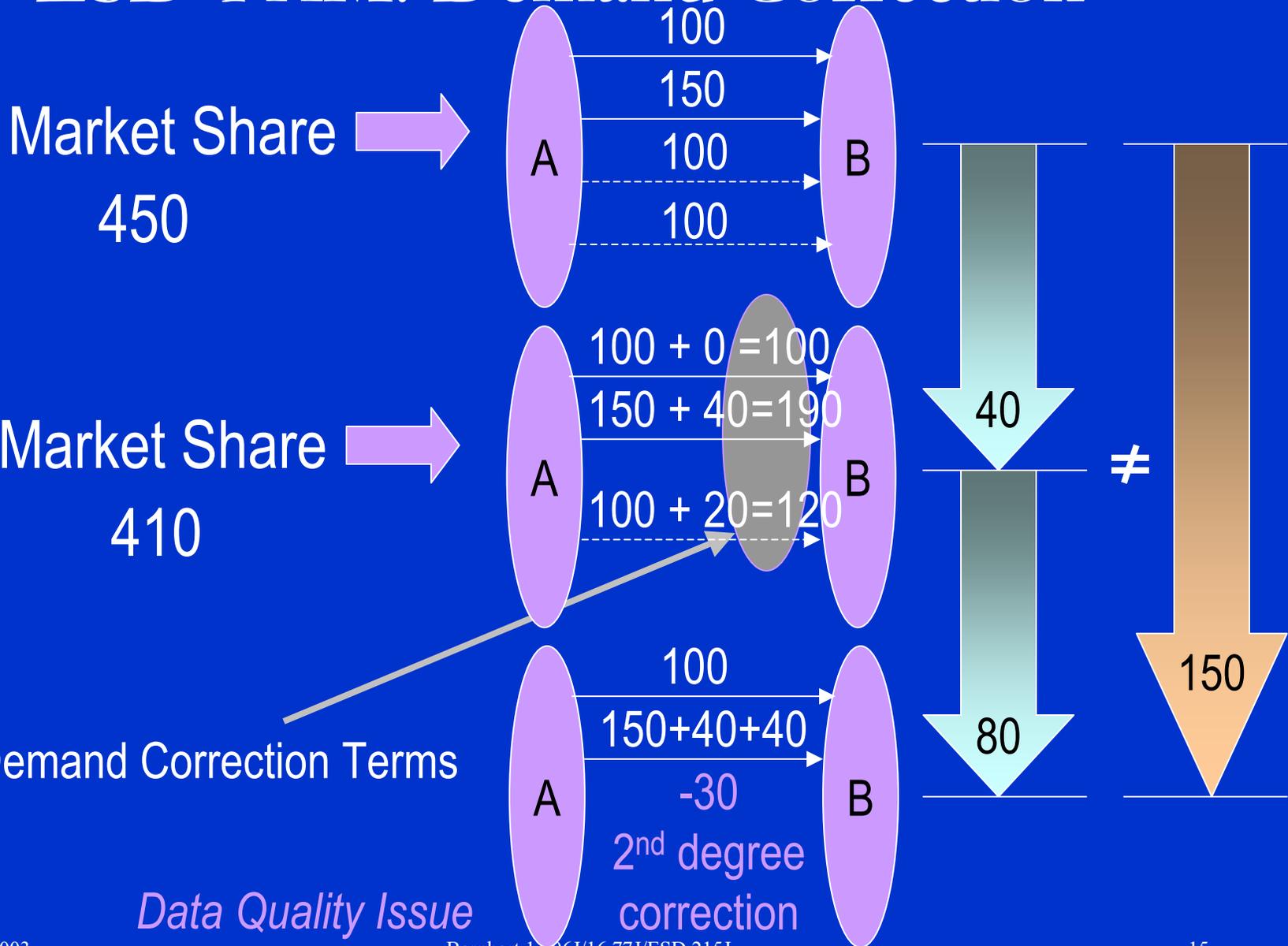
Spill + Recapture

$$t_p^r \geq 0 \quad f_{k,i} \in \{0,1\} \quad y_{k,o,t} \geq 0$$

Schedule Design: Variable Market Share Model

- Variable market share model
 - Extended Schedule Design and Fleet Assignment Model (ESD-FAM)
 - Utilize demand correction term to adjust demand explicitly

ESD-FAM: Demand Correction



ESD-FAM Formulation

$$\text{Min} \sum_{k \in K} \sum_{i \in L} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (\text{fare}_p - b_p^r \text{fare}_r) t_p^r + \sum_{q \in P^0} \left(\text{fare}_q D_q - \sum_{p \in P: p \neq q} \text{fare}_p \Delta D_q^p \right) (1 - Z_q)$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \quad \forall i \in L^F$$

$$\sum_{k \in K} f_{k,i} \leq 1 \quad \forall i \in L^O$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \quad \forall k, o, t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \quad \forall k \in K$$

$$\sum_{p \in P} \sum_{q \in P^0} \delta_i^p \Delta D_q^p (1 - Z_q) + \sum_{k \in K} f_{k,i} SEATS_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \geq Q_i \quad \forall i \in L$$

$$\sum_{q \in P^0} \Delta D_q^p (1 - Z_q) + \sum_{r \in P} t_p^r \leq D_p \quad \forall p \in P$$

$$Z_q - \sum_{k \in K} f_{k,i} \leq 0 \quad \forall i \in L(q)$$

$$Z_q - \sum_{i \in L(q)} \sum_{k \in K} f_{k,i} \geq 1 - N_q \quad \forall q \in P^0$$

$$f_{k,i} \in \{0,1\} \quad Z_q \in \{0,1\} \quad t_p^r \geq 0 \quad y_{k,o,t} \geq 0$$

ESD-FAM Formulation

ISD-FAM

Market Share Adjustment

$$f_{k,i} \in \{0,1\} \quad Z_q \in \{0,1\} \quad t_p^r \geq 0 \quad y_{k,o,t} \geq 0$$

ESD-FAM Formulation

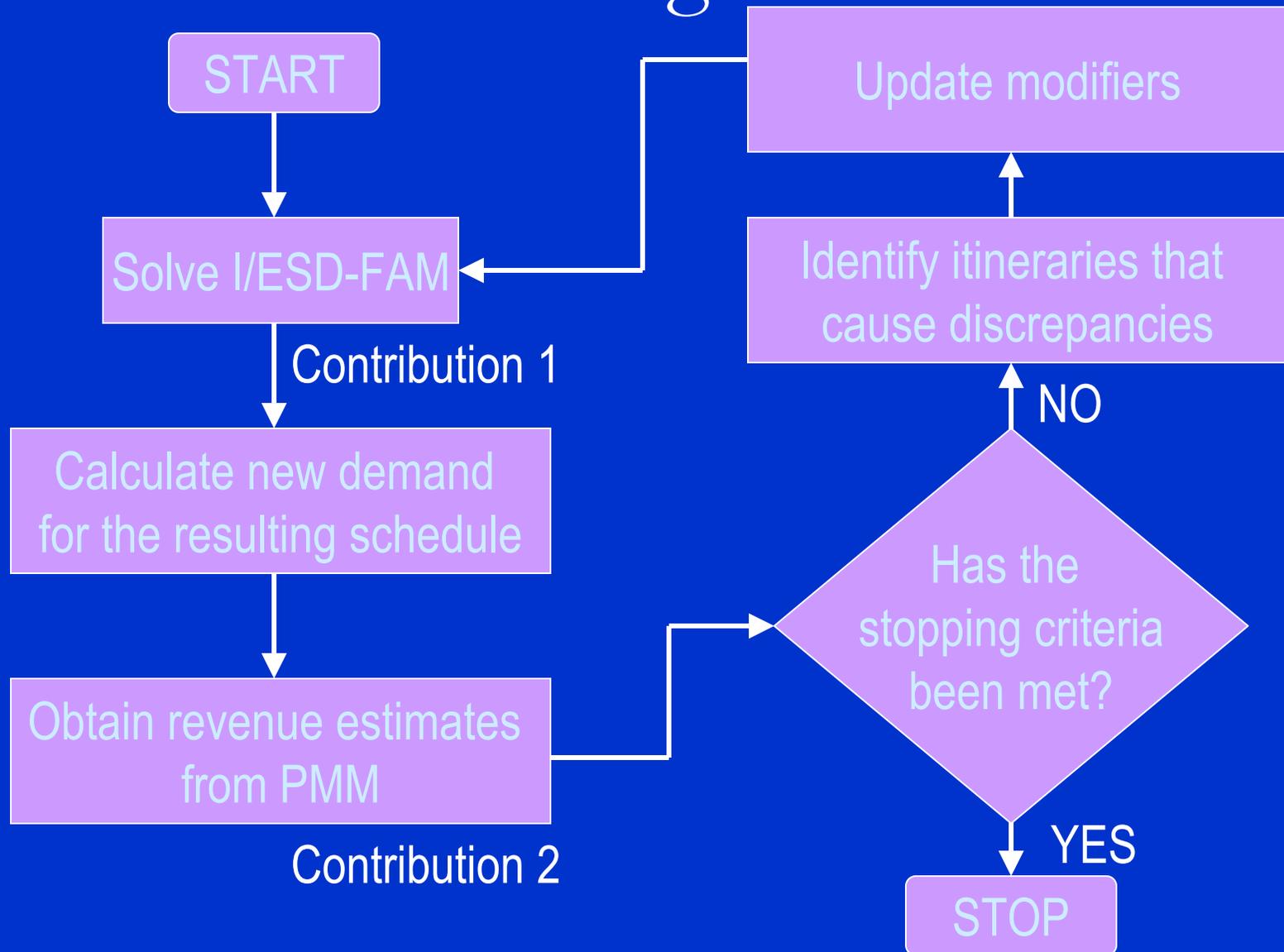
**Constant
Market Share**

**Schedule Design
& Fleet Assgn.**

Market Share Adjustment

$$J_{k,i} \in \{0,1\} \quad L_q \in \{0,1\} \quad v_p \geq 0 \quad J_{k,0,i} = 0$$

Solution Algorithm



State Of The Practice/ Theory

Practice:

- Most schedule decisions made without optimization
- At least one major airline uses Fleet Assignment with Time Windows
- Implementation of Incremental Schedule Design approach underway at a major airline

Theory:

- Models and algorithms for incremental schedule design have been developed and prototyped
- Validation in progress

Computational Experiences

- ISD-FAM requires long runtimes and large amounts of memory
 - ~ 40 minutes on a workstation class computer for medium size (800 legs) schedules
 - ~ 20 hours on a 6-processor workstation, running parallel CPLEX for full size (2,000 legs) schedules
- ESD-FAM takes even longer runtimes and exhausts the memory in some cases
 - 40 mins (ISD-FAM) vs. 12 hrs (ESD-FAM) on same medium size schedule

Schedule Design: Results

- Demand and supply interactions
 - ESD-FAM captures interactions more accurately
- Resulting schedules operate fewer flights
 - Lower operating costs
 - Fewer aircraft required
- ~\$100 - \$350 million improvement annually
 - Compared to *planners' schedules*
 - Exclude benefits from saved aircraft

Schedule Design Results

- Results are subject to several caveats
 - Plans are often disrupted
 - Competitors' responses
 - Underlying assumptions
 - Deterministic demand
 - Optimal control of passengers
 - Demand forecast
 - Recapture rates/Demand correction terms
- Nonetheless, significant improvements are achievable

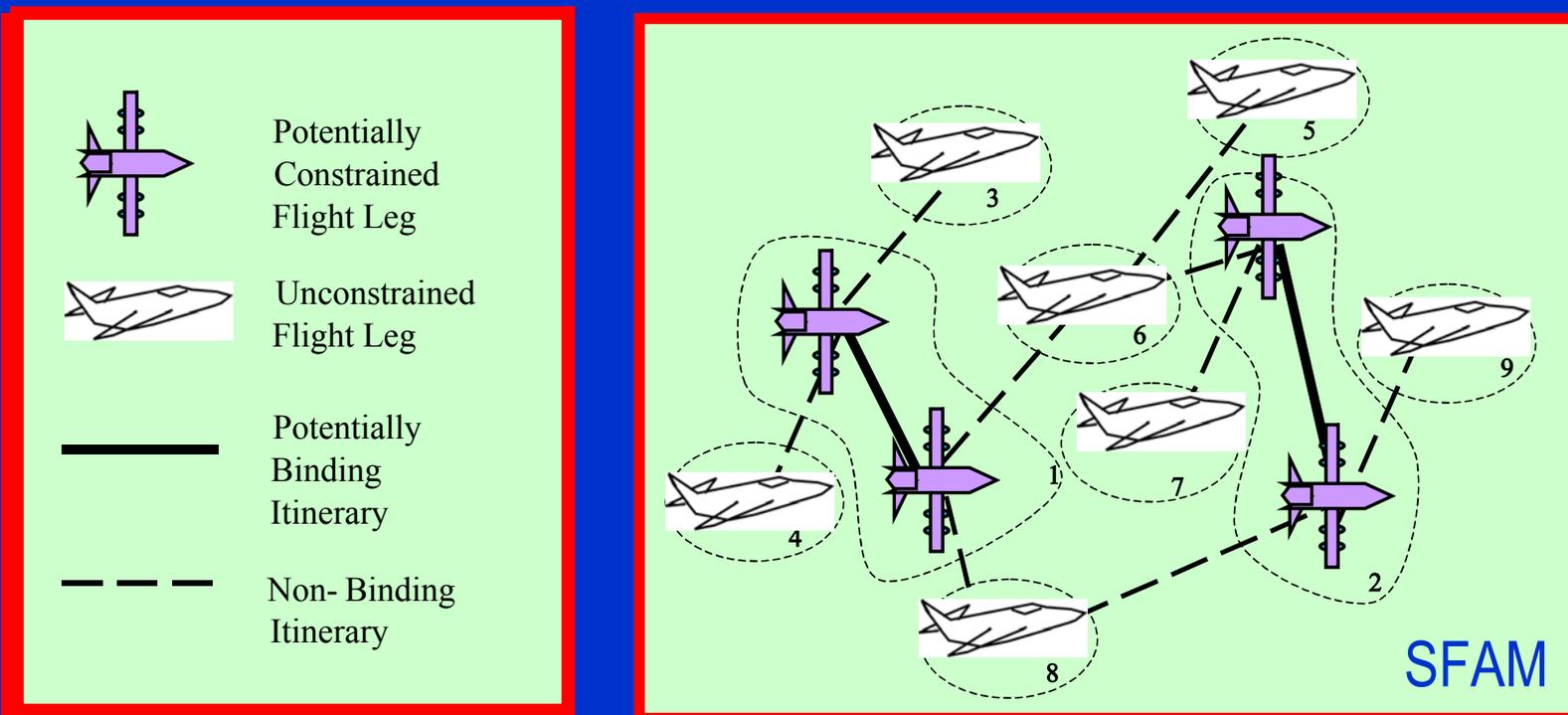
Potential for Improved Results

- Replace IFAM with SFAM

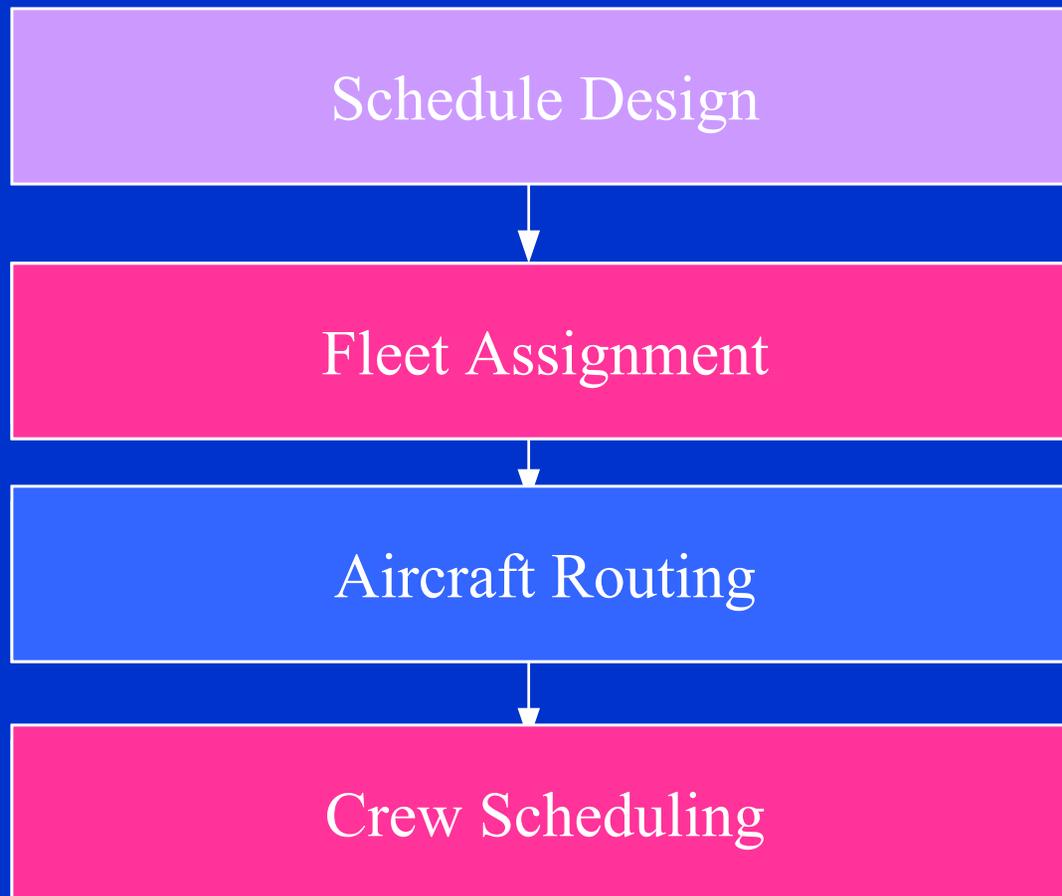
$$\begin{aligned}
 & \text{Min} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} (C_{\Pi^S}^m)_n (f_{\Pi^S}^m)_n \\
 \text{Subject to:} & \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} (\delta_{\Pi^S}^m)_n^i (f_{\Pi^S}^m)_n \leq 1 \quad \forall i \in L \\
 & y_{k,o,t^-} + \sum_{i \in I(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} (\kappa_{\Pi^S}^m)_n^{k,i} (f_{\Pi^S}^m)_n - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} (\kappa_{\Pi^S}^m)_n^{k,i} (f_{\Pi^S}^m)_n = 0 \quad \forall k,o,t \\
 & \sum_{o \in A} y_{k,o,t_n} + \sum_{i \in CL(k)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} (\gamma_{\Pi^S}^m)_n^k (f_{\Pi^S}^m)_n \leq N_k \quad \forall k \in K \\
 & (f_{\Pi^S}^m)_n \in \{0,1\} \quad y_{k,o,t} \geq 0
 \end{aligned}$$

SFAM Basic Concept

- Isolate network effects
 - Spill occurs only on *constrained legs*



A Look to the Future: Airline Schedule Planning Integration



Integrating crew scheduling and fleet assignment models yields:

- Additional 3% savings in total operating, spill and crew costs

- Fleeting costs increase by about 1%

- Crew costs decrease by about 7%

A Look to the Future: Real-time Decision Making

- For a typical airline, about 10% of scheduled revenue flights are affected by irregularities (like inclement weather, maintenance problems, etc.)
- According to the *New York Times*, irregular operations (due mostly to weather) result in more than \$440 million per year in lost revenue, crew overtime pay, and passenger hospitality costs
- Increasing use and acceptance of optimization-based decision support tools for operations recovery

A Look to the Future: Robust Scheduling

- Issue: Optimizing “plans” results in minimized *planned* costs, not *realized* costs
 - Optimized plans have little *slack*, resulting in
 - Increased likelihood of plan “breakage” during operations
 - Fewer recovery options
- Challenge: Building “robust” plans that achieve minimal realized costs