1.206J/16.77J/ESD.215J
Airline Schedule Planning

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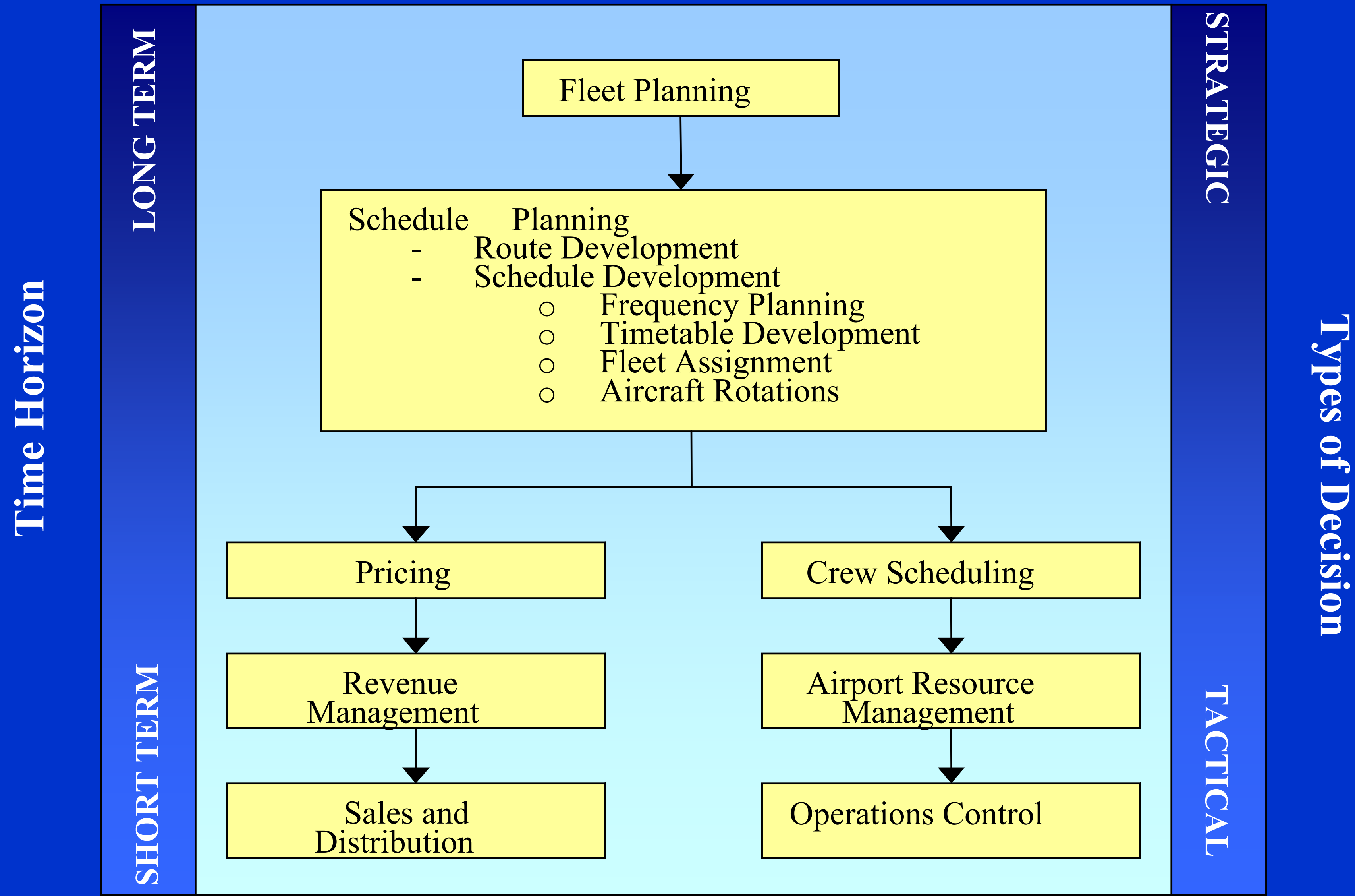
Outline

- Sign-up Sheet
- Syllabus
- The Schedule Planning Process
- Flight Networks
  - Time-line networks
  - Connection networks
- Acyclic Networks
- Shortest Paths on Acyclic Networks
- Multi-label Shortest Paths on Acyclic Networks


Time Horizon
LONG TERM
SHORT TERM
Fleet Planning
Schedule Planning
- Route Development
- Schedule Development
○ Frequency Planning
○ Timetable Development
○ Fleet Assignment
○ Aircraft Rotations
Pricing
Revenue Management
Sales and Distribution
Crew Scheduling
Airport Resource Management
Operations Control
STRATEGIC
TACTICAL
Types of Decision

Airline Schedule Planning

Schedule Design	Select optimal set of *flight legs* in a schedule
↓	A flight specifies origin, destination, and departure time
Fleet Assignment	Contribution = Revenue - Costs
↓	
Aircraft Routing	Assign crew (pilots and/or flight attendants) to flight legs
↓	
Crew Scheduling	

Airline Schedule Planning: Integration

Schedule Design

↓

Fleet Assignment

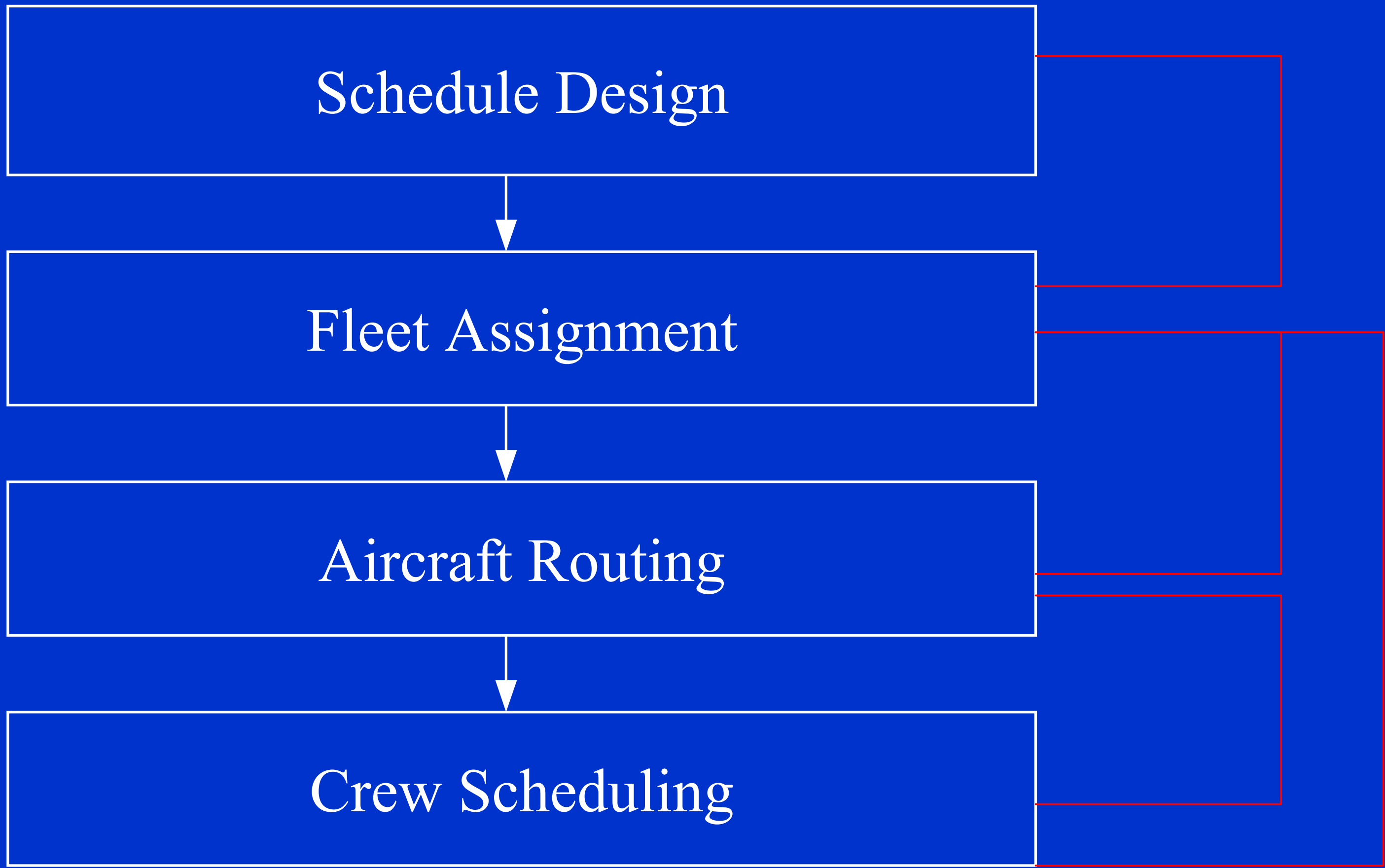
↓

Aircraft Routing

↓

Crew Scheduling

Airline Schedule Planning: Integration



Flight Schedule

- Minimum turn times = 30 minutes

Flight No.	Origin	Destin.	Dep. Time	Arrival Time
1	A	B	6:30	8:30
2	B	C	9:30	11:00
3	C	B	16:00	17:00
4	B	A	18:00	20:00

Time-Space Flight Network Nodes

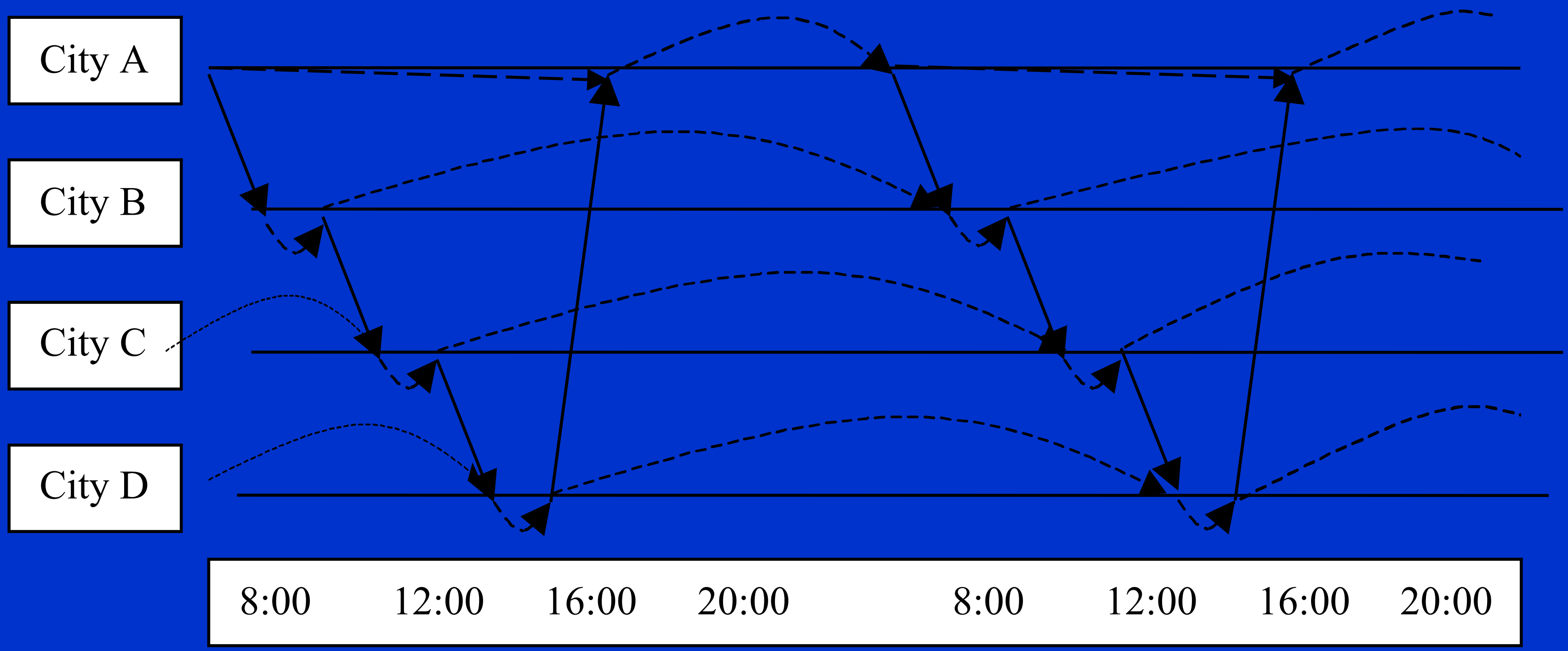
- Associated with each node *j* is a **location** ***l(j)*** and a **time** ***t(j)***
- **A Departure Node** ***j*** corresponds to a flight departure from location *l(j)* at time *t(j)*
- **An Arrival Node** ***j*** corresponds to a flight arrival at location *l(j)* at time *t(j)* – *min_turn_time*
  - *t(j)= arrival time of flight + min_turn_time = flight ready time*

Time-Space Flight Network Arcs

- Associated with each arc *jk* (with endnodes *j* and *k*) is an aircraft movement in space and time
- **A Flight Arc *jk*** represents a flight departing location *l(j)* at time *t(j)* and arriving at location *l(k)* at time *t(k)* – *min_turn_time*
- **A Ground Arc or Connection Arc *jk*** represents an aircraft on the ground at location *l(j)* (= *l(k)*) from time *t(j)* until time *t(k)*

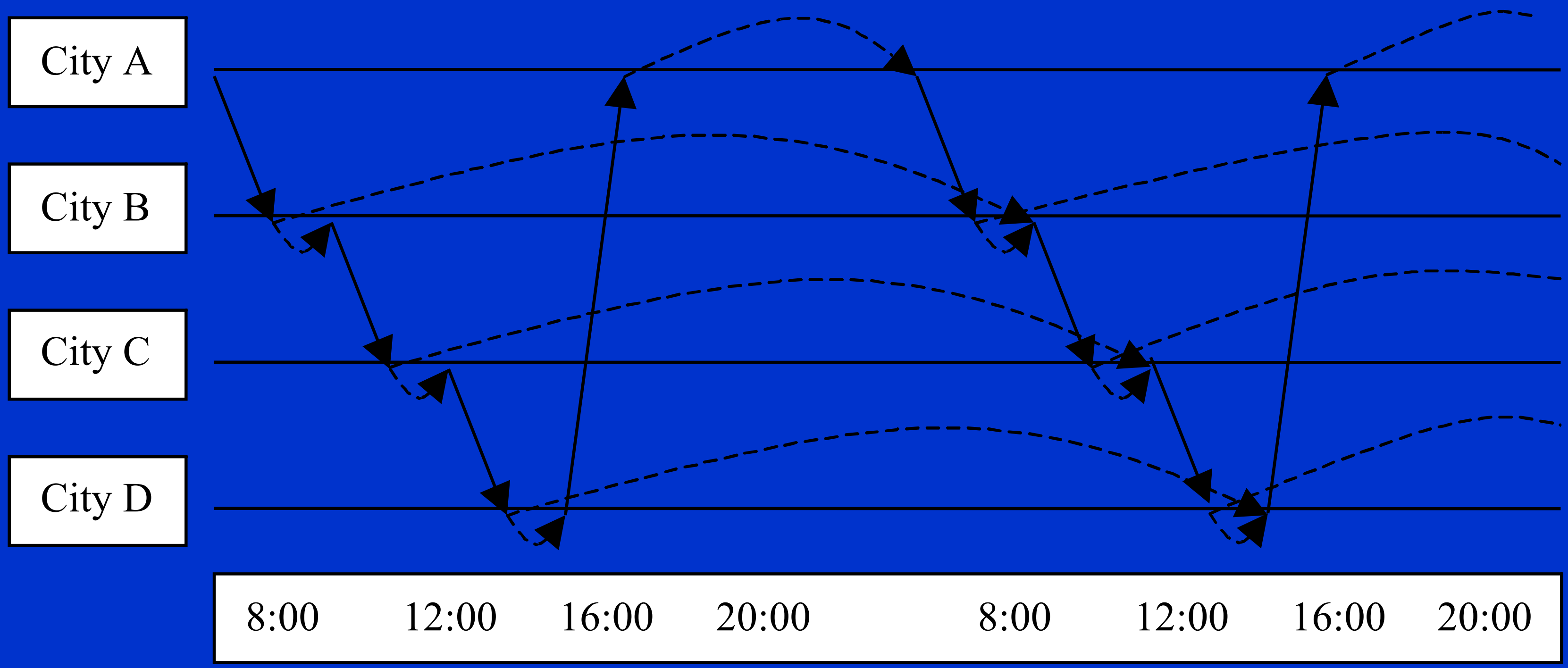
Time-Line Network

- Ground arcs



Connection Network

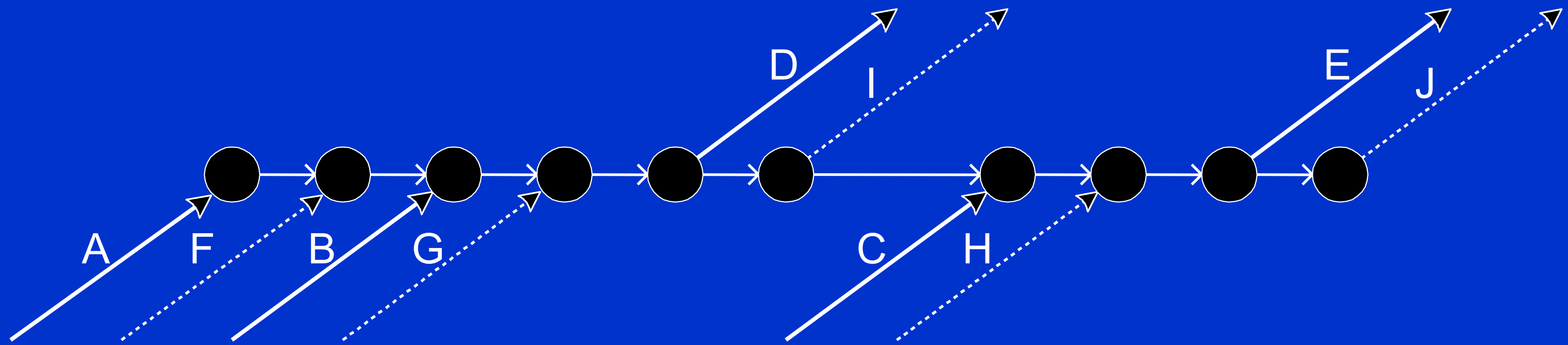
- Connection arcs



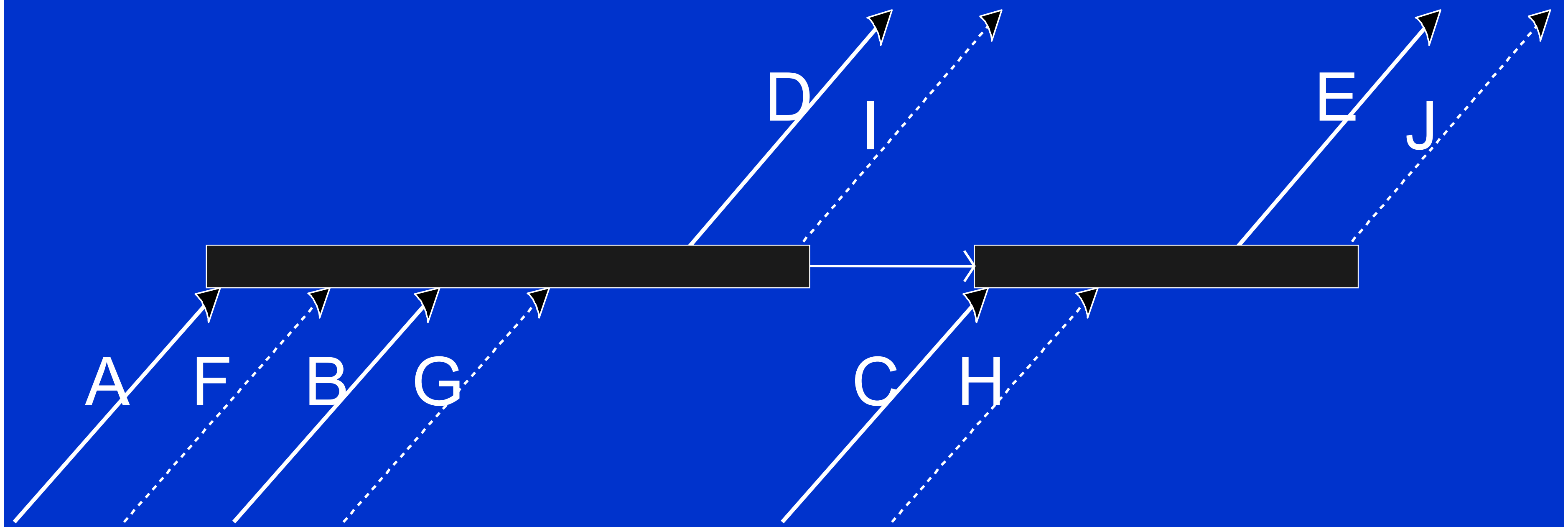
Time-Line vs. Connection Flight Networks

- For large-scale problems, time-line network has fewer ground arcs than connection arcs in the connection network
  - Further reduction in network size possible through "node consolidation"
- Connection network allows more complex relations among flights
  - Allows a flight to connect with *only a subset* of later flights

Time-Line Network



Node Consolidation



Flight Networks and Shortest Paths

- Shortest paths on flight networks correspond to:
  - Minimum cost itineraries for passengers
  - Maximum profit aircraft routes
  - Minimum cost crew work schedules (on crew-feasible paths only)

➢ Important to be able to determine shortest paths in flight networks

Shortest Path Challenges in Flight Networks

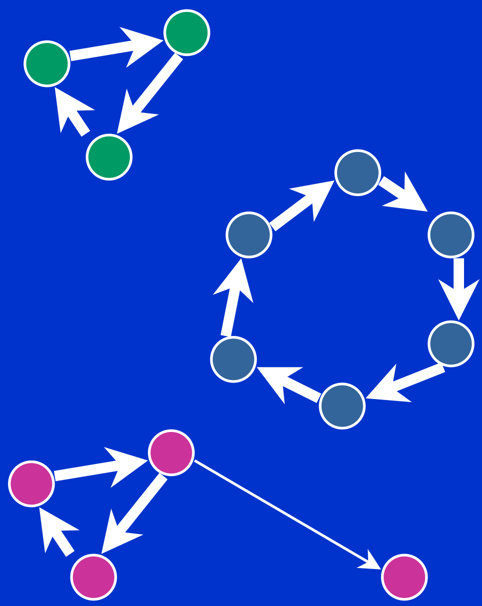
- Flight networks are large
  - Thousands of flight arcs and ground arcs; thousands of flight arcs and tens of thousands connection arcs
- For many airline optimization problems, repeatedly must find shortest paths
- Must consider only "feasible" paths when determining shortest path
  - "Ready time" (not "arrival time") of flight arrival nodes ensures feasibility of aircraft routes
  - Feasible crew work schedules correspond to a *small* subset of possible network paths
    - Identify the shortest "feasible" paths (i.e., feasible work schedules) using multi-label shortest path algorithms

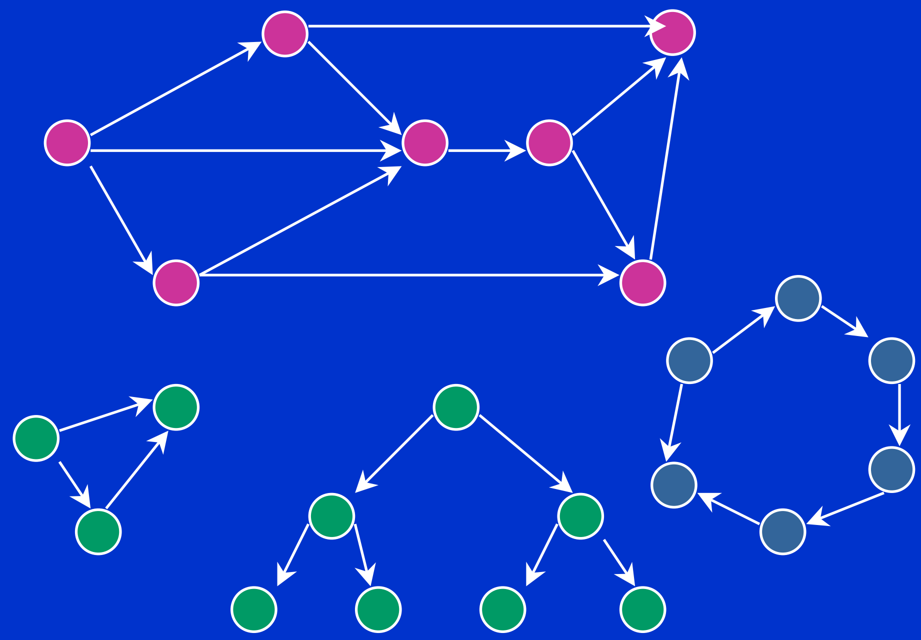
Acyclic Directed Networks

- Time-line and Connection networks are acyclic directed networks

Acyclic Networks

Cyclic Networks

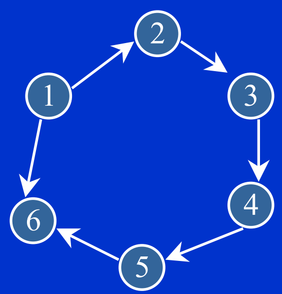


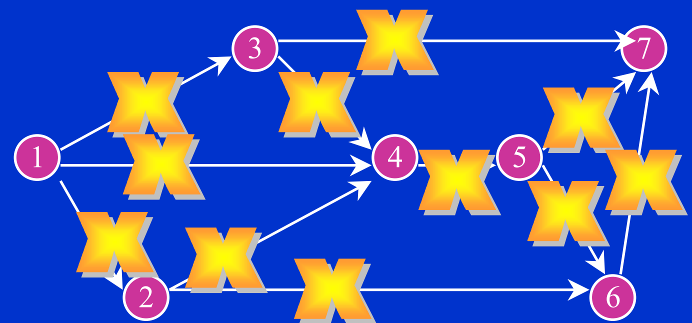


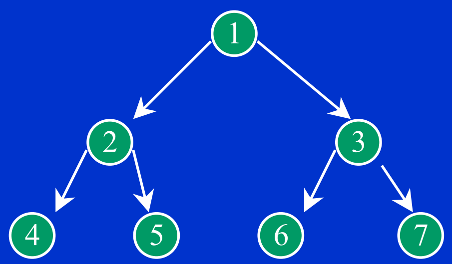
Acyclic Networks and Shortest Paths

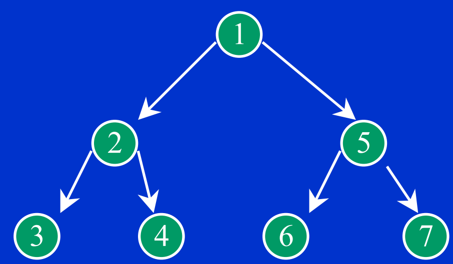
- Efficient algorithms exist for finding shortest paths on acyclic networks
  - Amount of work is directly proportional to the number of arcs in the network
  - Topological ordering necessary
    - Consider a network node j and let $n(j)$ denote its number
    - The nodes of a network G are topologically ordered if for each arc jk in G, $n(j) < n(k)$

Topological Orderings





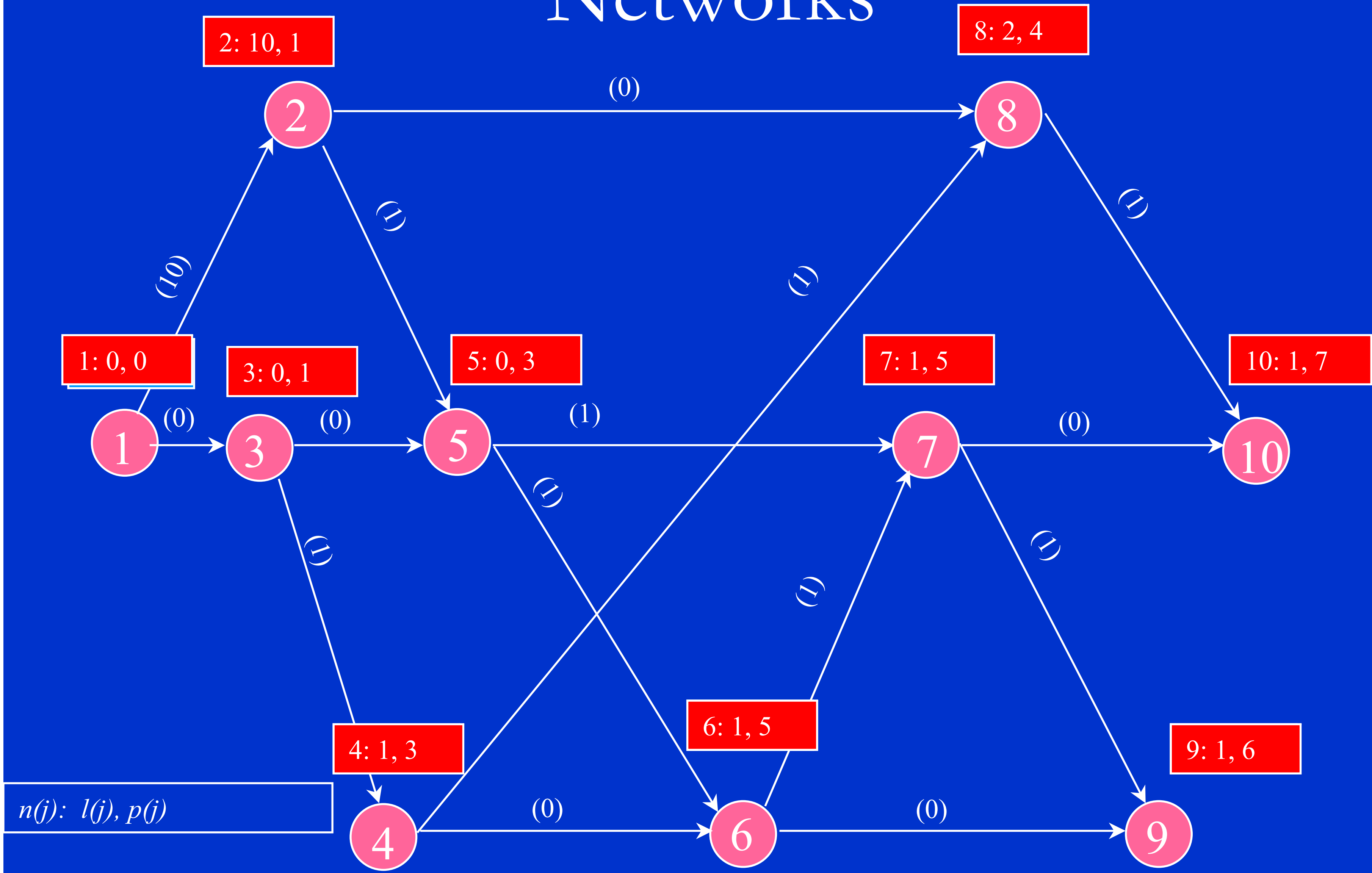




Topological Ordering Algorithm

- Given an acyclic graph G, let $n= 1$ and $n(j)=0$ for each node j in G
- Repeat until $n=|N|+1$ (where $|N|$ is the number of nodes in G)
  - Select any node j with no incoming arcs and $n(j) = 0$.
  - Let $n(j) = n$
  - Delete all arcs outgoing from j
  - Let $n = n+1$

Shortest Paths on Acyclic Networks



Shortest Path Algorithm for Acyclic Networks

- Given acyclic graph *G*, let *l(j)* denote the length of the shortest path to node *j*, *p(j)* denote the predecessor node of *j* on the shortest path and *c(jk)* the cost of arc *jk*
- Set *l(j)=infinity* and *p(j)= -1* for each node *j* in *G*, let *n=1*, *and* set *l(1)= 0* and *p(1)=0*
- For *n*<|*N*|+*1*
  - Select node *j* with *n(j)* = *n*
  - For each arc *jk* let *l(k)=min(l(k), l(j)+c(jk))*
    - If *l(k)=l(j)+c(jk)*, set *p(k)=j*
  - Let *n* = *n*+*1*

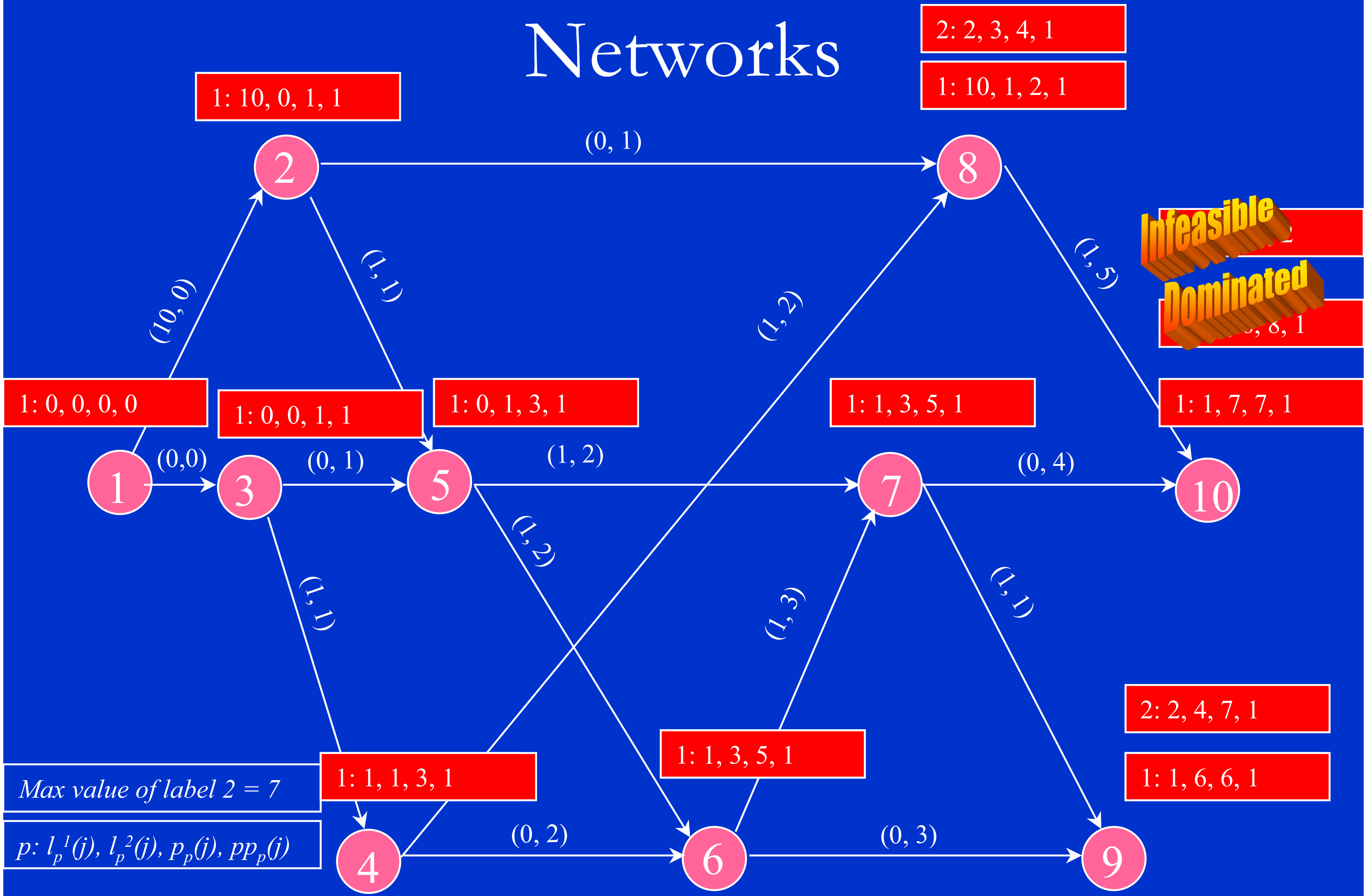
Multiple Label (Constrained) Shortest Paths on Acyclic Networks

- Consider the objective of finding the minimum cost path with flying time less than a specified value F
- Let label $t_p(j)$ denote the flying time on path p to node j and label $l_p(j)$ denote the cost of path p to node j, for any j
- Only paths with $t_p(j) < F$, at any node j, are considered (the rest are excluded)
- A label set must be maintained at node j for each non-dominated path to j
  - A path p' is dominated by path p at node j if $l_{p'}(j) > l_p(j)$ and $t_{p'}(j) > t_p(j)$
  - If p' is not dominated by any path p at node j, p' is non-dominated at j
  - In the worst case, a label set is maintained at each node j for each path p into j

Constrained Shortest Paths and Crew Scheduling

- Label sets are used to ensure that the shortest path is a "feasible" path
  - Labels are used to count the number of work hours in a day, the number of hours a crew is away from their home base, the number of flights in a given day, the number of hours rest in a 24 hour period, etc…
  - In some applications, there are over 2 dozen labels in a label set
- Many paths are non-dominated
- Exponential growth in the number of label sets (one set for each non-dominated path) at each node

Shortest Paths on Acyclic Networks



Constrained Shortest Path Notation for Acyclic Networks

- Given acyclic graph G,
  - $l_p^k(j)$ denotes the value of label k (e.g., length, flying time, etc.) on label set p at node j
  - $p_p(j)$ denotes the predecessor node for label set p at node j
  - $pp_p(j)$ denotes the predecessor label set for label set p at node j
  - $c(jk)$ denotes the cost of arc jk
  - m denotes the maximum possible number of non-dominated label sets at any node j
  - $np(j)$ denotes the number of non-dominated label sets for node j

Constrained Shortest Path Algorithm for Acyclic Networks

- For $p = 1$ to m, let $l_p^k(j)=infinity$ for each k, and $np(j)=0$, $p_p(j)= -1$ and $pp_p(j)= -1$ for each node j in G
- Let $n=1$ *and* set $np(1)=1$, $l_1^k(1)= 0$ for each k, $p_1(1)=0$ and $pp_1(1)=0$
- For $n<|N|+1$
  – Select node i with $n(i) = n$
  – For each non-dominated p at node i
    - For each arc ij, let $np(j)= np(j)+1$, $p_{np(j)}(j)=i$, $pp_{np(j)}(j)=p$
      – For each k, let $l_{np(j)}^k(j) =l_p^k(i)+c(ij)$
    - If $l_{np(j)}^k(j)>l_s^k(j)$ for some $s=1,..,np(j)-1$, then dominated and set $np(j)= np(j)-1$
  – Let $n = n+1$