1.206J/16.77J/ESD.215J
Airline Schedule Planning

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1.206J/16.77J/ESD.215J
The Fleet Assignment Problem

- Outline
  - Problem Definition and Objective
  - Fleet Assignment Network Representation
  - Fleet Assignment Model
  - Fleet Assignment Solution
    - Branch-and-bound
  - Results

Airline Schedule Planning

Schedule Design

Fleet Assignment

Aircraft Routing

Crew Scheduling
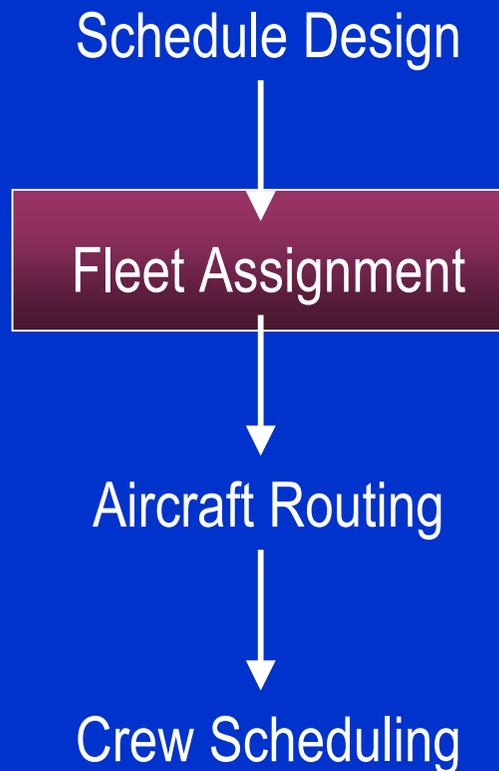
Select optimal set of *flight legs* in a schedule

Assign aircraft types to flight legs such that *contribution* is maximized

Contribution = Revenue - Costs

Assign crew (pilots and/or flight attendants) to flight legs

Problem Definition

Given:

- Flight Schedule
  - Each flight covered exactly once by one fleet type
- Number of Aircraft by Equipment Type
  - Can't assign more aircraft than are available, for each type
- Turn Times by Fleet Type at each Station
- Other Restrictions: Maintenance, Gate, Noise, Runway, etc.
- Operating Costs, Spill and Recapture Costs, Total Potential Revenue of Flights, by Fleet Type

Problem Objective

Find:

– Cost minimizing (or profit maximizing) assignment of aircraft fleets to scheduled flights such that maintenance requirements are satisfied, conservation of flow (balance) of aircraft is achieved, and the number of aircraft used does not exceed the number available (in each fleet type)

Definitions (again)

- Spill
  - passengers that are denied booking due to capacity restrictions

- Recapture
  - passengers that are recaptured back to the airline after being spilled from another flight leg

- For each fleet - flight combination:

  Cost ≡ Operating cost + Spill cost

Fleet Assignment References

- Abara (1989), Daskin and Panayotopoulos (1989), Hane, Barnhart, Johnson, Marsten, Neumhauser, and Sigismondi (1995)

- Hane, et al. "The Fleet Assignment Problem, Solving a Large Integer Program," *Mathematical Programming*, Vol. 70, 2, pp. 211-232, 1995

Network Representation

- Topologically sorted time-line network
  - Nodes:
    - Flight arrivals/ departures (time and space)
  - Arcs:
    - Flight arcs: one arc for each scheduled flight
    - Ground arcs: allow aircraft to sit on the ground between flights

Time-Line Network

- Ground arcs
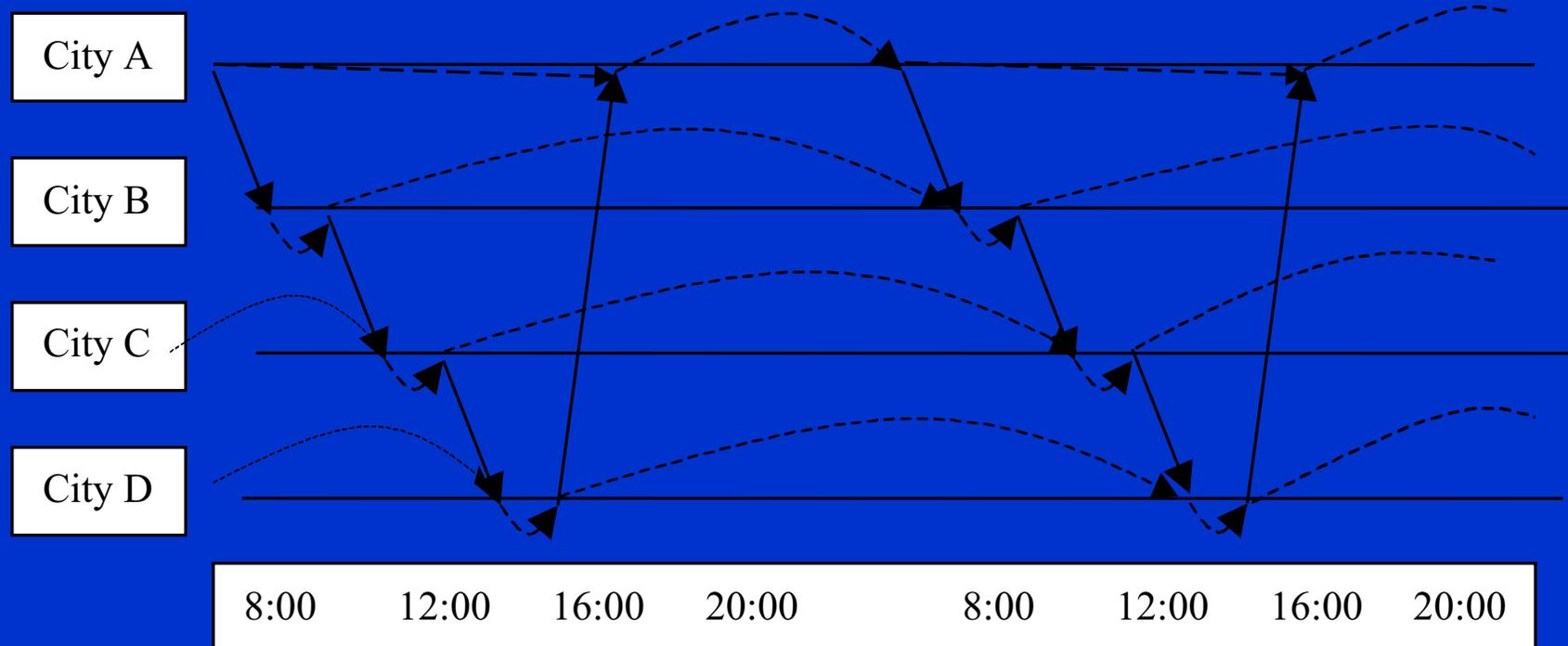
Time-Line Network

- "Daily" problem
  – Wrap-around (or overnight) arcs

Time

Washington, D.C.

Baltimore

New York

Boston
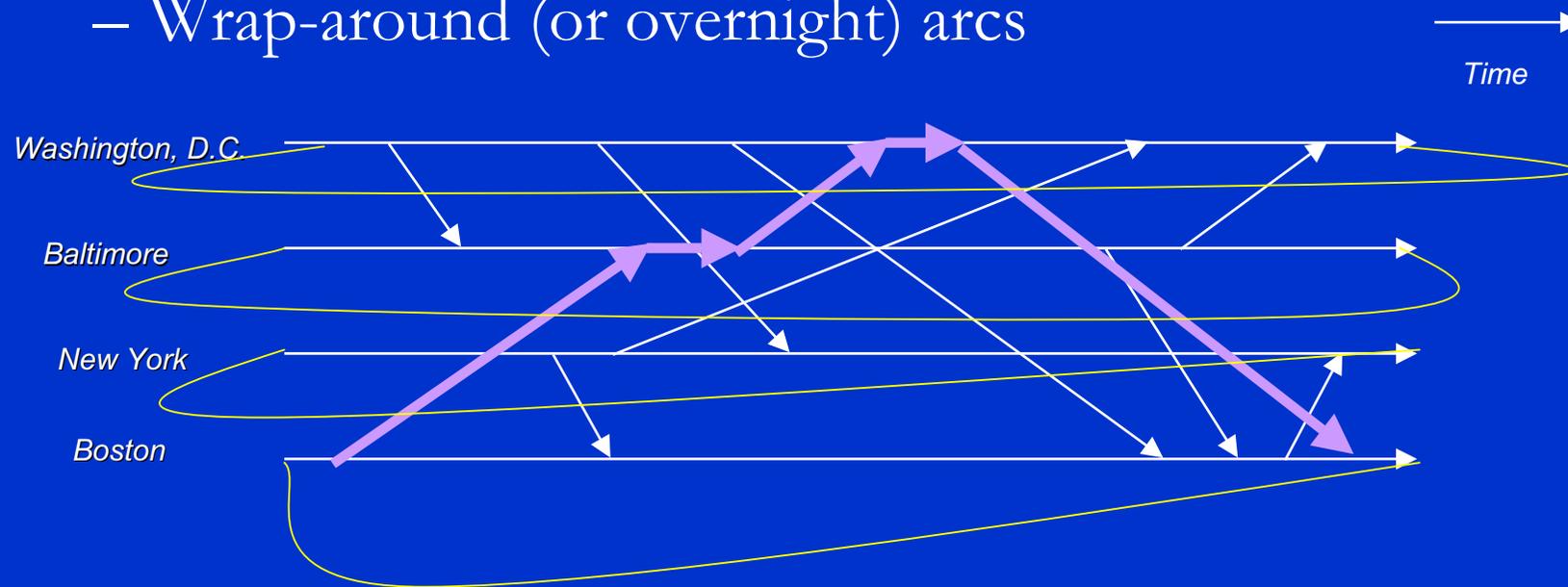
Constraints

- Cover Constraints
  - Each flight must be assigned to exactly one fleet
- Balance Constraints
  - Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing
- Aircraft Count Constraints
  - Number of aircraft of a fleet type used cannot exceed the number available

Objective Function

For each fleet - flight combination: Cost $\equiv$ Operating cost + Spill cost

- Operating cost associated with assigning a fleet type k to a flight leg j is relatively straightforward to compute
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning "infinite" costs
- Spill cost for flight leg j and fleet assignment k = average revenue per passenger on j * MAX(0, unconstrained demand for j – number of seats on k)
  - Unclear how to compute revenue for flight legs, given revenue is associated with itineraries

Spill Cost Computation and Underlying Assumption

Given:

– Spill cost for flight leg j and fleet assignment k = average revenue per passenger on j * MAX(0, unconstrained demand for j – number of seats on k)

Implication:

– A passenger might be spilled from some, but not all, of the flight legs in his/ her itinerary

FAM Spill Calculation Heuristics

- ## Fare Allocation
  - Full fare - the full fare is assigned to each leg of the itinerary
  - Partial fare - the fare divided by the number of legs is assigned to each leg of the itinerary
  - Shared fare - the fare divided by the number of *capacitated* legs is assigned to each *capacitated* leg in the itinerary

- ## Spill Cost for each variable
  - Representative Fare
    - A "spill fare" is calculated; each passenger spilled results in a loss of revenue equal to the spill fare
  - Integration
    - Sort each itinerary by fare, spill costs are sum of x lowest fare passengers, where $x = max\{0, demand - capacity\}$

An Illustrative Example

Fleet Type	Seats
A	100
B	200

flight 1   flight 2

X → Y → Z

Market	Itinerary	Average Fare	No. of Pax
X-Y	1	$200	75
Y-Z	2	$225	150
X-Z	1-2	$300	75

Fleet Assign. Fl. 1- Fl. 2	Partial Alloc. Spill	Full Alloc. Spill	Actual Opt. Spill	Spilled Pax
A-A	$30,000	$38,125	31,875	50 X-Z, 75 Y-Z
A-B	$11,250	$15,625	12,500	25 X-Z, 25 X-Y
B-A	$22,500	$28,125	28,125	125 Y-Z
B-B	$3,750	$5,625	5,625	25 Y-Z

Spill Calculation: Results

- For a 3 fleet, 226 flights problem:
  - The best representative fare solution results in a gap with the optimal solution of $2,600/day
  - Using a shared fare scheme and integration approach, we found a solution with an $8/day gap.
- By simply modifying the basic spill model, significant gains can be achieved

FAM-PMIX: Measures the Spill Approximation Error

FAM

PMIX

Fleeting decision

Net revenues

Operating costs

Fleeting contributions
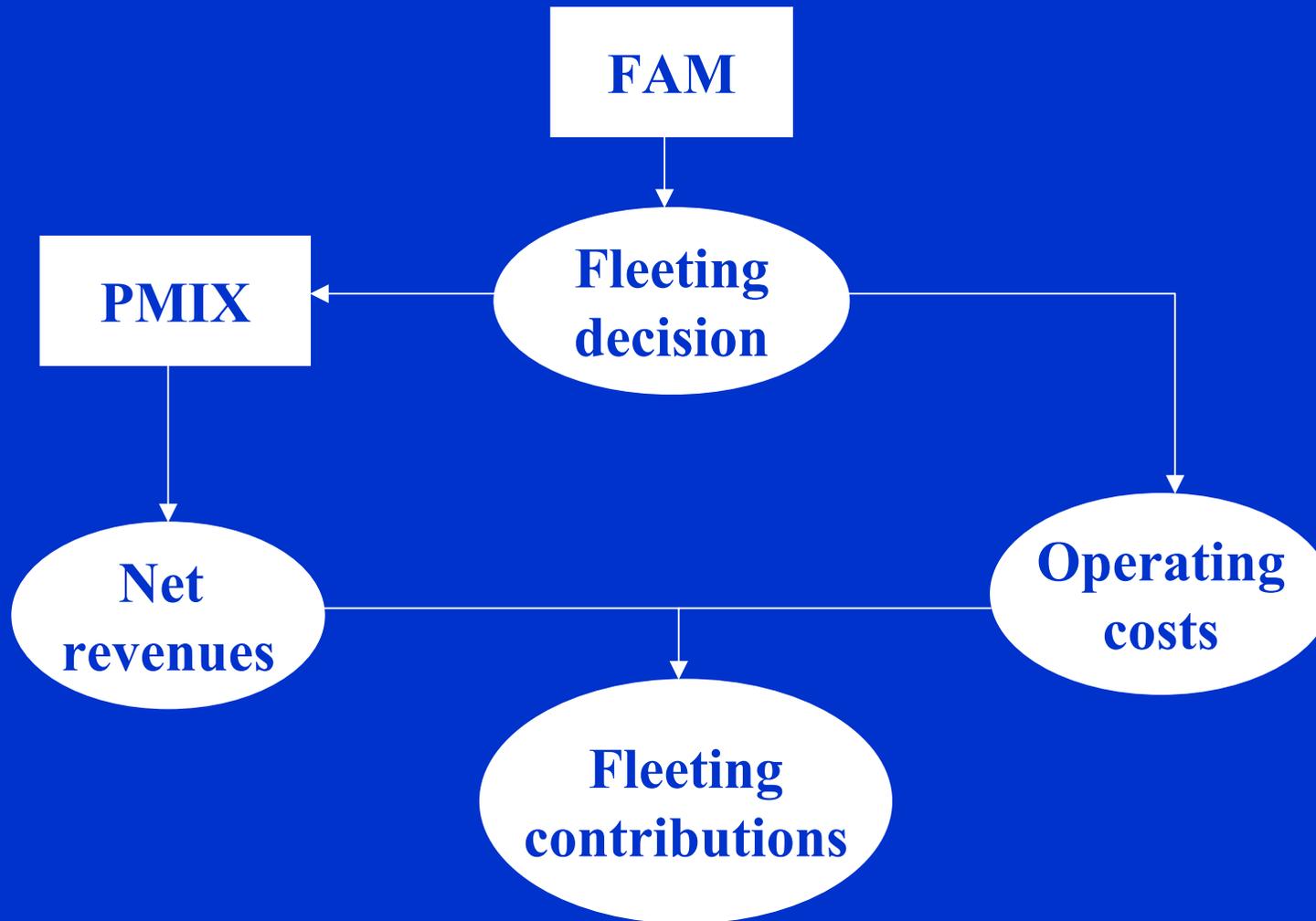
Passenger Mix

- Passenger Mix Model (PMIX)
    - Kniker (1998)
    - Given a fixed, fleeted schedule, unconstrained passenger demands by itinerary (requests), and recapture rates find maximum revenue for passengers on each flight leg

 Network Effects and Recapture

FAM Notations

- Decision Variables
  - $f_{k,i}$ equals 1 if fleet type k is assigned to flight leg i, and 0 otherwise
  - $y_{k,o,t}$ is the number of aircraft of fleet type k, on the ground at station o, and time t

- Parameters
  - $C_{k,i}$ is the cost of assigning fleet k to flight leg i
  - N_k is the number of available aircraft of fleet type k
  - t_n is the "count time"

- Sets
  - L is the set of all flight legs i
  - K is the set of all fleet types k
  - O is the set of all stations o
  - $CL(k)$ is the set of all flight arcs for fleet type k crossing the count time

Fleet Assignment Model (FAM)

$$Min \sum_{k \in K} \sum_{i \in L} c_{k,i} f_{k,i}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \le N_k \qquad \forall k \in K$$

$$f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \ge 0$$

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)

FAM Solution

- Exploitation of problem structure and understanding context are important
  - Node consolidation
  - Islands

- Branch-and-Bound

Time-Line Network
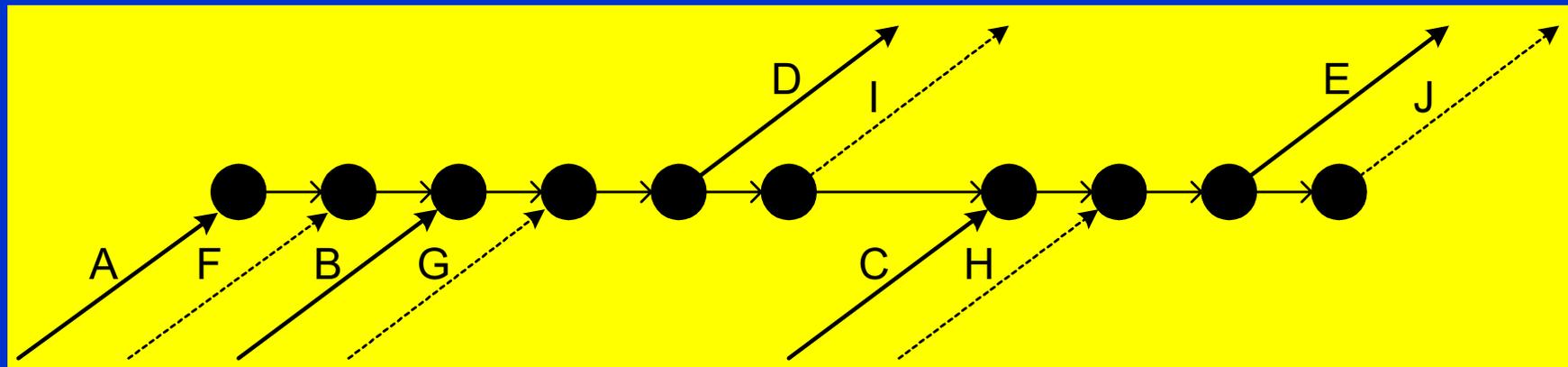
Node Consolidation
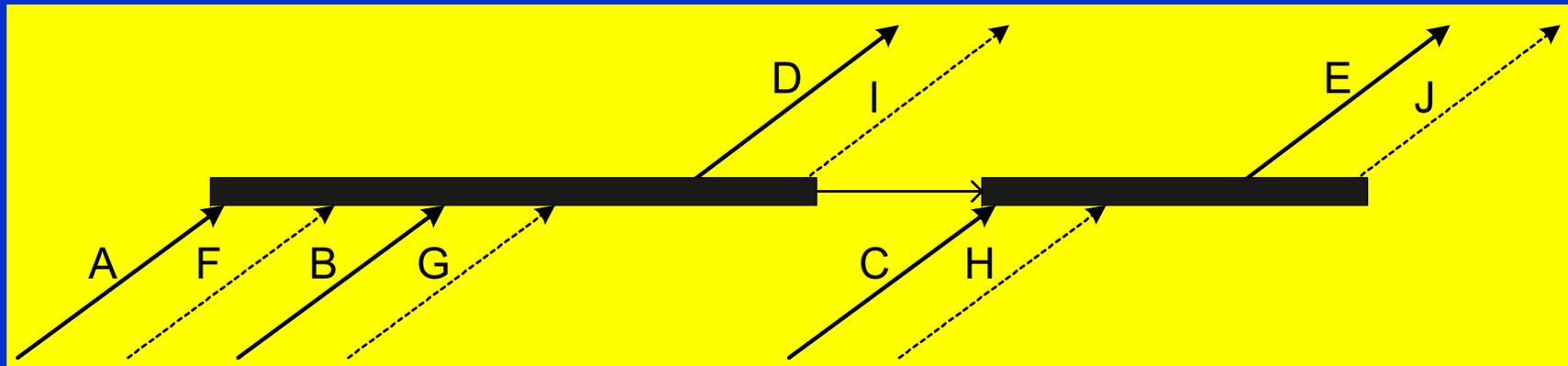
Islands

- For non-maintenance stations, the minimum number of aircraft on the ground at some point in time during the day is 0
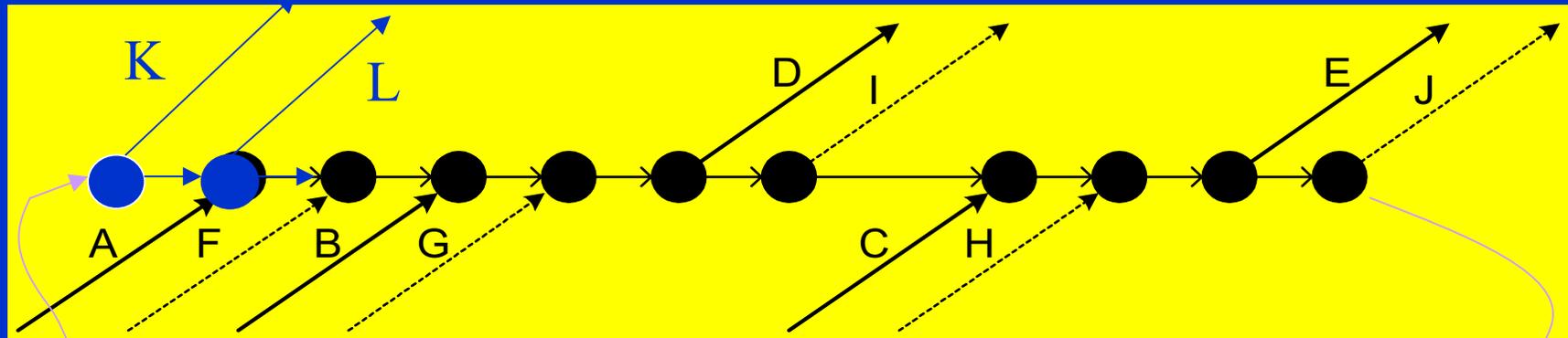
Fleet Assignment Model and Islands (FAM)

- Implications to number of ground variables and "required throughs"
  - Required through: same aircraft (type) must fly a sequence of flights

Branch-and-Bound: FAM Branching Strategies

- Variable branching
  - Set $x_i^k = 0$ or $x_i^k = 1$
  - "Unbalanced" branches: $x_i^k = 0$ branch is not as effective as $x_i^k = 1$ branch
  - "Small" decisions
    - Set one variable at a time… might have to solve a number of LPs
- Special ordered set branching
  - Set $x_1^k + x_2^k + … + x_m^k = 0$ or $x_1^k + x_2^k + … + x_m^k = 1$
  - More "balanced" branches
  - "Larger" decisions
    - Allow LP maximal flexibility to select solution, might need to solve fewer LPs

Branch-and-Bound Termination Criteria

- Branch-and-bound finds a provable optimal solution when all branches are pruned

- Branch-and-bound can be terminated prematurely if solution time limits exist or optimality is not the objective
  - Terminate the algorithm when the lower bound on the optimal solution for a minimization problem is close enough to the incumbent IP solution
    - Stop when integrality gap is small

Solution

- Solve fleet assignment problems for large domestic carriers (10-14 fleets, 2000-3500 flights) within 10-20 minutes of computation time on workstation class computers

- Hane, et al. "The Fleet Assignment Problem, Solving a Large Integer Program," *Mathematical Programming*, Vol. 70, 2, pp. 211-232, 1995

FAM Shortcomings: Network Effects

A → B (80, $200)

B → C (90, $250)

A → C (50, $400)

(Demand, Fare)
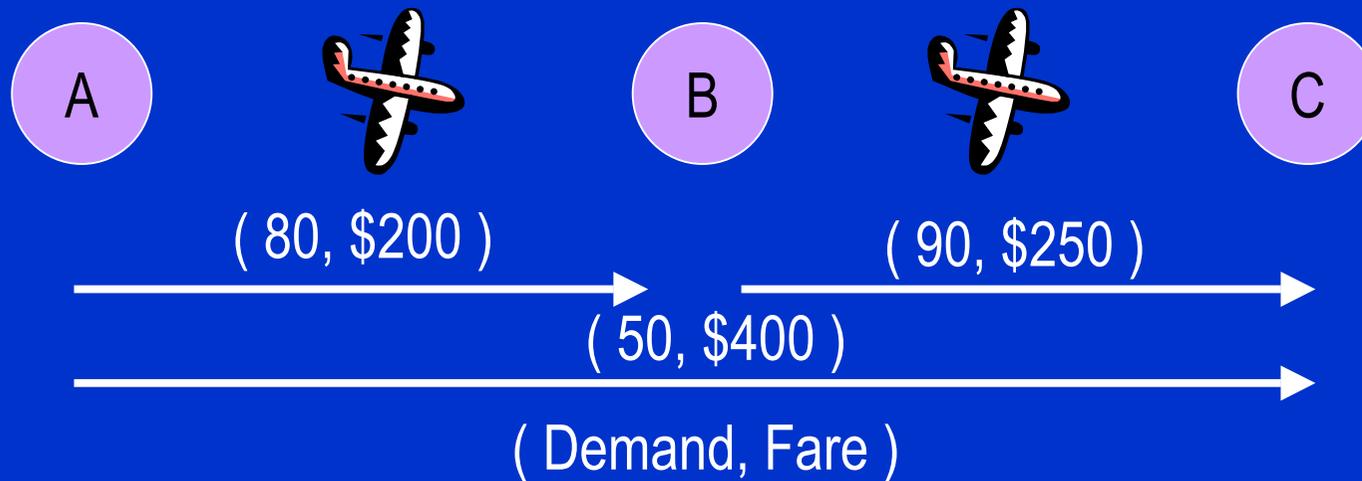
Fleet Type	Capacity	Spill Cost
i	80	?
ii	100	?
iii	120	?
iv	150	$0

Leg Interdependence

Network Effects

FAM Shortcomings: NO Recapture

100 seats   100 seats

A   ✈   B   ✈   C

(80, $200)   (90, $250)

(50, $400)

9AM

20

30 X (0.3) = 9 recaptured passengers

29

(20, $400)

10AM

(Demand, Fare)

Recapture Rate
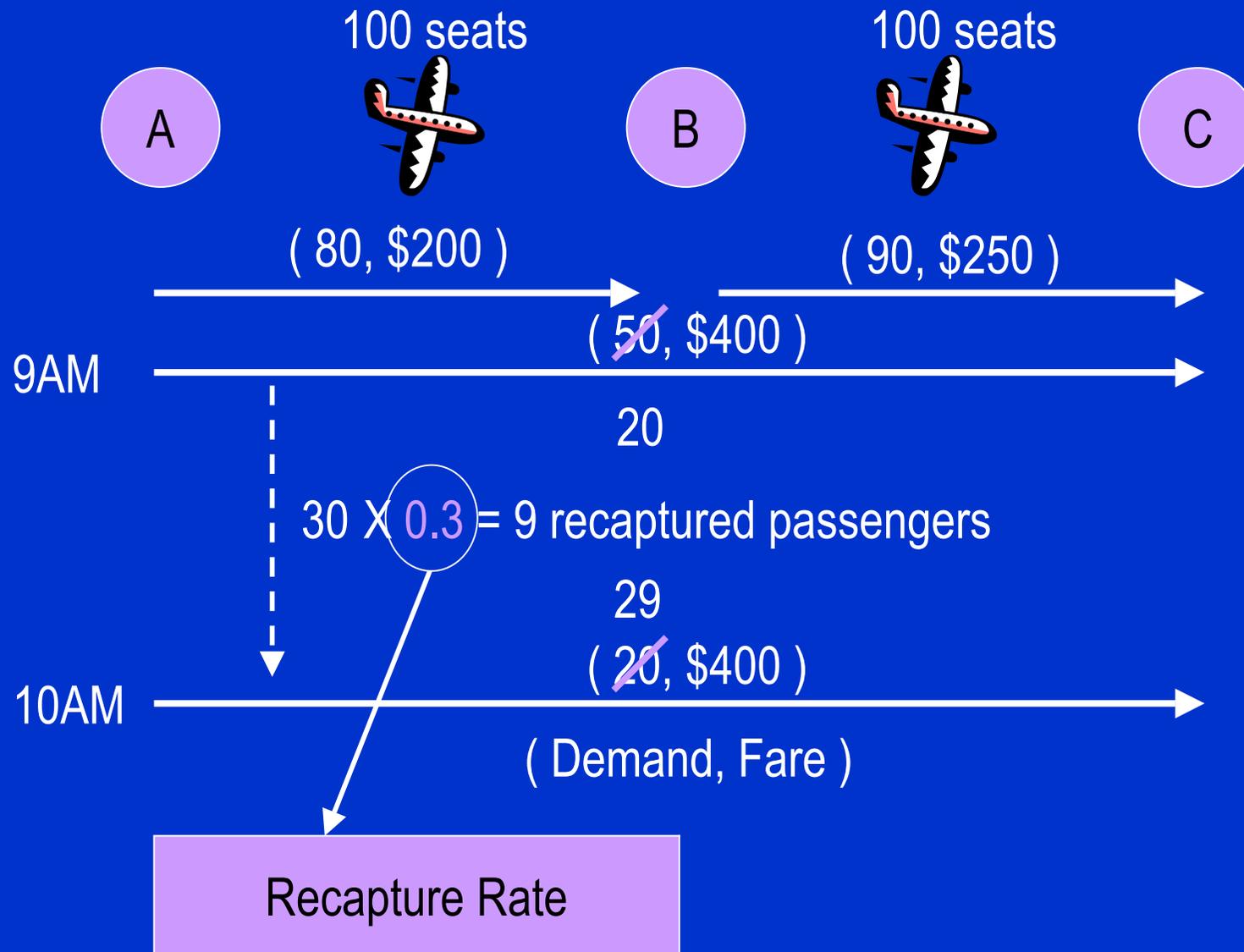
Itinerary-Based Fleet Assignment

- Impossible to estimate airline profit exactly using link-based costs
- Enhance basic fleet assignment model to include passenger flow decision variables
  - Associate operating costs with fleet assignment variables
  - Associate revenues with passenger flow variables (PMIX)

Itinerary-based Fleet Assignment Definition

- Given

  – a fixed schedule,

  – number of available aircraft of different types,

  – unconstrained passenger demands by itinerary, and

  – recapture rates,

  Find maximum contribution

   Network effects

Itinerary-Based FAM (IFAM)

$$Min \sum_{k \in K} \sum_{i \in L} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r) \ t_p^r$$

Subject to:
$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \qquad \forall k \in K$$

$$\sum_k f_{k,i} SEATS_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \geq Q_i \qquad \forall i \in L$$

$$\sum_{r \in P} t_p^r \leq D_p \qquad \forall p \in P$$

$$t_p^r \geq 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

Kniker (1998)

Itinerary-Based FAM (IFAM)

Fleet Assignment

Consistent Spill + Recapture

$$t_p^r \geq 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

Kniker (1998)

Itinerary-Based FAM (IFAM)

$$Min \sum_{k \in K} \sum_{i \in L} \tilde{c}_{k,i} f_{k,i} + \sum_{p \in P} \sum_{r \in P} (fare_p - b_p^r fare_r) \boxed{t_p^r} \quad 3$$

$$Subject\ to: \qquad \sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \qquad \forall k \in K$$

$$1 \quad \sum_k f_{k,i} SEATS_k + \sum_{r \in P} \sum_{p \in P} \delta_i^p t_p^r - \sum_{r \in P} \sum_{p \in P} \delta_i^p b_p^r t_p^r \geq Q_i \qquad \forall i \in L$$

$$2 \quad \sum_{r \in P} t_p^r \leq D_p \qquad \forall p \in P$$

$$t_p^r \geq 0 \qquad f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

Kniker (1998)

IFAM Solution Algorithm

START

Build Restricted Master Problem (RMP)

1

Preprocessing

IFAM LP

Solve LP Relax.

2

Row Generation

3

Column Gen.

Any rows or columns added ?

YES

NO

Branch and Bound

Feas ?

NO

YES

STOP
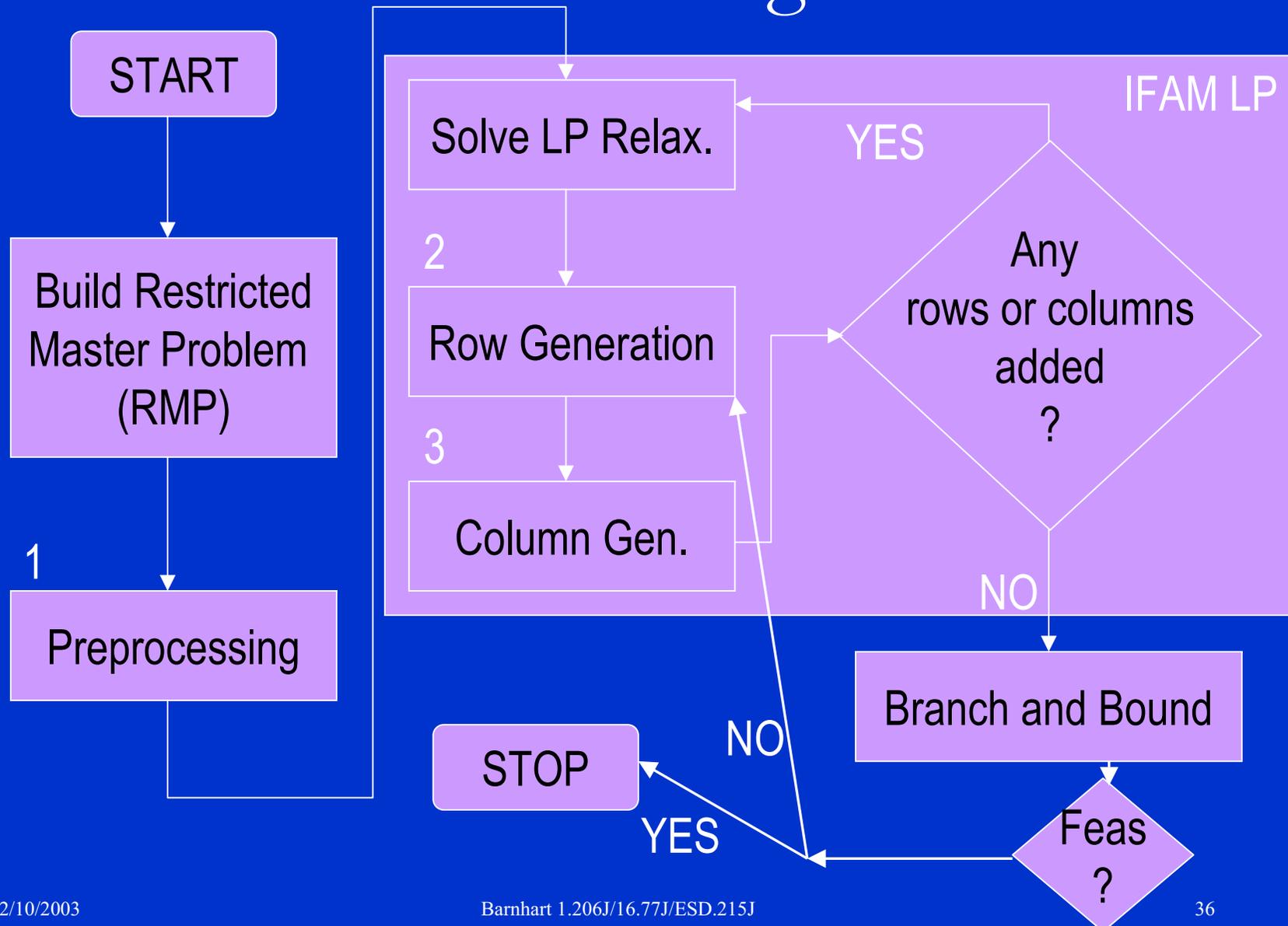
Implementation Details

- Computer
  - Workstation class computer
  - 2 GB RAM
  - CPLEX 6.5
- Full size schedule
  - ~2,000 legs
  - ~76,000 itineraries
  - ~21,000 markets
  - 9 fleet types

- RMP constraint matrix size
  - ~77,000 columns
  - ~11,000 rows
- Final size
  - ~86,000 columns
  - ~19,800 rows
- Solution time
  - LP: > 1.5 hours
  - IP: > 4 hours

88%   Saving from Row Generation
> 95%   Saving from Column Generation

IFAM Contributions

- Annual improvements over basic FAM
  - Network Effects: ~$30 million
  - Recapture: ~$70 million
- These estimates are upper bounds on achievable improvements
- Actual improvements will be smaller

Caveats

2. Deterministic Demand

4. Optimal Control of Paxs

3. Demand Forecast Errors

1. Recapture Rate Errors

Recapture Rate Sensitivity

Specified Recapture Rate → IFAM → Fleeting Decision

Fleeting Decision → Solve PMM with varied recapture rates → Estimated Revenue → Fleeting Contribution

Fleeting Decision → Operating Cost → Fleeting Contribution

- ☀ PMM flows passengers on fleeted schedule assuming full knowledge of passenger choices

Recapture Rate Sensitivity

Recapture Rate Sensitivity
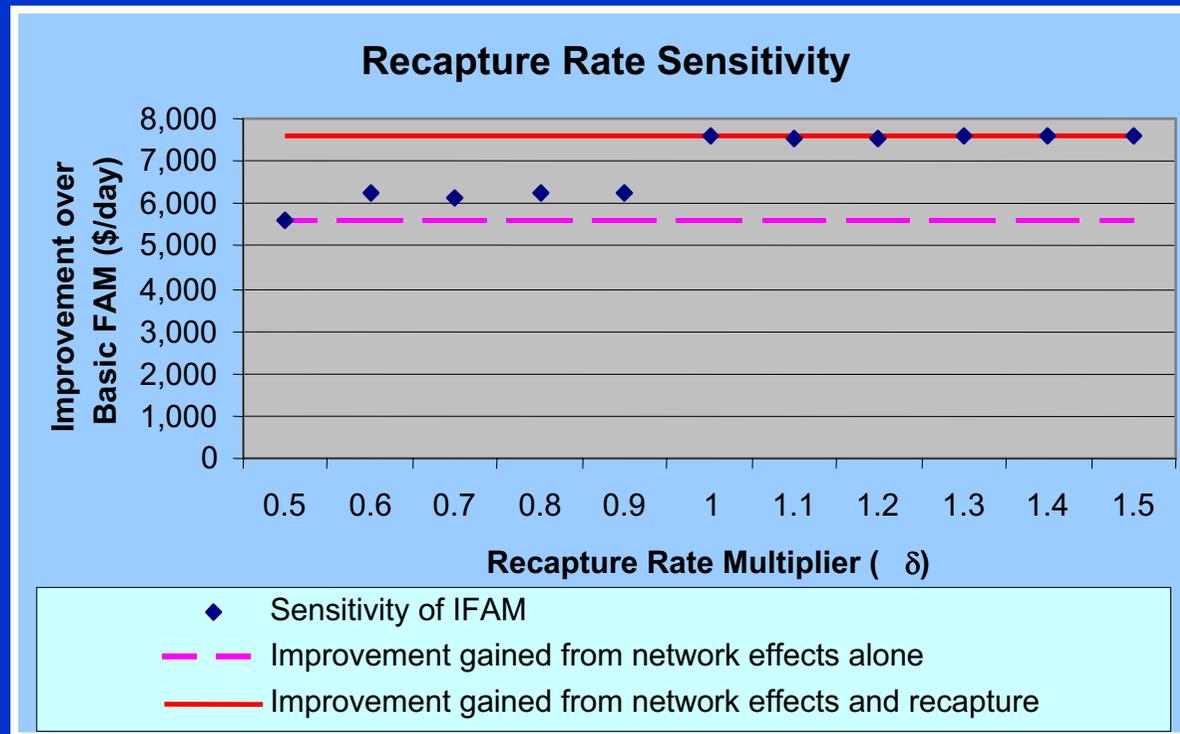
Y-axis: Improvement over Basic FAM ($/day) — 0, 1,000, 2,000, 3,000, 4,000, 5,000, 6,000, 7,000, 8,000

X-axis: Recapture Rate Multiplier (δ) — 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, 1.3, 1.4, 1.5

- ◆ Sensitivity of IFAM
- — — Improvement gained from network effects alone
- —— Improvement gained from network effects and recapture

IFAM Sensitivity Analysis

- ## Simulations
  - ✹ Simulate 500 realizations of demand based on Poisson distributions

Demand Variations

Realizations

Average Demand → FAM or IFAM

Fleeting Decision

Passenger Allocation Simulation

Operating Cost

Estimated Revenue

Fleeting Contribution

IFAM Sensitivity Analysis

- ## Simulations
  - ☀ Simulate 500 realizations of demand based on Poisson distributions

Demand Variations

Realizations

Average Demand → FAM or IFAM

Fleeting Decision

Passenger Allocation Simulation

Operating Cost

Estimated Revenue

Fleeting Contribution
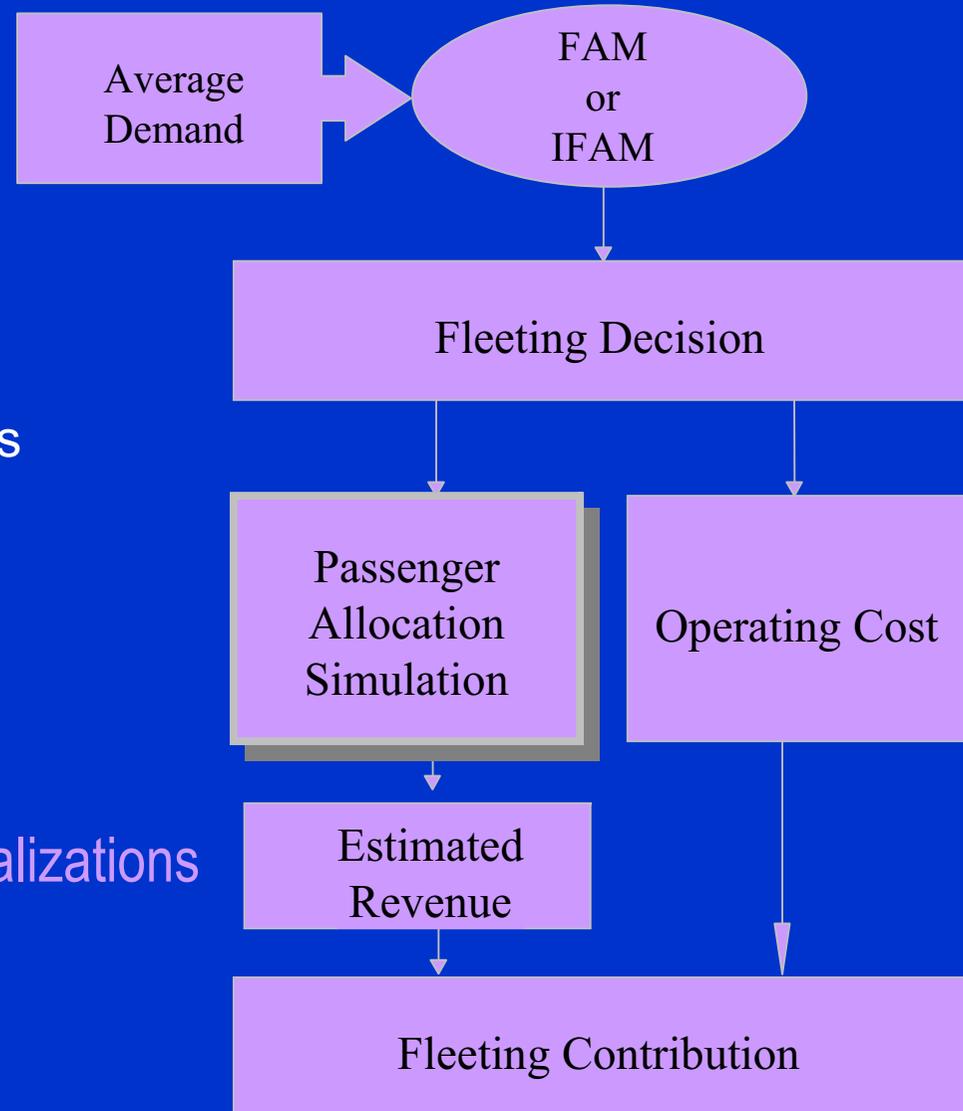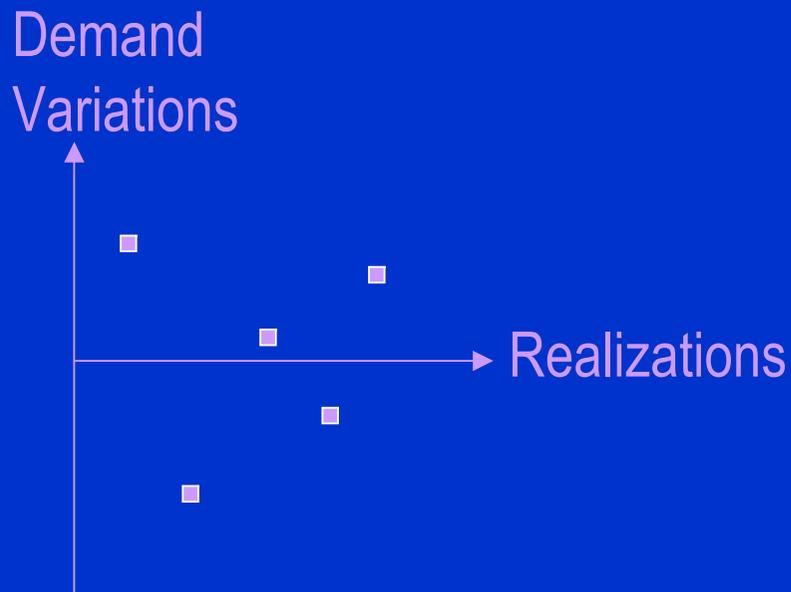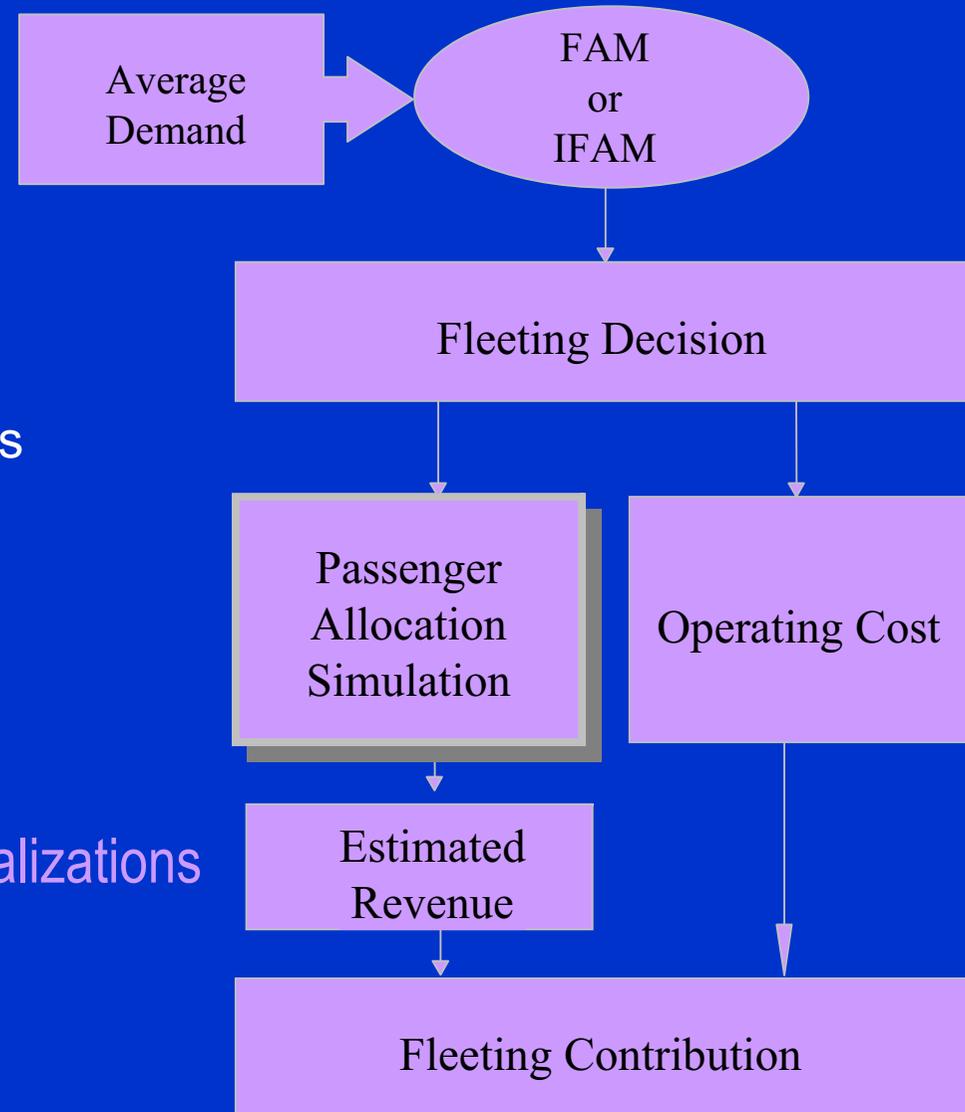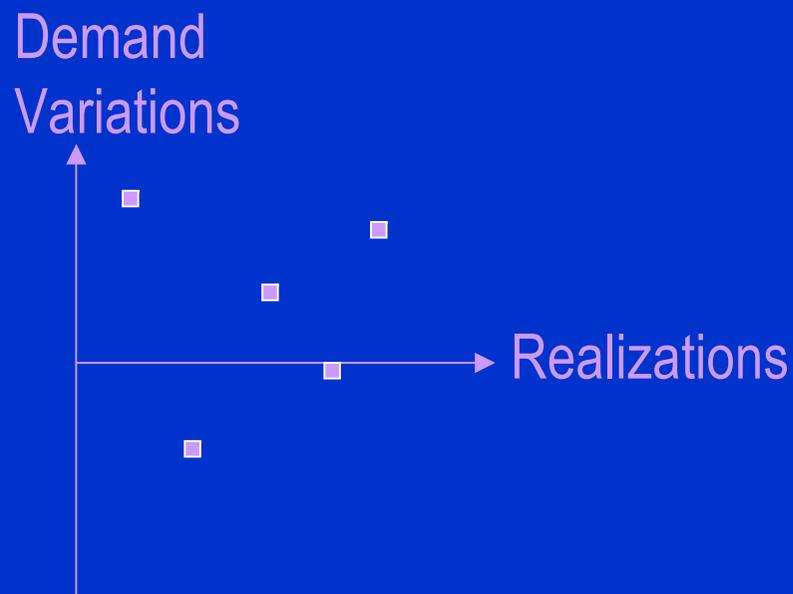
IFAM vs. FAM

Demand Stochasticity

Demand Variations

Realizations

Demand deviation ~14%
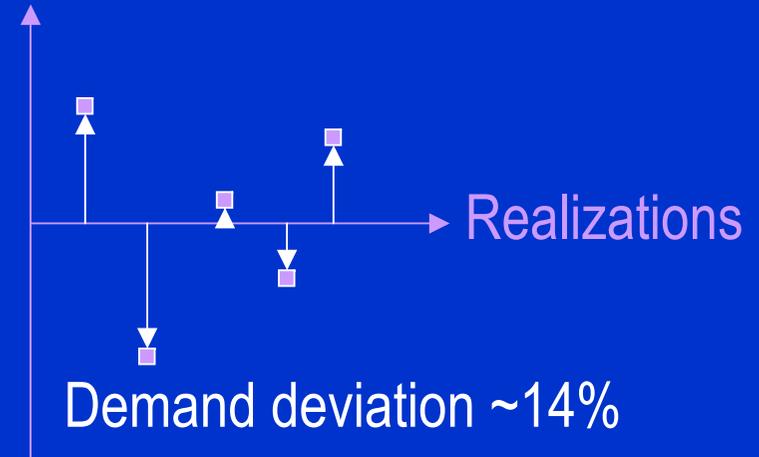
	FAM	IFAM	Difference (IFAM-FAM)
Problem 1N-3A			
Revenue	$ 4,858,089	$ 4,918,691	$ 60,602
Operating Cost	$ 2,020,959	$ 2,021,300	$ 341
Contribution	$ 2,837,130	$ 2,897,391	$ 60,261
Problem 2N-3A			
Revenue	$ 3,526,622	$ 3,513,996	$ (12,626)
Operating Cost	$ 2,255,254	$ 2,234,172	$ (21,082)
Contribution	$ 1,271,368	$ 1,279,823	$ 8,455
			$/day

IFAM vs. FAM

Demand Stochasticity

Forecast Errors

Data Quality Issue

Demand Variations

Realizations

Demand deviation ~15%
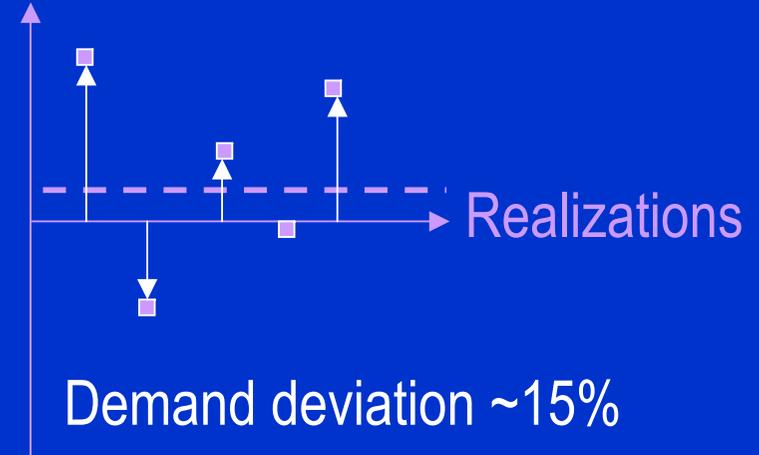
Model Sensitivity to Demand Forecast Errors

Contributions ($ million/day) vs Simulated Demands (% of Forecasted Demand)
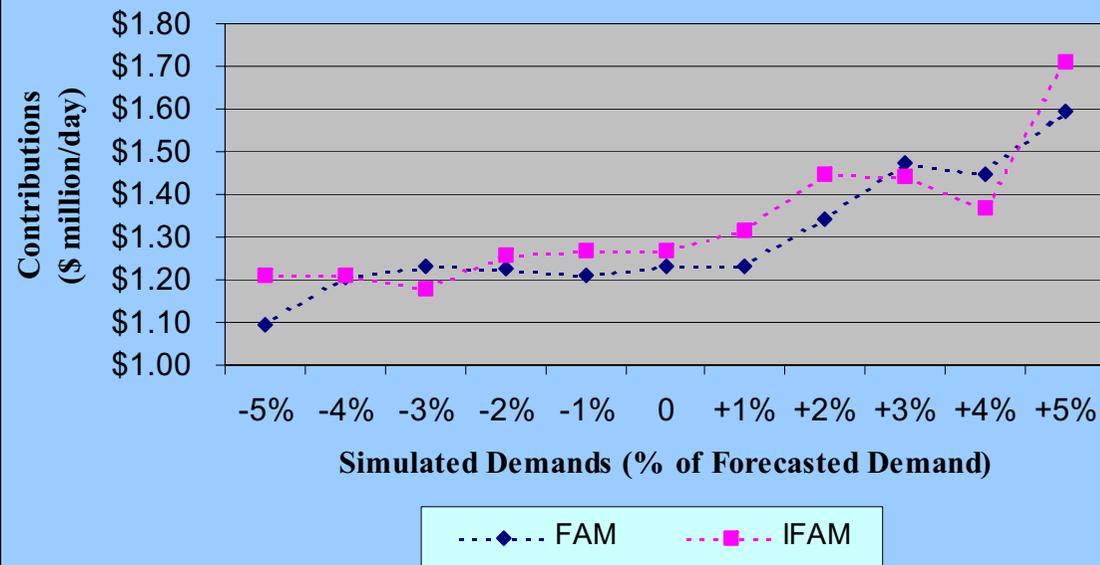
FAM — IFAM

IFAM vs. FAM

Demand Stochasticity

Forecast Errors

Optimal Control of Passengers

	FAM	IFAM	Difference (IFAM-FAM)
Problem 1N 3A			
Operating Cost	$ 2,266,261	$ 2,261,172	$ (21,082)
Contribution	$ 1,246,346	$ 1,268,977	$ 22,631
			$/day

From our analysis, there is evidence suggesting that network effects dominate demand uncertainty in hub-and-spoke fleet assignment problems.

Another Fleet Assignment Model and Solution Approach…

Subnetwork-Based FAM

- IFAM has tractability issues

- Limited opportunities for further IFAM extension

- Need alternative kernel

  - Capture network effects

  - Maintain tractability
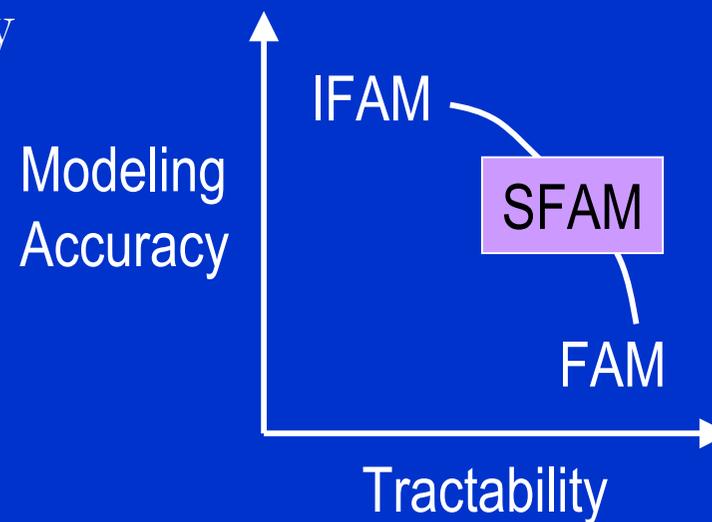
Basic Concept

- Isolate network effects
  - Spill occurs only on *constrained legs*

Legend:
- Potentially Constrained Flight Leg
- Unconstrained Flight Leg
- Potentially Binding Itinerary
- Non-Binding Itinerary

SFAM
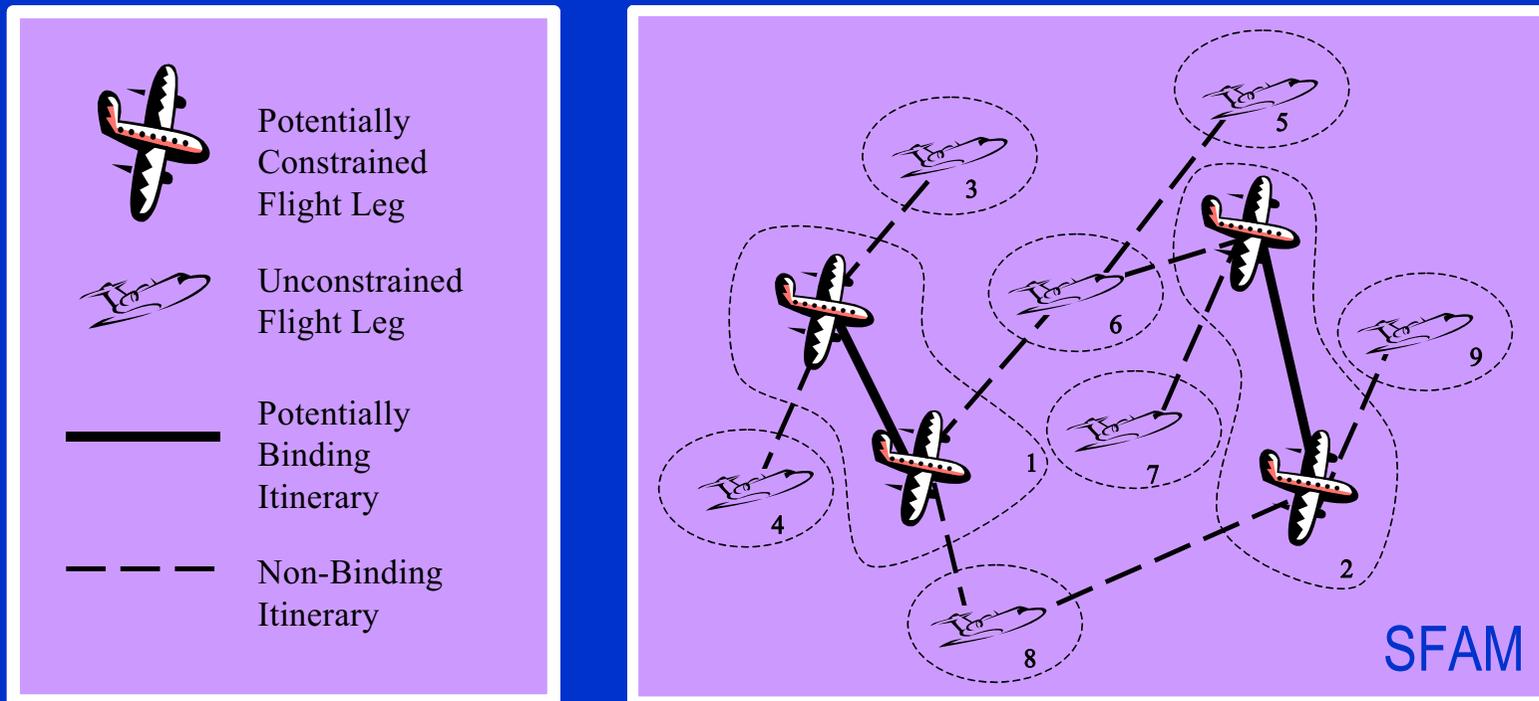
- ❊ < 30% of total legs are potentially constrained
- ❊ < 6% of total itineraries are potentially binding

Modeling Challenges

- Utilize composite variables (Armacost, 2000; Barnhart, Farahat and Lohatepanont, 2001)
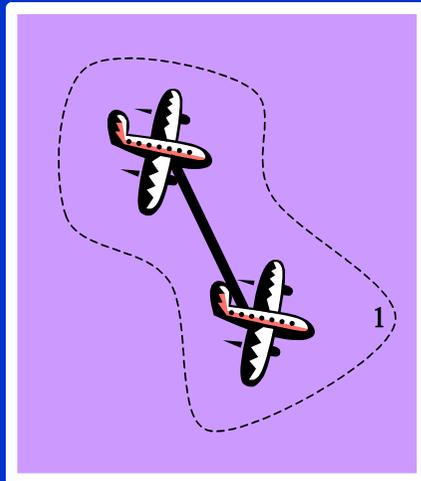
1	2	3	4	5	6	7	8	9
A	A	A	B	B	B	C	C	C
A	B	C	A	B	C	A	B	C

3 Fleet Types: A, B, and C

☀ **Challenges**

☀ Efficient column enumeration

Implementation

- Partition construction
  - Construct a *complete partition*
  - Subdivide the complete partition
- Parsimonious column enumeration
  - Potentially constrained leg might become *unconstrained* if assigned bigger aircraft
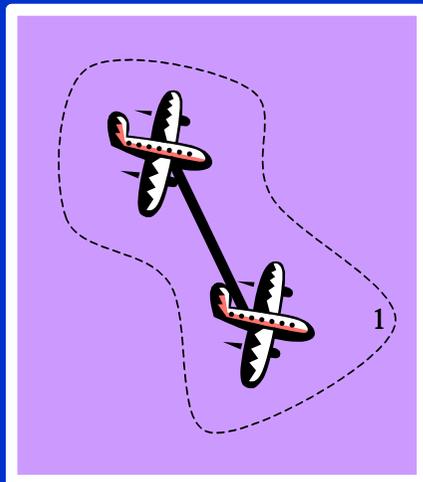
1	2	3	4	5	6	7	8	9
A	A	A	B	C				C
A	B	C			A	B	C	C

3 Fleet Types: A, B, and C

Remove up to 97% of otherwise necessary columns

SFAM Formulation

$$Min \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(C_{\Pi^S}^m \right)_n \left(f_{\Pi^S}^m \right)_n$$

Subject to:

$$\sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\delta_{\Pi^S}^m \right)_n^i \left(f_{\Pi^S}^m \right)_n = 1 \quad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\kappa_{\Pi^S}^m \right)_n^{k,i} \left(f_{\Pi^S}^m \right)_n - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\kappa_{\Pi^S}^m \right)_n^{k,i} \left(f_{\Pi^S}^m \right)_n = 0 \quad \forall k,o,t$$

$$\sum_{o \in A} y_{k,o,t_n} + \sum_{i \in CL(k)} \sum_{m=1}^{M^S} \sum_{n=1}^{\eta_{\Pi^S}^m} \left(\gamma_{\Pi^S}^m \right)_n^k \left(f_{\Pi^S}^m \right)_n \leq N_k \quad \forall k \in K$$

$$\left(f_{\Pi^S}^m \right)_n \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

FAM solution algorithm applies

Results

- 1,888 Flights
- 9 Fleet Types
- 75,484 Itineraries

Partition	Max No. of Flight Legs	No. of Subnetworks
Π^1	4	2,021
Π^2	4	2,013
Π^3	4	2,001
Π^4	4	1,986
Π^5	4	1,972
Π^6	6	1,922
Π^7	6	1,920
Π^8	7	1,915

Partition Construction

- Allow "spill dependent" subnetworks
  - Merge spill dependent subnetworks when solution has a spill calculation error

Potentially Constrained Flight Leg

Unconstrained Flight Leg

Potentially Binding Itinerary

Non-Binding Itinerary
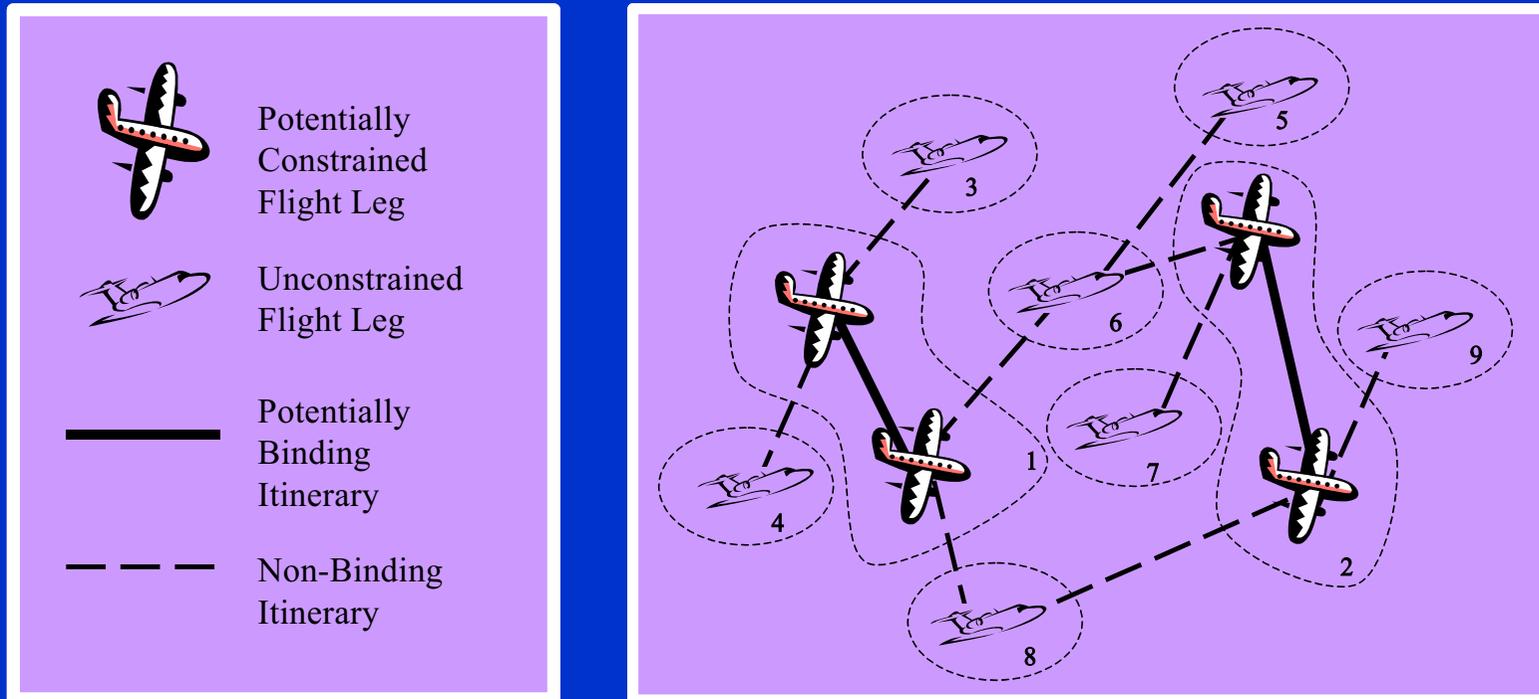
Runtime

	Runtime (sec)			
	Preprocess	LP	IP	Total
FAM	-	97	893	990
SFAM(Π^1)	6	103	419	528
SFAM(Π^2)	9	103	631	743
SFAM(Π^3)	11	121	485	617
SFAM(Π^4)	16	109	815	940
SFAM(Π^5)	22	111	376	509
SFAM(Π^6)	285	153	1,495	1,933
SFAM(Π^7)	342	203	2,480	3,025
SFAM(Π^8)	1,007	249	1,187	2,443
IFAM	-	100	6,831	6,931

Solution Quality

	Contribution ($/day)		IP-LP Gap
FAM		21,178,815	36
SFAM(Π^1)	+	48,407	2,811
SFAM(Π^2)	+	48,928	191
SFAM(Π^3)	+	50,180	162
SFAM(Π^4)	+	50,331	69
SFAM(Π^5)	+	50,344	53
SFAM(Π^6)	+	50,333	543
SFAM(Π^7)	+	50,241	758
SFAM(Π^8)	+	50,232	198
IFAM	+	48,691	5,132

SFAM: Results & Conclusions

- Testing performed on full size schedules
- SFAM can achieve optimal solutions equivalent to IFAM's
- Because of formulation structure, SFAM produces tighter LP relaxations
- Tighter LP relaxations lead to quicker solution times
- SFAM has great potential for further integration or extension
  - Time windows
  - Stochastic demand
  - Schedule design

Extending Fleet Assignment Models to Include "Incremental" Schedule Design…

Airline Schedule Planning Process

Schedule Design

↓

Fleet Assignment

↓

Aircraft Routing

↓

Crew Scheduling
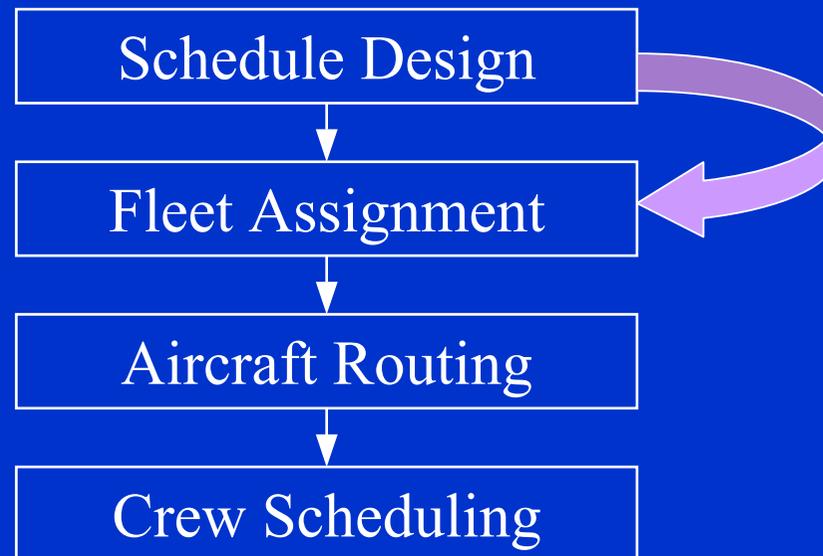
Fleet Assignment with Time Windows: *A step to integrate schedule design and fleet assignment*

Fleet Assignment with Time Windows (FAMTW)

- Assume that departure times (and arrival) times are **NOT** fixed for each flight, instead there is a *time window* for departures

  - Publication of schedule is several months out

  - Passenger forecasts won't change for minor re-timings

  - Produce a better fleet assignment

    - Save money (operating costs, spill costs)
    - Free up aircraft by "tightening" the schedule

Time Window Flight Network
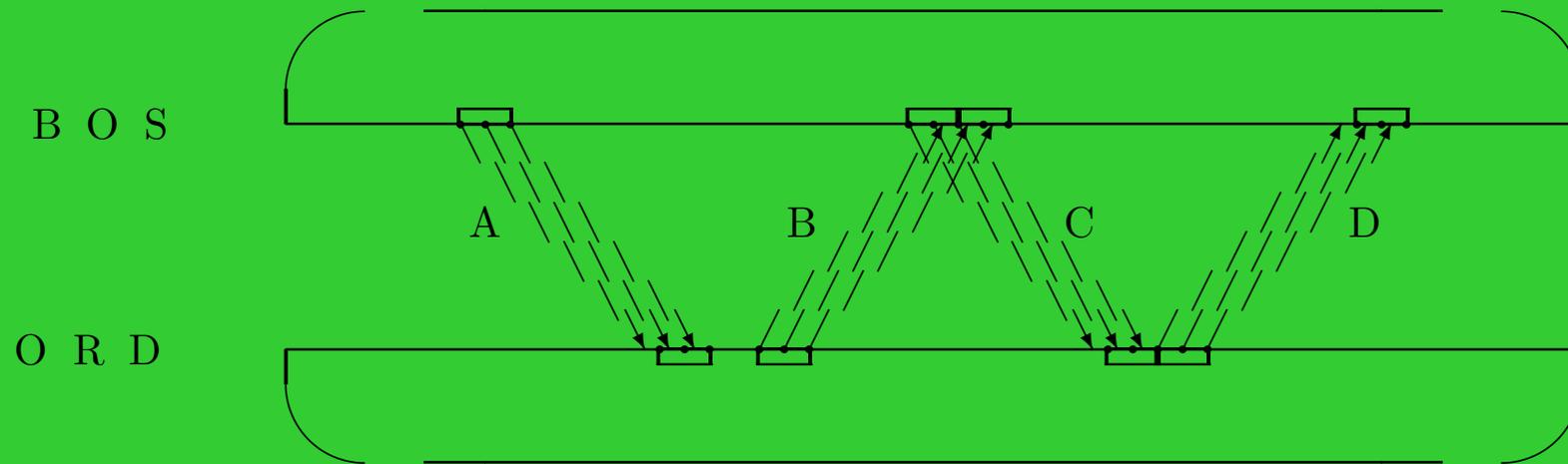
F i g u r e   1 :   T i m e   w i n d o w   fl i g h t   n e t w o r k

The New Model

- Replace single flight arc with cluster of flight "copies"
  - Try various window widths and copy intervals
  - Maintain bank structure to ensure appropriate passenger connection times are still met
- Change cover constraints to accommodate flight copies

Modified Notations for FAMTW

- Decision Variables
  - $f_{n,k,i}$ equals 1 if fleet type k is assigned to copy n of flight leg i, and 0 otherwise
- Parameters
  - $C_{n,k,i}$ is the assignment cost of assigning fleet k to copy n of flight leg i
- Sets
  - N_{ki} is the set of all copies of flight leg i

Fleet Assignment with Time Windows Model (FAMTW)

$$Min \sum_{i \in L} \sum_{k \in K} \sum_{n \in N_{ki}} C_{n,k,i} f_{n,k,i}$$

$$\sum_{k \in K} \sum_{n \in N_{ki}} f_{n,k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{(i,n) \in I(k,o,t)} f_{n,k,i} - y_{k,o,t^+} - \sum_{(i,n) \in O(k,o,t)} f_{n,k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{(i,n) \in CL(k)} f_{n,k,i} \leq N_k \qquad \forall k \in K$$

$$f_{n,k,i} \in \{0,1\}$$

$$y_{k,o,t} \geq 0$$

Network Pre-Processing To Reduce Model Size

- Node consolidation

- Redundant flight copies elimination

- Islands

Direct Solution Technique (DST)

- Branch-and-bound with Specialized Branching
- Specialized branching:
    - Special ordered sets (SOS)
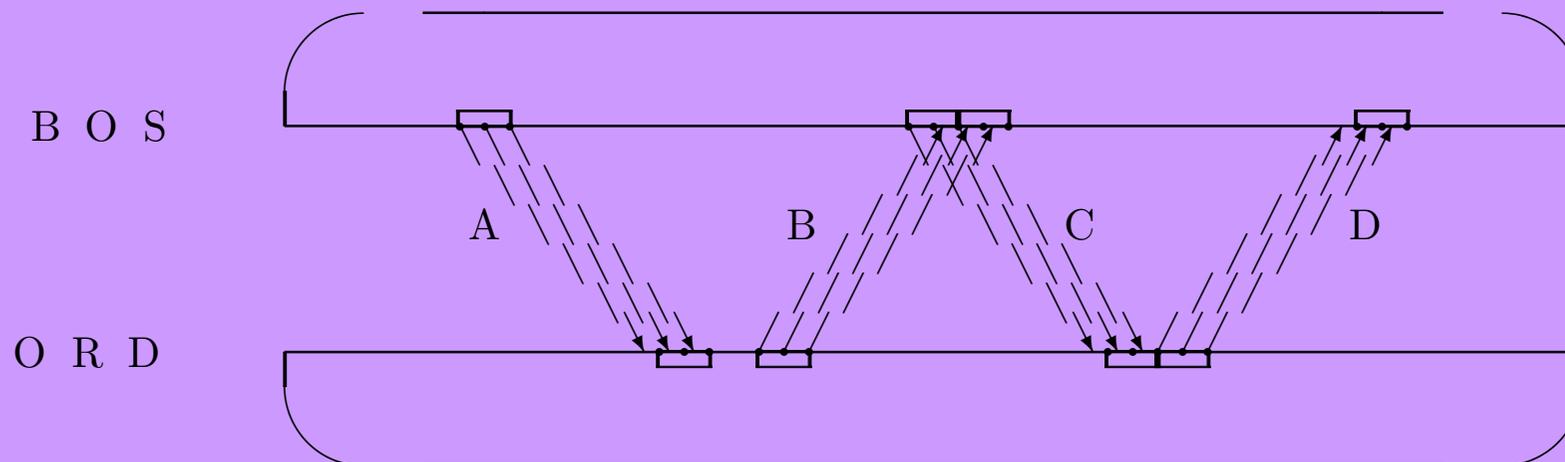
Iterative Solution Technique (IST)
Motivation

- Not all flights need multiple flight copies, generate as needed

- Solve larger problems, perhaps more quickly than the Direct Solution Technique (DST)

- Make the problem smaller -- this may be useful if we would like to merge FAMTW with other models

Solution Analysis
Time Window Width

		width = 0 Daily Cost	width = 20 Improvement	width = 40 Improvement
P1	**Total**	*28,469,066*	*126,553*	*212,873*
	Operation	*25,792,433*	*16,751*	*243,789*
	Spill	*2,676,632*	*(37,198)*	*(30,916)*
P2	**Total**	*29,339,882*	*68,107*	*92,084*
	Operation	*26,081,053*	*12,777*	*5,406*
	Spill	*3,258,829*	*55,330*	*86,678*

Solution Analysis
Flight Copy Interval

Improvements in optimal objective function value when using 20-minute time windows

	Interval = 20	Interval = 5	Interval = 1
P 1	*122,743*	*126,553*	*126,882*
P 2	*67,320*	*67,819*	*67,139*

Solution Analysis
Re-fleeting and Re-timing

	P1-20.5	P2-20.5
# Re-fleeted	236	207
% Re-fleeted	14.56%	10.16%
# Re-timed	129	111
% Re-timed	7.96%	5.45%
Avg. time Shift	8.84 min.	8.41 min

Solution Analysis
Aircraft Utilization

Do time windows allow us to save aircraft?

	TW = 0		TW = 20	
	a/c req'd	cost	a/c req'd	cost
P1	365	28,261,302	363	28,114,913
P2	428	29,000,175	426	28,965,409

Free Flight

FAMTW Application to Free Flight

Data Sets

Problem	# of Fleets	# of Flights	# of Aircraft
P1	2	456	112
P2	3	1,445	299
P3	7	2,037	432

Results

	P1				P2				P3			
	Base	Base w/ RBT	M in # A/C	M in A/C w/ RBT	Base	Base w/ RBT	M in # A/C	M in A/C w/ RBT	Base	Base w/ RBT	M in # A/C	M in A/C w/ RBT
FAM												
Cost (million $/day)	8.23	8.21	8.23	8.22	20.15	20.12	20.15	20.14	28.96	28.89	29.00	28.96
Cost Increase ($/day)		-21,406	-407	-15,263		-32,365	0	-6,830		-73,124	41,415	1,320
# of Aircraft saved			-	-			-	3			4	10
FAM TW												
Cost (million $/day)	8.21	8.20	8.21	8.21	20.12	20.08	20.12	20.11	28.89	*	28.95	28.91
Cost Increase ($/day)		-11,613	0	-7,043		-31,729	6,283	-8,839		-	59,807	17,097
# of Aircraft saved			-	1			1	5			9	17

* Did not reach solution.

Conclusions

- Time windows can provide significant cost savings, as well as a potential for freeing aircraft

- Good run times for DST, especially because copies need not be placed at a fine interval

- IST provides problem size "capacity" so that FAMTW may be enhanced, integrated with other models, etc.

- Applications: Don't underestimate value of modeling