

1.224 Carrier Systems, 2003  
Problem Set 2 -- SOLUTIONS

1.

(a) see excel spreadsheet

(b)

- All demand for a given region must be satisfied by a single source

$$\sum_i y_{ij} = 1, \forall j \in J$$

- Each city can only have a maximum of 2 shipping lanes open.

$$\sum_j y_{ij} \leq 2, \forall i \in I$$

- If lane 1 to A is open, then lane 2 to A has to be open

$$y_{2A} \geq y_{1A}$$

- If lanes 1 to B and 2 to B are open, then lane 3 to B has to be closed

$$y_{1B} + y_{2B} + y_{3B} \leq 2$$

- If either lane 1 to C or 2 to C is open, lane 3 to C must be open

$$y_{3C} \geq y_{1C}$$

$$y_{3C} \geq y_{2C}$$

**Problem 2 (1999 Exam Question 2)**

Consider the following integer problem:

$$MIN \quad 6x_1 + 9x_2 + 4x_3 + x_4$$

s.t.

$$2x_1 + 4x_2 + 3x_3 + x_4 \geq 6$$

$$x_i \in \{0,1\}, \forall i$$

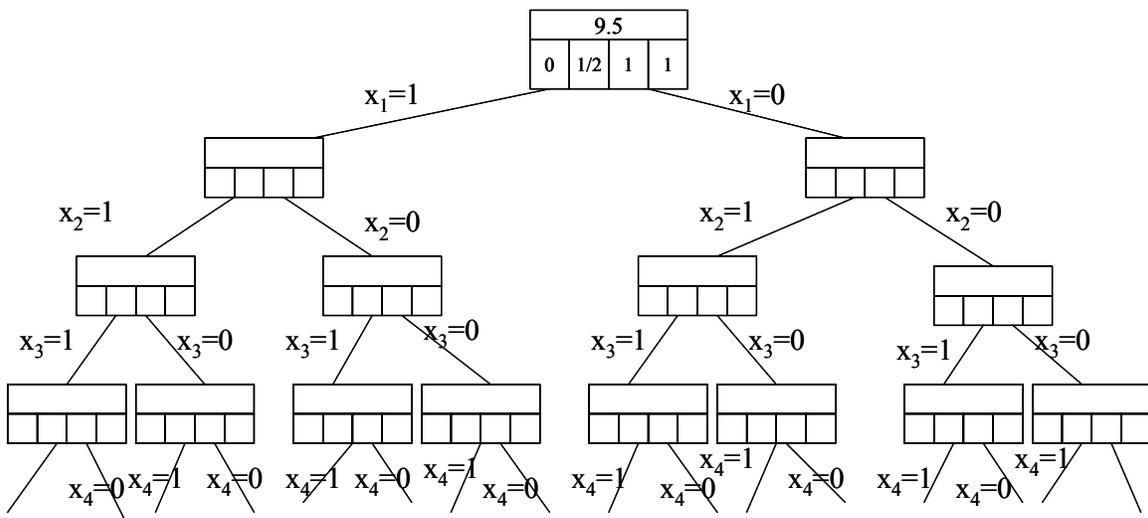
Consider the following lower bound algorithm:

- Step 0: Set  $x_i=0$  for each  $i=1,2,3,4$ . Let the candidate list contain items 1, 2, 3 and 4.
- Step 1: Determine the ratio of each item's cost to its constraint "contribution"
- Step 2: Select the item in the candidate list with the smallest ratio. Call it  $i$  and remove  $i$  from the candidate list.
- Step 3: Given  $x_j$  values for each item  $j$  removed from the candidate list; set the value of  $x_i$  to the minimum of
  - $x_i=1$
  - $x_i$  such that  $2x_1 + 4x_2 + 3x_3 + x_4 = 6$
- Step 4: If  $2x_1 + 4x_2 + 3x_3 + x_4 = 6$ , STOP! Otherwise, return to Step 2.

Example:

The ratios are  $6/2$ ,  $9/4$ ,  $4/3$ , and  $1/1$ . Item 4 has the smallest ratio. We set  $x_4$  as large as possible ( $=1$ ). The next smallest ratio is for item 3. We set  $x_3=1$  and the left hand side of the constraint sums to 4. The next item is item 2. If we set  $x_2=1/2$ , then the constraint is satisfied and we stop. The simple algorithm solves the LP relaxation and yields a solution (lower bound with cost of 9.5).

- (a) Find the upper bound on the optimal integer solution and describe how you obtained it.
- (b) Using the lower bound algorithm, fill-in the attached branch and bound tree to find the optimal integer solution. Clearly mark the branches you prune and indicate the reason for pruning.



- (c) Next, we add the following constraint to the formulation.  
 $10x_1 + 8x_2 + 4x_3 + 7x_4 \leq 17$

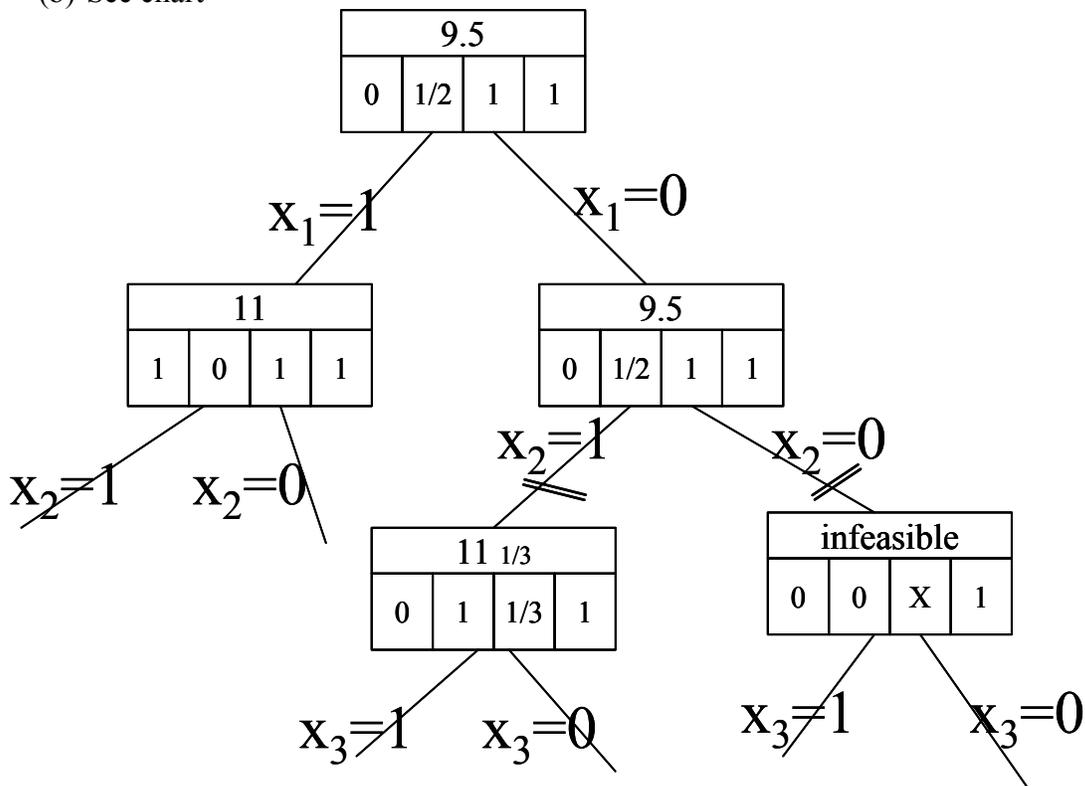
When you solve the LP relaxation and print it out, the printer fails and you have the following incomplete output. Fill in the missing entries and explain your reasoning.

Variable	Value	Red. Cost
X2	0.5	-----
Constraint	Value	Shadow price
Constr1	-----	-2.25
Constr2	15	----

(d) Based on this output, what is the most you would be willing to pay to reduce the first constraint from 6 to 5?

(a) An upper bound on the optimal integer solution is provided by any feasible integer solution. The simplest (and worst) bound is provided by setting all the variables to 1. This yields an upper bound of 20.

(b) See chart



(c)

Variable	Value	Red. Cost
X2	0.5	0

<b>Constraint</b>	<b>Value</b>	<b>Shadow price</b>
Constr1	<b>6</b>	-2.25
Constr2	15	<b>0</b>

- Reduced cost must be 0 by the optimality condition (Red.cost (var)=0)
- Since we have a non-zero shadow price for constraint 2, constraint 2 must be tight and its value must equal the right hand side value (i.e. 6).
- Since constraint 3 is not tight, it must have a shadow price of zero.

(d) The most we'd be willing to pay is 2.25.