1.206J/16.77J/ESD.215J
Airline Schedule Planning

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Spring 2003

The Extended Crew Pairing Problem with Aircraft Maintenance Routing

Outline

- Review of Individual Problems
- Interdependence and motivation for an alternative approach
- Sequential Approaches
- Integrated Approaches
- Comparison of Models

The Maintenance Routing Problem (**MR**)

- Given:
  - Flight Schedule for a single fleet
    - Each flight covered exactly once by fleet
  - Number of Aircraft by Equipment Type
    - Can't assign more aircraft than are available
  - FAA Maintenance Requirements
  - Turn Times at each Station
  - Through revenues for pairs or sequences of flights
  - Maintenance costs per aircraft

MR Problem Objective

- Find:
  - Revenue maximizing assignment of aircraft of a single fleet to scheduled flights such that each flight is covered exactly once, maintenance requirements are satisfied, conservation of flow (balance) of aircraft is achieved, and the number of aircraft used does not exceed the number available

MR String Model: Variable Definition

- A string is a sequence of flights beginning and ending at a maintenance station with maintenance following the last flight in the sequence
  – Departure time of the string is the departure time of the first flight in the sequence
  – Arrival time of the string is the arrival time of the last flight in the sequence + maintenance time

MR String Model: Constraints

- Maintenance constraints
  - Satisfied by variable definition
- Cover constraints
  - Each flight must be assigned to exactly one string
- Balance constraints
  - Needed only at maintenance stations
- Fleet size constraints
  - The number of strings and connection arcs crossing the count time cannot exceed the number of aircraft in the fleet

MR String Model: Solution

- Integer program
  - Branch-and-bound with too many variables to consider all of them
  - Solve Linear Program using **Column Generation**
- Branch-and-Price
  - Branch-and-bound with bounding provided by solving LP's using column generation at each node of the branch-and-bound tree

Crew Pairing Problem (**CP**)

- Given:
  - Flight Schedule for a *fleet family*
    - Each flight covered exactly once
    - Usually daily or weekly schedule
  - FAA and Collective Bargaining Agreements
    - Rest
    - Maximum duty, sit, flying times in a duty
    - 8-in-24 rule
    - Maximum time-away-from-base
    - Brief/debrief
  - Crew base locations
  - Minimum connection times between aircraft at each station
  - Number of crews at each crew base

CP Cost Function

- Duty cost is maximum of:
  - Flying time
  - f_1 * elapsed duty time
  - Minimum duty pay
- Pairing cost is maximum of:
  - Sum of duty costs
  - f_2 * time-away-from-base
  - f_3 * number of duties

CP Problem Objective

- Find:
  - Cost minimizing assignment of crews to scheduled flights such that each flight is covered exactly once and all collective bargaining and FAA work rules are satisfied (and the number of crews assigned does not exceed the number available)

CP Set Partitioning Model: Variables and Constraints

- A variable is a *pairing*, that is, a sequence of flights beginning and ending at the same crew base and satisfying all work rules
  – Binary variables: = 1 if pairing is assigned to a crew; = 0 if pairing not flown
- Set partitioning constraints (crew size constraints often ignored) requiring each flight to be covered exactly once

CP Set Partitioning Model: Solution

- Integer program
  - Branch-and-bound with too many variables to consider all of them
  - Solve Linear Program using **Column Generation**
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MR and Its Impact on CP

- Maintenance routing problem (MR) finds a ***feasible*** assignment of aircraft to flights to ensure adequate maintenance opportunities
- Crews need enough time between two sequential flights to travel through the terminal -- ***minimum connect time***
- If both flights are covered by the same aircraft, this connection time can be reduced -- tighter connections can be permitted
- A ***short connect*** is a connection that is crew-feasible only if both flights are covered by the same aircraft

Research Objective

- Our goal is to improve crew scheduling by incorporating relevant maintenance routing decisions
- Exploit the fact that only a subset of the maintenance routing decisions impact crew scheduling
  - To decrease problem size

Motivation

- Crew costs are the second largest operating expense faced by airlines
- Small improvements in efficiency can have significant financial impact
- Scheduling options are limited by maintenance routing decisions made earlier in the airline planning process

MR then CP

Sequential Approach

- Current practice:
  - Solve MR
  - If flight *A* is followed by flight *B* in a routing string, *B* can follow *A* in a crew pairing, even if the connection is shorter than the minimum connect time
  - Output from MR is input to CP
  - All other crew connections must satisfy maximum connection time
  - Restricts set of feasible CP solutions

Sequential Solution Approach

CP then MR

Sequential Approach

- Klabjan, Johnson, and Nemhauser
  - CP costs dominate MR revenues
  - Solve CP in which *all* short connects are permitted
  - Solve MR, enforcing short connects used by CP
  - May lead to infeasibility

Integrated Approach

- Solve both problems simultaneously to find optimal solution that is feasible to both problems
- Short connects are the only link -- crew can't fly a tight connection unless the flights share an aircraft in routing solution
- Cordeau, Stojković, Soumis, and Desrosiers
  - Directly integrate string-based models
  - Basic maintainance routing and crew pairing variables and constraints, plus linking constraints
  - Benders decomposition approach using a heuristic branching strategy
  - Promising computational results

Maintenance Feasibility

- Because crew costs dominate, we will focus on ***maintenance feasibility***, rather than on through revenues
- Problem is to minimize crew pairing costs subject to maintenance feasibility
- Approaches can be easily extended to include through revenues

String Based Approach (**MRCP**)

- If y_r is the variable for a routing string and x_p is the variable for a crew pairing, linking constraint for short connect t is

$$\sum \delta_{tr} y_r - \sum \delta_{tp} x_p \geq 0$$

where δ_{tr} is 1 if routing string r contains short connect t and 0 otherwise, and δ_{pt} is 1 if crew pairing p contains short connect t and 0 otherwise

MRCP Problem

0	$\Sigma c_p x_p$
MR	**0**
0	**CP**
$\sum \delta_{tr} y_r - \sum \delta_{tp} x_p \geq 0$	

MRCP Problem Size

- Variables:
  - One for each routing string
  - One for each crew pairing
- Constraints:
  - Maintenance cover constraints
  - Maintenance balance constraints (maintenance stations only)
  - Maintenance aircraft count
  - Crew cover constraints
  - One linking constraint for each short connect

Solving MRCP

- Too many columns to enumerate explicitly -- *branch-and-price*
- Column generation:
  - Denote by τ_t the dual for the linking constraint of short connect t
  - Reduced cost of a routing string or a pairing is the same as in the original models, except add τ_t for each short connect t included
  - Can modify pricing network by adding $-\tau_t$ to the connection arc representing this turn

➢ TRACTABILITY ISSUES…

Our Objectives

- Guarantee maintenance feasibility
- Allow the user the flexibility to trade off between solution time and quality
- Leverage the fact that only a portion of the maintenance routing decisions are relevant to the crew pairing problem

Approach

- In the sequential approach, the crew scheduler is given an MR solution and solves the corresponding CP
- We'd like to allow the crew scheduler to choose from a collection of MR solutions the one which contains the most useful set of short connects
- Problem: We don't want to solve one CP for each MR solution
- Solution: Extended crew pairing model (ECP)

The Extended Crew Pairing Model (ECP)

- In addition to choosing crew pairings, select one maintenance routing solution from a given set of feasible solutions
- Add constraints that prohibit pairings containing a short connect from being selected unless the chosen maintenance solution also contains that short connect

Notation

- P^k is the set of feasible pairings for fleet type k
- F^k is the set of daily flights assigned to fleet type k
- T^k is the set of short connects for the flights assigned to fleet type k
- S^k is the set of feasible MR solutions for the flights assigned to fleet type k

Notation, cont.

- δ_{fp} is defined to be 1 if flight f is included in pairing p, else 0
- α_{ts} is defined to be 1 if MR solution s includes short connect t, else 0
- β_{tp} is defined to be 1 if short connect t is contained in pairing p, else 0
- c_p is the cost of pairing p

Notation, cont.

- y_s is a binary decision variable – value 1 indicates that MR solution s is chosen, else 0
- x_p is a binary decision variable – value 1 indicates that pairing p is chosen, else 0

General Formulation

$$\min \sum_{p \in P^k} c_p x_p$$

$$st$$

Flights:

$$\sum_{p \in P^k} \delta_{fp} x_p = 1 \qquad \forall f \in F^k$$

short connects:

$$\sum_{s \in S^k} \alpha_{ts} y_s - \sum_{p \in P^k} \beta_{tp} x_p \geq 0 \qquad \forall t \in T^k$$

Convexity:

$$\sum_{s \in S^k} y_s = 1$$

$$y_s, x_p \in \{0,1\} \qquad \forall s, p$$

An Example

- Flights:

A B C D E F G H

- short connects:

A-B A-C C-D

- MR solution (y_1) uses short connects A-C and C-D
- MR solution (y_2) uses short connect A-B

- Potential pairings:
  - A-C-D-F (x_1) - \$1
  - A-B-E-F (x_2) - \$2
  - C-D-G-H (x_3) - \$4
  - B-E-G-H (x_4) - \$6
- Crew pairing solutions:
  - y_1 => pairings 1, 4 -- \$7
  - y_2 => pairings 2, 3 -- \$6

Matrix Representation

	y_1	y_2	x_1	x_2	x_3	x_4		rhs	
Flights:	0	0	1	1	0	0	=	1	A
	0	0	0	1	0	1	=	1	B
	0	0	1	0	1	0	=	1	C
	0	0	1	0	1	0	=	1	D
	0	0	0	1	0	1	=	1	E
	0	0	1	1	0	0	=	1	F
	0	0	0	0	1	1	=	1	G
	0	0	0	0	1	1	=	1	H
short connects:	0	1	0	−1	0	0	≥	0	A-B
	1	0	−1	0	0	0	≥	0	A-D
	1	0	−1	0	0	0	≥	0	D-G
Convexity:	1	1	0	0	0	0	=	1	Conv.

Solving ECP

- Too many columns to enumerate explicitly -- *branch-and-price*
- Column generation:
  - Doesn't change for crew variables:
    - Denote by τ_t the dual for the linking constraint of short connect t
    - Reduced cost of a routing string or a pairing is the same as in the original models, except add τ_t for each short connect t included
    - Can modify pricing network by adding $-\tau_t$ to the connection arc representing this turn
  - Generating a MR solution variable is the same as solving MR with modified costs
    - Minimize negative of duals on short connect connection arcs

Comparison of Models

- Time to solve maintenance pricing problems
  - MRCP generates routing strings
  - ECP generates routing solutions
    - Might require generating routing strings!
    - Re-optimizing with new objective function; initial column set and known feasible solution

Comparison of Models (cont.)

- Size of restricted master problems
  - MRCP has one column for each routing string
  - ECP has one column for each routing solution -- many more columns?!
    - Redundancy
    - Dominance
    - Example
      - 14 flights
      - 104 feasible routing strings
      - 16 maintenance routing solutions
      - 11 unique short connect sets
      - 6 dominant short connect sets

ECP Enhancements

- Dramatically reduce the **number** of MR columns:
  - **Uniqueness**: Eliminate redundant columns
    - Example: 41 flights, single set of short connects => >>8,700 solutions => 1 column
  - **Maximal independence**: Eliminate dominated columns
- Only need one column per *unique*, *maximally independent*, *maintenance feasible* short connect set
- Theoretical bounds and computational observations
  - Example: 61 flights => >> 25,000 solutions => 4 required columns

ECP Enhancements, cont.

- Relax the **integrality** of MR columns:
  - Same number of binary variables as original CP

- LP relaxation of ECP is tighter than LP relaxation of a basic integrated approach

Generating MR Solutions

- Algorithm to generate UMI columns -- can generate k UMI columns in at most the time needed to solve k MR problems
- Can generate columns using column generation to take advantage of dual information -- pricing problem still yields UMI columns
- Can still use existing crew pairing generators

Computational Experiment

- **Problem A**:

Lower bound:	31,396.10
ECP with 16 MR columns:	31,396.10
Optimality gap:	0%

- **Problem B**:

Lower bound:	25,076.60
ECP with 20 MR columns:	25,498.60
Optimality gap:	1.7%

Observations

- Zero optimality gap in instance A doesn't necessarily imply sequential approach would yield an optimal solution – many equivalent crew pairing solutions, some maintenance feasible and some not
- Number of short connects in an optimal solution is small relative to total number – UMI sets often capture many of them
  - A: 58 max; ~38 per column; 9 used in solution
  - B: 68 max; ~37 per column; 10 used in solution

Benefits of ECP

- Ensures maintenance feasibility
- Can be solved heuristically or to optimality, allowing user to trade off solution time and quality
- Leverages the fact that only short connect decisions from the maintenance routing problem impact crew pairing

Benefits of ECP, cont.

- No more binary variables than the basic crew pairing model alone
- Tighter LP relaxation than a basic integrated approach
- Flexible
  - Can take advantage of advances in maintenance routing solvers and crew pairing generators
  - Can incorporate new maintenance constraints

Conclusions

- Crew scheduling is critical to airline profitability but quality can be compromised by making maintenance routing decisions independently
- A direct integration can be inflexible and difficult to solve
- ECP provides an alternative approach that exploits the fact that only some maintenance routing information is relevant