1.206J/16.77J/ESD.215J
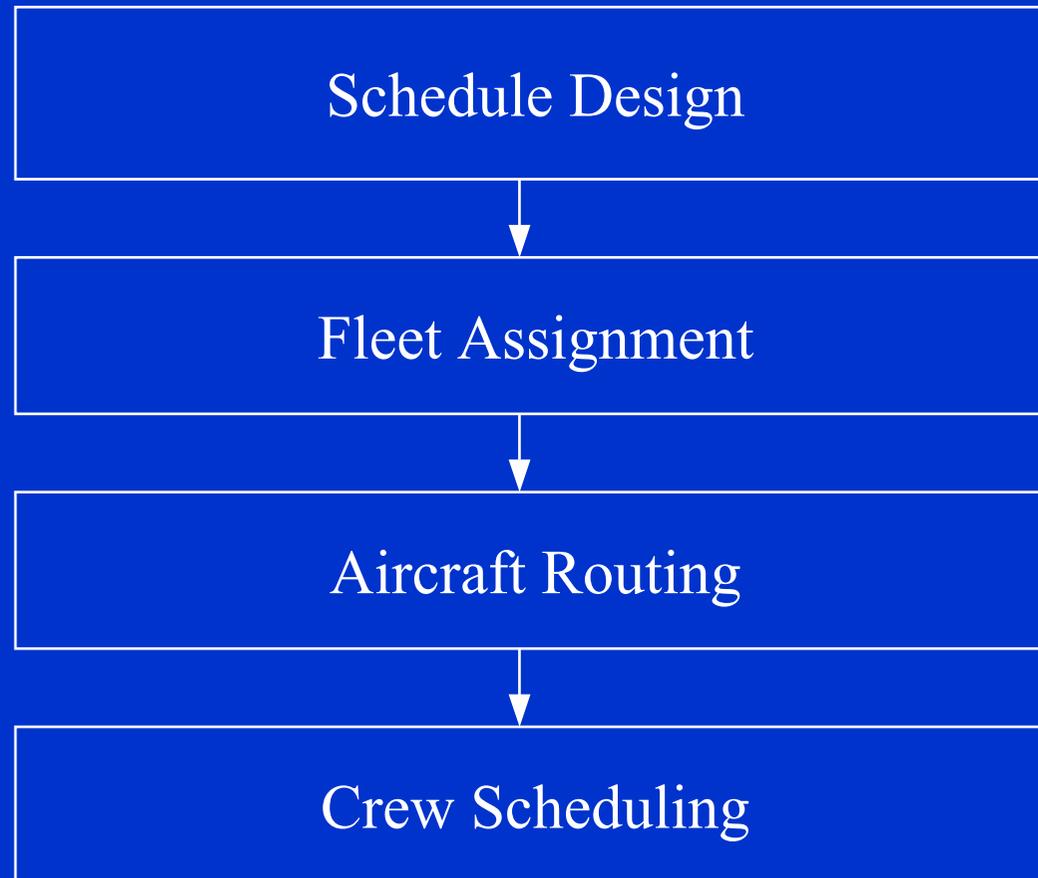# Airline Schedule Planning

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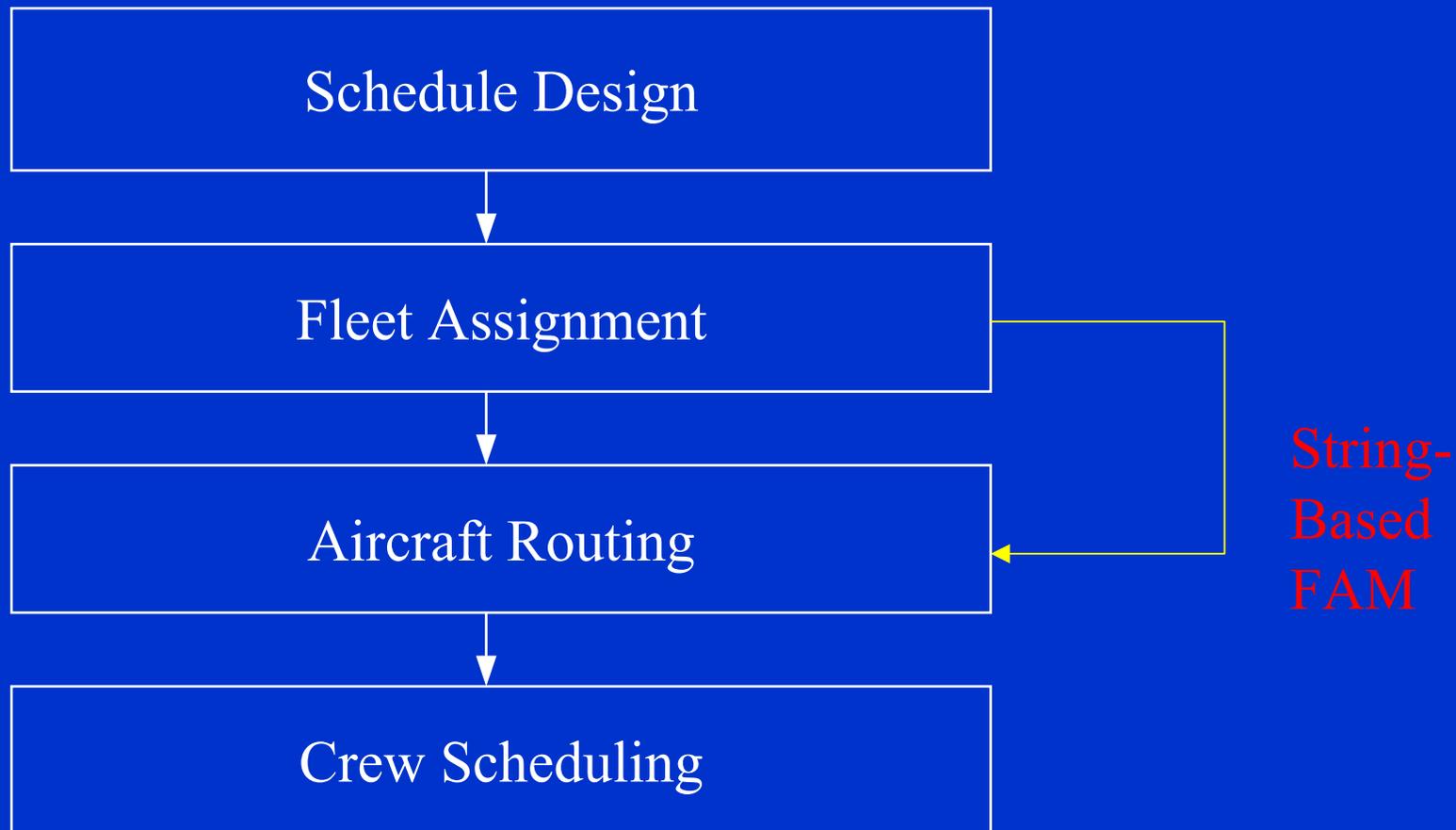# Aircraft Maintenance Routing

Outline

- Problem Definition and Objective

- Network Representation

- String Model

- Solution Approach

- Branch-and-price

- Extension: Combined Fleet Assignment and Aircraft Routing

Airline Schedule Planning

Schedule Design

↓

Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling

Airline Schedule Planning

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┌─────────────────────────────┐
│      Schedule Design         │
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│      Fleet Assignment        │────┐
└─────────────────────────────┘    │
              │                     │
              ▼                     │    String-
┌─────────────────────────────┐    │    Based
│      Aircraft Routing        │◄───┘    FAM
└─────────────────────────────┘
              │
              ▼
┌─────────────────────────────┐
│      Crew Scheduling         │
└─────────────────────────────┘
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Problem Definition

- Given:
    - Flight Schedule for a single fleet
        - Each flight covered exactly once by fleet
    - Number of Aircraft by Equipment Type
        - Can't assign more aircraft than are available
    - FAA Maintenance Requirements
    - Turn Times at each Station
    - Through revenues for pairs or sequences of flights
    - Maintenance costs per aircraft

Problem Objective

- Find:

  – Revenue maximizing assignment of aircraft of a single fleet to scheduled flights such that each flight is covered exactly once, maintenance requirements are satisfied, conservation of flow (balance) of aircraft is achieved, and the number of aircraft used does not exceed the number available

FAA Maintenance Requirements

- "A" Checks
  - Maintenance required every 60 hours of flying
  - Airlines maintain aircraft every 40-45 hours of flying with the maximum time between checks restricted to three to four calendar days

FAM Representation of Maintenance Constraints

- Maintenance arcs for fleet k included in time line network at each maintenance station for k

- Each arc
  - Begins at an aircraft arrival + turn time
  - Spans minimum maintenance time

- Constraints added to FAM for each aircraft type k, requiring a minimum number of aircraft of type k on the set of "maintenance arcs"
  - Ensures that sufficient maintenance opportunities exist
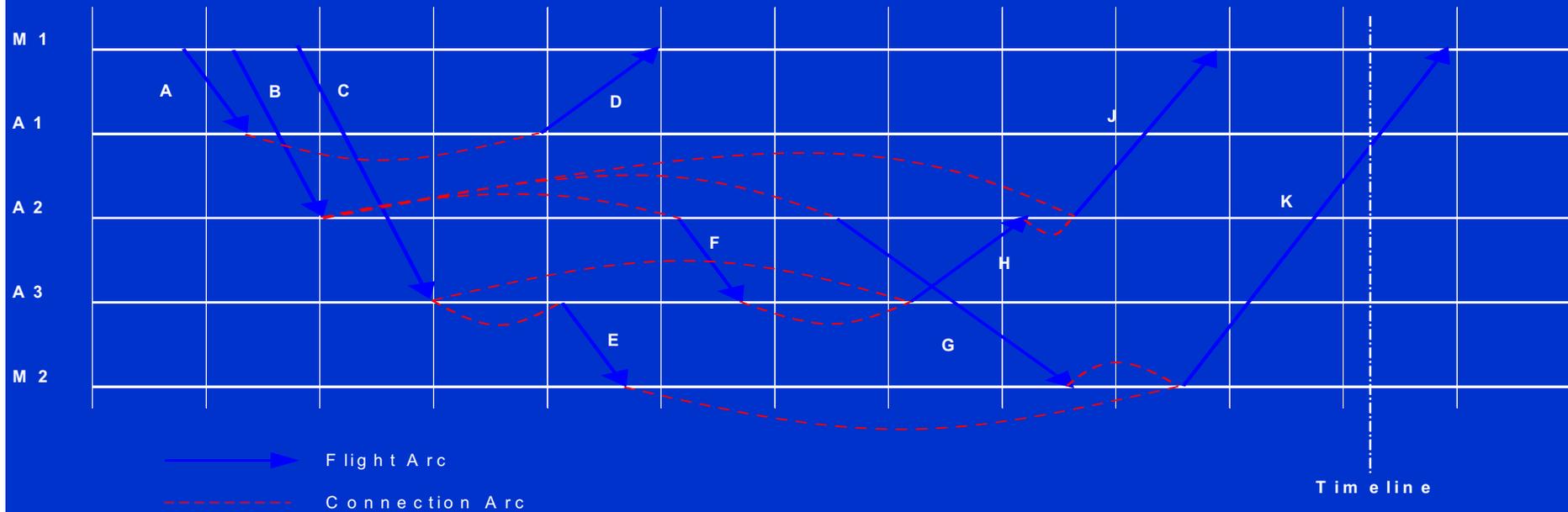 - One aircraft might be serviced daily and others not at all

Hub-and-Spoke vs. Point-to-Point Networks

- Domestic U.S. carriers with hub-and-spoke networks find that *approximate* maintenance constraints result in maintenance feasible routings
  - Sufficient number of opportunities at hubs to *swap* aircraft assignments so that aircraft get to maintenance stations as needed

- Approximate maintenance constraints often do not result in maintenance feasible routings for point-to-point networks
  - Flying time between visits to maintenance stations often too long

Network Representation

- Connection network
  - Nodes:
    - Flight arrivals/ departures (time and space)
  - Arcs:
    - Flight arcs: one arc for each scheduled flight
    - Connection arcs: allow aircraft to connect between flights

Connection Network

M 1

A   B   C
A 1   D   J

A 2   K
                                    F
                                                        H
A 3

         E   G
M 2

──────▶ F l i g h t A r c

---------- C o n n e c t i o n A r c

T i m e l i n e

Connection vs. Time-line Network

- Network Size
  - Time-line network typically has   more arcs than connection network

- Model capabilities
  - Connection network provides richer modeling possibilities
    - Through revenues can be captured easily
    - Disallowed or forced connections can be modeled easily

String Model: Variable Definition

- A string is a sequence of flights beginning and ending at a maintenance station with maintenance following the last flight in the sequence
  - Departure time of the string is the departure time of the first flight in the sequence
  - Arrival time of the string is the arrival time of the last flight in the sequence + maintenance time

String Model: Constraints

- Maintenance constraints
  - Satisfied by variable definition
- Cover constraints
  - Each flight must be assigned to exactly one string
- Balance constraints
  - Needed only at maintenance stations
- Fleet size constraints
  - The number of strings and connection arcs crossing the count time cannot exceed the number of aircraft in the fleet

NOTE: If the problem is daily, each string can contain a flight at most once! Assume we focus on weekly problem.

Model Strengths and Weaknesses

- Nonlinearities and complex constraints can be handled relatively easily

- Model size
  - Number of variables
  - Number of constraints

Notations

K is the set of fleets

x^k is an augmented string variable equal to 1 if $s \in S$ is flown by fleet k in the solution, and equal to 0 otherwise

y^k counts the number of aircraft of fleet k on the ground at maintenance stations

$e^k_{i;a}$ ($e^k_{i;d}$) is the event number for fleet k corresponding to the arrival (departure) of flight i at some maintenance station

$e^{+;k}_{i;a}$ ($e^{+;k}_{i;d}$) is the next event for k at that station after the arrival (departure) of i

$e^{-;k}_{i;a}$ ($e^{-;k}_{i;d}$) is the event at that station preceding the arrival (departure) of i

G^k is the set of ground variables for fleet k

S^-_i is the set of augmented strings ending with flight i and maintenance

S^+_i is the set of augmented strings beginning with flight i

a_{is} equals 1 if flight $i \in F$ is in augmented string s, and equals 0 otherwise

c^k_s is the cost of flying augmented string s with fleet k

r^k_s is the number of times (possibly greater than 1) augmented string s assigned to fleet k crosses the count time

p^k_j is the number of times (0 or 1) ground arc $j \in G^k$ for fleet k crosses the count time

N^k is the number of planes in fleet k

Aircraft Maintenance Routing: String Model

$$\min \quad \sum_{k \in K} \sum_{s \in S} c_s^k x_s^k$$

$$\sum_{k \in K} \sum_{s \in S} a_{is} x_s^k = 1, \qquad \forall i \in F$$

$$-\sum_{s \in S_i^+} x_s^k - y_{(e_{i,d}^{-k}, e_{i,d}^k)}^k + y_{(e_{i,d}^k, e_{i,d}^{+k})}^k = 0; \qquad \forall i \in F, \forall k \in K$$

$$-\sum_{s \in S_i^-} x_s^k - y_{(e_{i,a}^{-k}, e_{i,a}^k)}^k + y_{(e_{i,a}^k, e_{i,a}^{+k})}^k = 0, \qquad \forall i \in F, \forall k \in K$$

$$\sum_{s \in S} r_s^k x_s^k + \sum_{j \in G^k} p_j^k y_j^k \le N^k, \qquad \forall k \in K$$

$$y_j^k \ge 0, \qquad \forall j \in G^k, \forall k \in K$$

$$x_s^k \in \{0,1\}, \qquad \forall s \in S, \forall k \in K.$$

Model Solution

- Integer program
  - Branch-and-bound with too many variables to consider all of them
  - Solve Linear Program using **Column Generation**

- Branch-and-Price
  - Branch-and-bound with bounding provided by solving LP's using column generation, at each node of the branch-and-bound tree

LP Solution: Column Generation

- Step 1: Solve Restricted Master Problem

- Step 2: Solve Pricing Problem (generate columns)

- Step 3: If columns generated, return to Step 1; otherwise STOP

Generating The Right Variables

- Find a Variable with "Negative-Reduced Cost"
  - From Linear Programming Theory
    - reduced cost of each string between two nodes = cost - sum of the dual variables associated with flights in string + constant
  - Not feasible to compute reduced cost for each variable, instead exploit problem structure

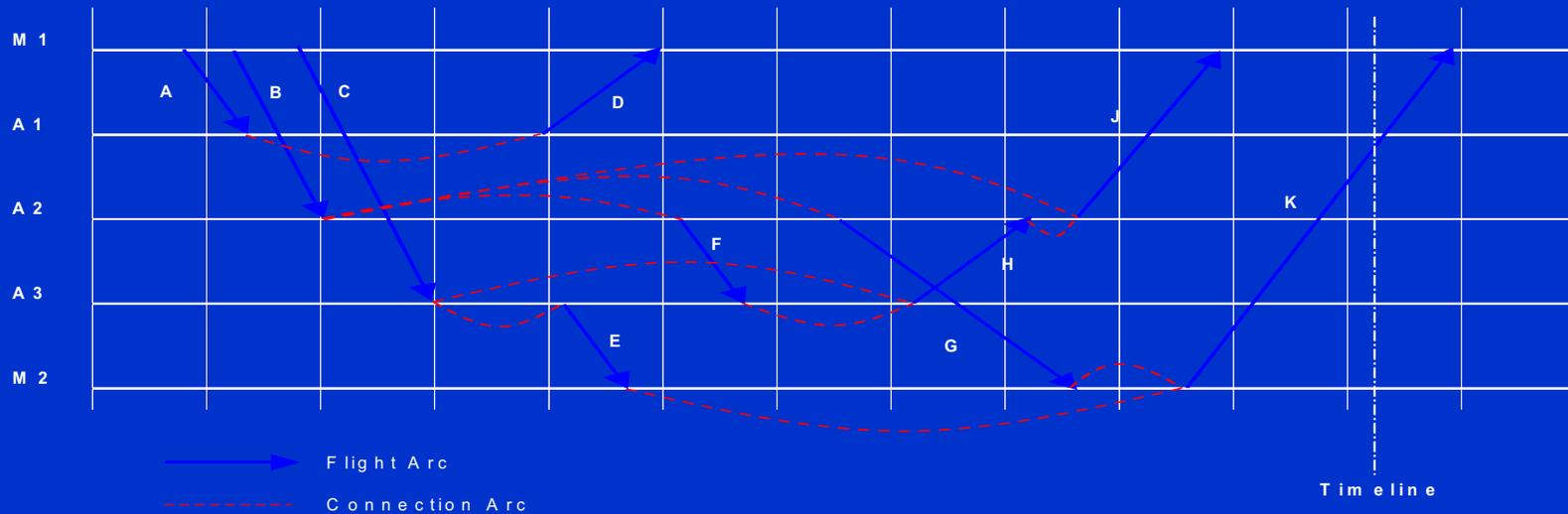# The Pricing Problem: A Constrained Shortest Path Problem

Time-Line Network

Time →

Washington, D.C.

Baltimore

New York

Boston
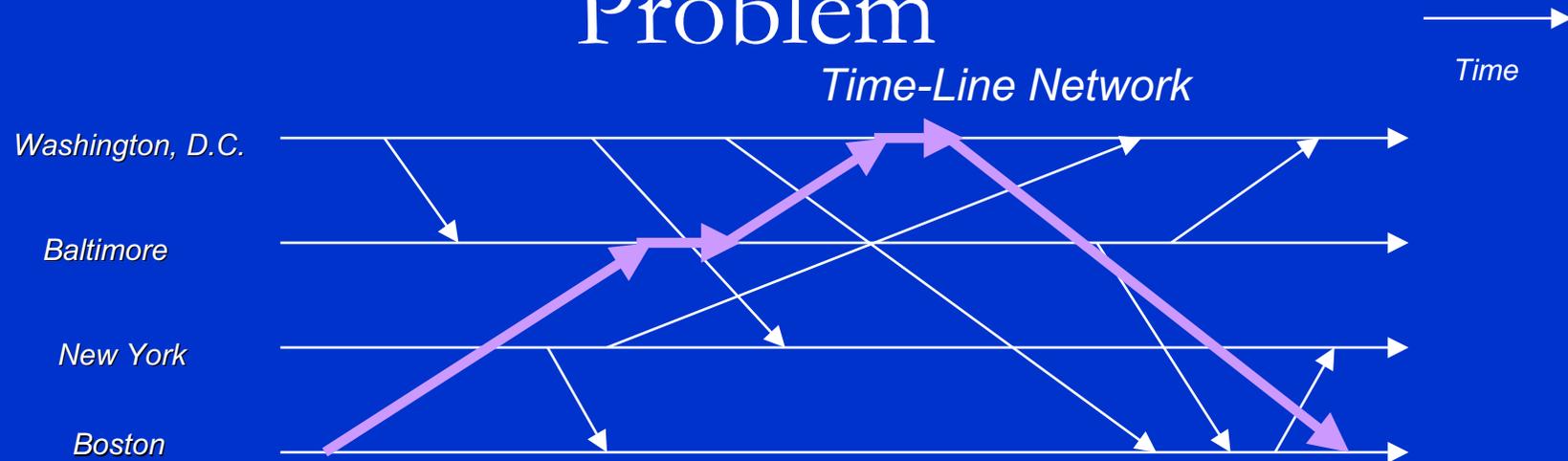
- If the "Length" of the shortest path is
  - <u>negative</u>: a variable has been identified for inclusion in the LP;
  - <u>non-negative</u>: the optimal LP solution is found

Pricing Problem Solution

- Find negative reduced cost route between maintenance stations

- Price-out columns by running a shortest path procedure with costs on arcs modified

- Ensure that shortest path solution satisfies maintenance constraints

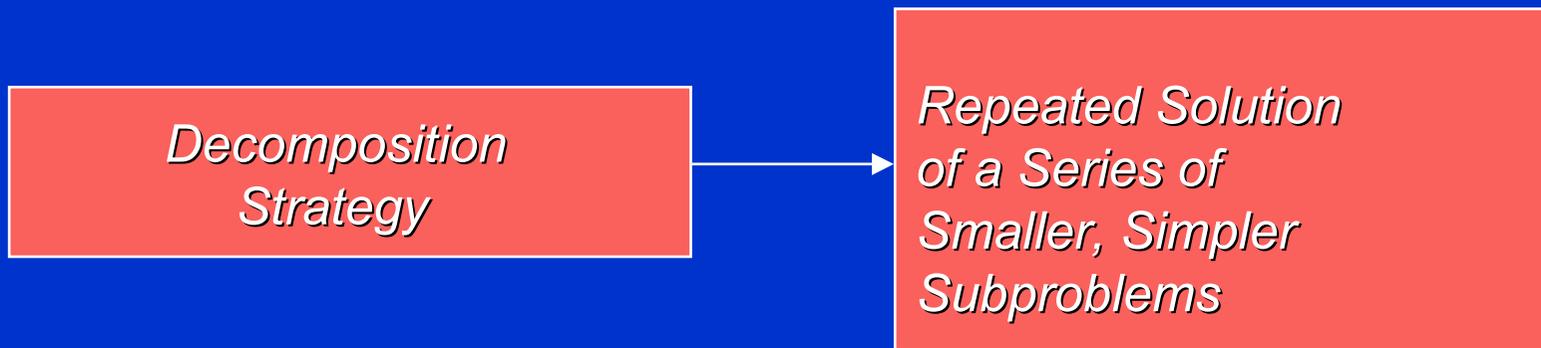# Path Lengths in Modified Network

Maintenance Requirements and the Pricing Problem Solution

- *Maximum Elapsed Time Requirement*
  - Unconstrained, simple shortest path computation
  - Computationally inexpensive
- *Maximum Flying Time Requirement*
  - Constrained shortest path procedure
    - Additional labels need to be maintained
  - Computationally expensive

Maximum Flying Time Requirement: Multiple Labels and Dominance

- Label i has cost $c(i)$ and elapsed flying time $t(i)$
- Label i dominates label $i+1$ if $c(i)$ _____ $c(i+1)$ and $t(i)$ _____ $t(i+1)$

Column Generation Solution Approach

Decomposition Strategy → **Repeated Solution of a Series of Smaller, Simpler Subproblems**

Allows Huge Programs To Be Solved Without Evaluating All Variables

Success of Approach Depends on Ease of Solution of Subproblems

Branch-and-Price Challenge

- Devise Branching Strategy that:
  - Is Compatible with Column Generation Subproblem
    - Does not destroy tractability of pricing problem
    - Conventional branching based on variable dichotomy does not work

Branching Strategy

- Branch on follow-ons:
  - Identify two fractional strings *s* and *s'*
  - Select flight *i(1)* contained in both strings
    - One exists because _____
  - Select flight *i(2)* contained in *s* but not *s'*
    - One exists because _____
  - Select *i(1)-i(2)* pair such that *i(1)* is followed immediately by *i(2)* in *s*
    - Such a pair exists

      _____

Branches

- Left branch: force flight *i(1)* to be followed by *i(2)* if *i(1)* and *i(2)* are in the string
  - To enforce: eliminate any connection arcs including *i(1)* or *i(2)* but not both
- Right branch: do not allow flight *i(1)* to be followed by *i(2)*
  - To enforce: eliminate any connection arcs from *i(1) to i(2)*

Summary: Branch-and-Price

- At each node of the Branch-and-Bound tree
  - Column generation used to solve LP
- Column Generation
  - Allows huge LPs to be solved by considering only a subset of all variables (columns)
  - Solves shortest path subproblem repeatedly to generate additional columns as needed
- Nontrivial Implementations
  - New branching strategies necessary
  - Ongoing research

Extensions: Combined Fleeting and Routing

- Motivation: Long-haul applications
  - Approximate maintenance constraints in FAM might result in "maintenance-infeasible" solutions
  - Exact maintenance constraints necessary
    - Need to model individual aircraft routings

Objective

- Minimize operating costs plus approximate spill costs minus through revenues

Model Solution

- Branch-and-Price

  - Branch on "fleet-flight" pairs

    - Provides a partition of flights to fleets

    - Enforce by

    _____

  - For each resulting fleet specific aircraft routing problem

    - Branch on follow-ons

Case Study

- Data provided by a major US long haul airline
  - 1162 flights per week serving 55 cities worldwide
  - 11 fleet types and 75 aircraft
  - 8 maintenance stations

- Subproblems generated to perform scenario analyses quickly

Problem Sizes

Problem Name	# Fleets	# Flights
P1	2	126
P2	2	403
P3	4	360
P4	4	536
P5	7	546
P6	11	1162

Effect of Maintenance Requirements

Problem name	# flights (# fleets)	Shortest Path Problem	Solution Time	Solution Value
P 1	126 (2)	Unconstrained	7 s	2960369.00
		Constrained	17 s	2960369.00
P 2	408 (2)	Unconstrained	2 m 1 s	10611441.00
		Constrained	6 m 43 s	10641120.00
P 4	536 (4)	Unconstrained	38 m 38 s	13624637.45
		Constrained	4 h 14 m 22 s	13563334.35
P 5	546 (7)	Unconstrained	23 m 11 s	16286255.29
		Constrained	1 h 45 m 46 s	16179416.08

Summary

- New model and solution approach for the combined fleet assignment and aircraft routing problems

- Computational experiments showed:
  - Near-optimal solutions in reasonable run times
  - Maintenance feasible solutions ensured
  - Through revenues captured

- One step closer to integration of overall airline scheduling process