+Sheets A, B & C

III INTERPARTICLE FORCES: Components and Interaction

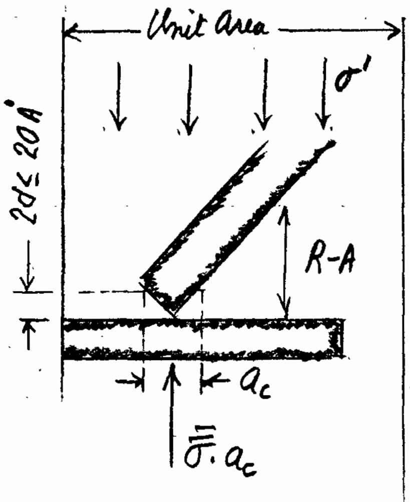
(1.361 Reference)

1 COMPONENTS OF EFFECTIVE STRESS (II 2-25)

1.1 Physico-Chemical Effective Stress Eqn (Ladd 1961)

1) Objective - general idea of how σ' is transmitted between particles in cohesive soil.

2) Assume "contact" forces act when $2d \leq 20\text{Å}$ (rather arbitrary) over area ratio a_c & long range double layer type forces act at $2d > 20\text{Å}$



3) Eqn: σ' = net contact stress + net long range stress

$$= \bar{\bar{\sigma}} \cdot a_c \quad + \quad R - A$$

$$= (\bar{\bar{\sigma}}_r - \bar{\bar{\sigma}}_a)\, a_c \quad + \quad R - A$$

4) Long range stresses ($2d > 20\text{Å}$)

- Double layer (osmotic) repulsion $R = f(P_r)$ - Sheets A & B
- Long range van der Waals attraction $A = f(P_a)$

$$P_a \text{(parallel particles)} = \frac{A''}{48\pi}\left(\frac{1}{d^3} + \frac{1}{(d+\delta)^3} - \frac{2}{(d+\frac{\delta}{2})^3}\right)$$

A'' = Hamaker constant $\simeq 2 \pm 0.5 \times 10^{-20}$ J (JKM '93, p 124)
δ = particle thickness
d = half spacing between particles

$\therefore A \propto 1/d^3$, increases w/ δ & thought to be indep. of pore fluid

5) Contact stresses ($2d < 20$ Å)

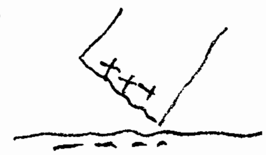
Repulsion $\bar{\bar{\sigma}}_r$ = displacement of "adsorbed" water (pore fluid when not H_2O)

+ Born repulsion of mineral contact

(+ edge to face repulsion for neg. edge charge)

Attractive $\bar{\bar{\sigma}}_a$ = short range van der Waals

+ edge-to face electrostatic attraction (for +edge & neg. face)



+ primary valence bonding of mineral contact (ionic & covalent)

+ cementation (like carbonates, iron oxides..)

} put together

1.2 Discussion

1) Components for granular soils

• Is there a DL? • $\sigma' = \bar{\bar{\sigma}} \cdot a_c$ (10,000s atm)

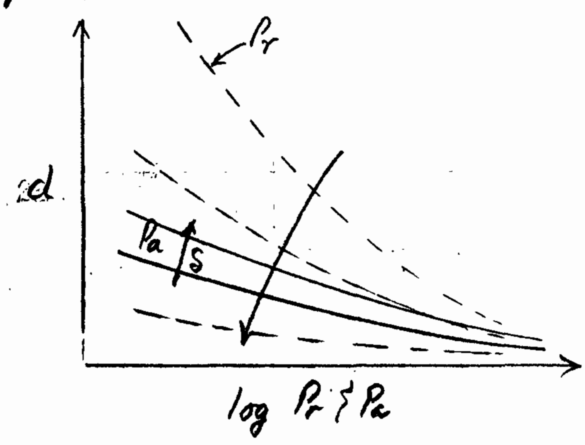
2) Effects of pore fluid on R-A for cohesive soils with "high" SSA

Decreasing $R = f(P_r)$ for

(1) EC valence ν

(2) Bulk c_0

(3) Dielectric const. D



• For clay in sea water ($35\,g/l \approx 1.1M \rightarrow c_0 = 0.6M$): $R \gtrless A$?

• For clay in alcohol ($D = 20$ vs 80 for H_2O): $R \gtrless A$?

3) Effects of pore fluid on $\bar{\bar{\sigma}} \cdot a_c$ for cohesive soils

What trends for

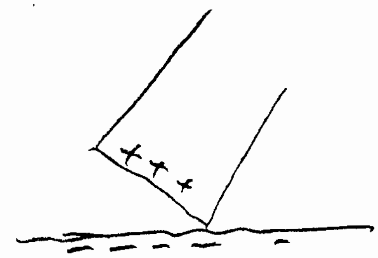
$\bar{\bar{\sigma}}_r$	$\bar{\bar{\sigma}}_a$
Adsorbed H_2O (Fluid)	vdWaals
Born	E/F +/-
	prim. val. bonding

(1) Incr. pH

(2) Incr. c_o

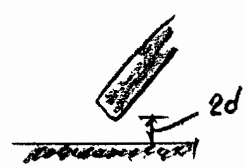
(3) V. high anion valence

(4) Decr. D



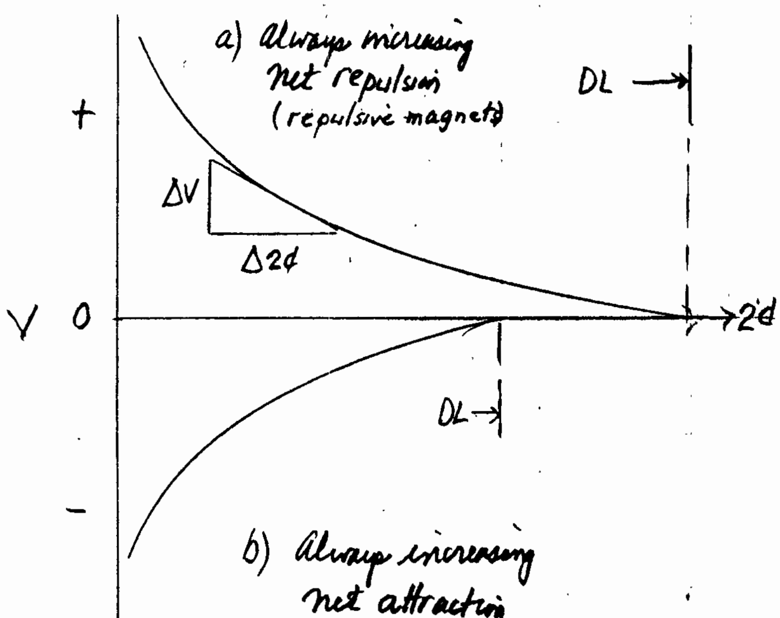
2. PARTICLE INTERACTION

2.1 Energy Diagrams



V = energy/unit area to change minimum distance (2d) between two particles

Net stress = $-\Delta V / \Delta 2d$

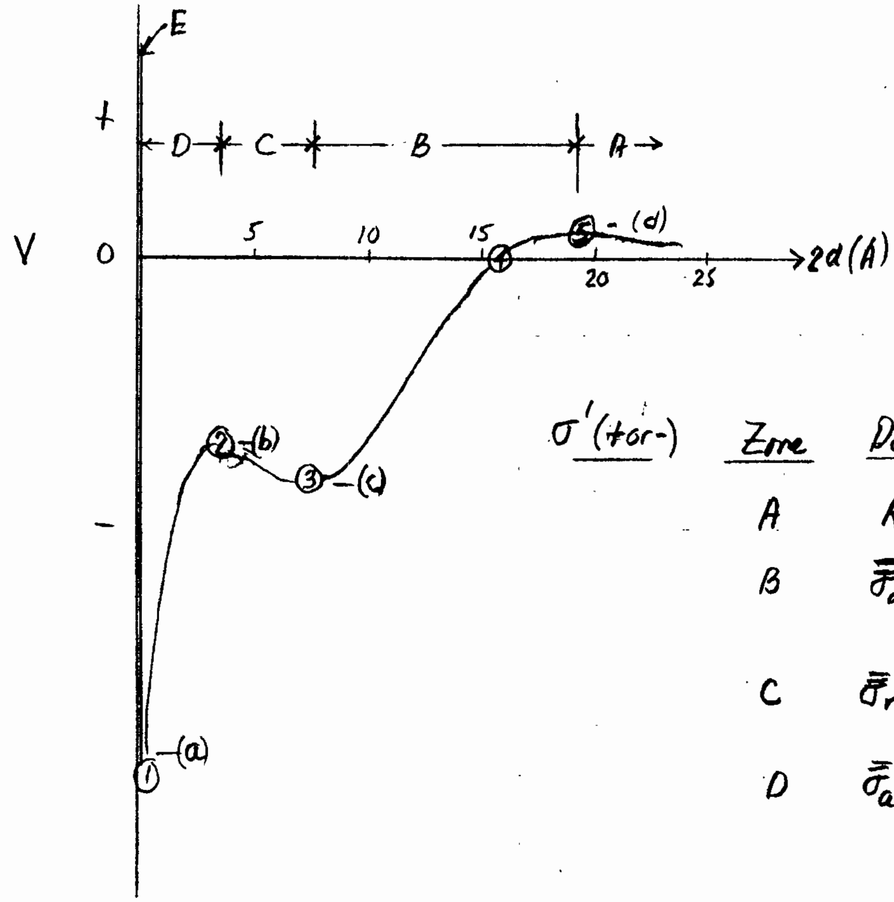


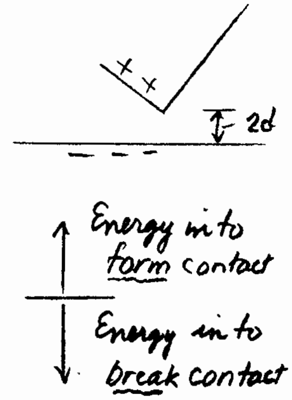
Need to apply energy to DECREASE spacing

Need to apply energy to INCREASE spacing

2.2 Energy Diagram for Hypothecial Contact (Sheet C)

1) Overview for Illitic clay at low salt conc.





σ′ (+or-)	Zone	Dominant Component
	A	R = osmotic repulsion
	B	$\bar{\bar{\sigma}}_a$ due to electrostatic + to - attraction & van der Waals
	C	$\bar{\bar{\sigma}}_r$ due to adsorbed H_2O (fluid)
	D	$\bar{\bar{\sigma}}_a$ due to v.d. Waals & then primary valence
	E	$\bar{\bar{\sigma}}_r$ due to Born repulsion

2) Significance of numbered pts

② & ⑤ Metastable

$\sigma' =$, but slight $\Delta V \rightarrow$

① & ③ Energy sink

$\sigma' =$, but need significant $+\Delta V \rightarrow$

2.3 Source of True Cohesion

Must put energy into system to increase spacing

① → ② $\Delta V = b - a$

③ → ⑤ $\Delta V = d - c$

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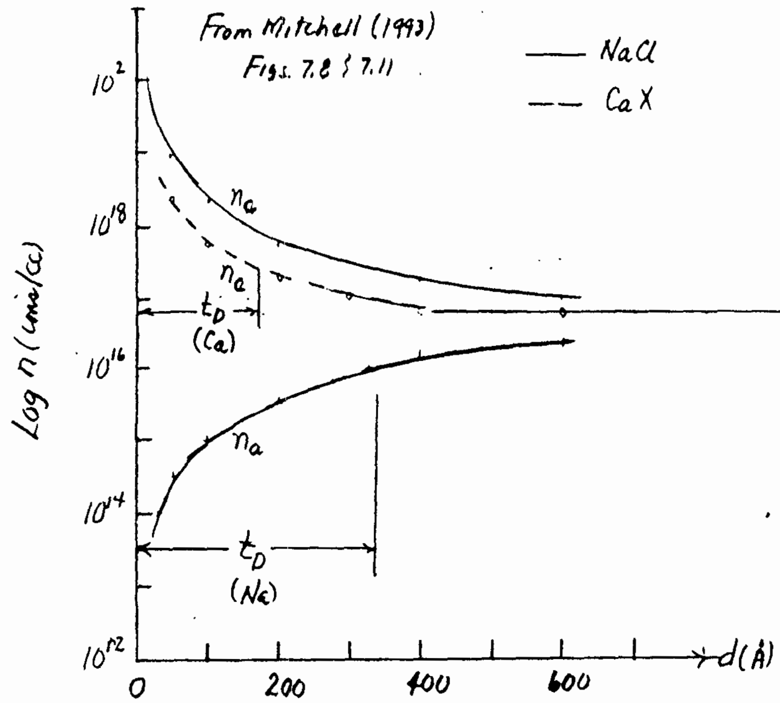
Supplement on Double Layer Repulsion

Single Double Layer: Ion conc. vs. distance Na & Ca Montmorillonite

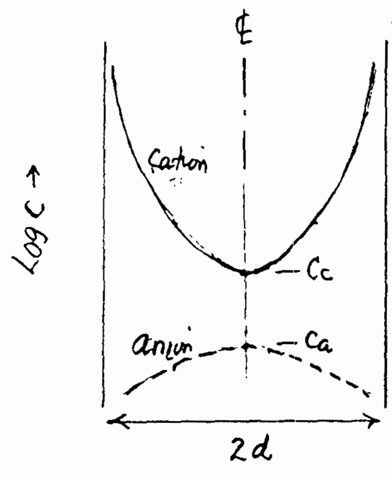


t_D = Debye thickness
= 333 Å Na
= 167 Å Ca

$C_0 = 0.83 \times 10^{-4}$ M
$\times 6 \times 10^{23} \times \frac{1}{1000}$
=
5×10^{16}

M = moles/l
=
6×10^{23} anions & cations

Interacting Double Layer (valence $v = v_c = v_a$; C_0 = bulk concentration, M = moles/l of anions + cations)



$$P_r = R_g T\,(\underbrace{C_c + C_a}_{\text{mid-plane}} - \underbrace{2C_0}_{\text{Bulk}})$$

$T = 273 + °C$

$R_g = 8.314 \frac{J}{mol.°K}$

C = M, moles/l

$$\boxed{P_r(bar) = 24.37\,(C_c + C_a - 2C_0) \quad \text{at } 20°C,\ D = 80 \text{ for } H_2O}$$

Fig 2-1
1.361 Part II-2

C_0(M)	v	2d(Å)	P_r(bar≈atm)	
10^{-3}	1	200	0.2	$C_0 \to 0.06$ g/l NaCl
		100	0.8	
		50	3.3	
		25	12	
10^{-3}	2	100	0.2	≈ ½ spacing
		50	0.8	
		25	3.3	
0.1	1	50	0.6	Large decr. at larger spacings
		25	7	
	2	25	0.6	

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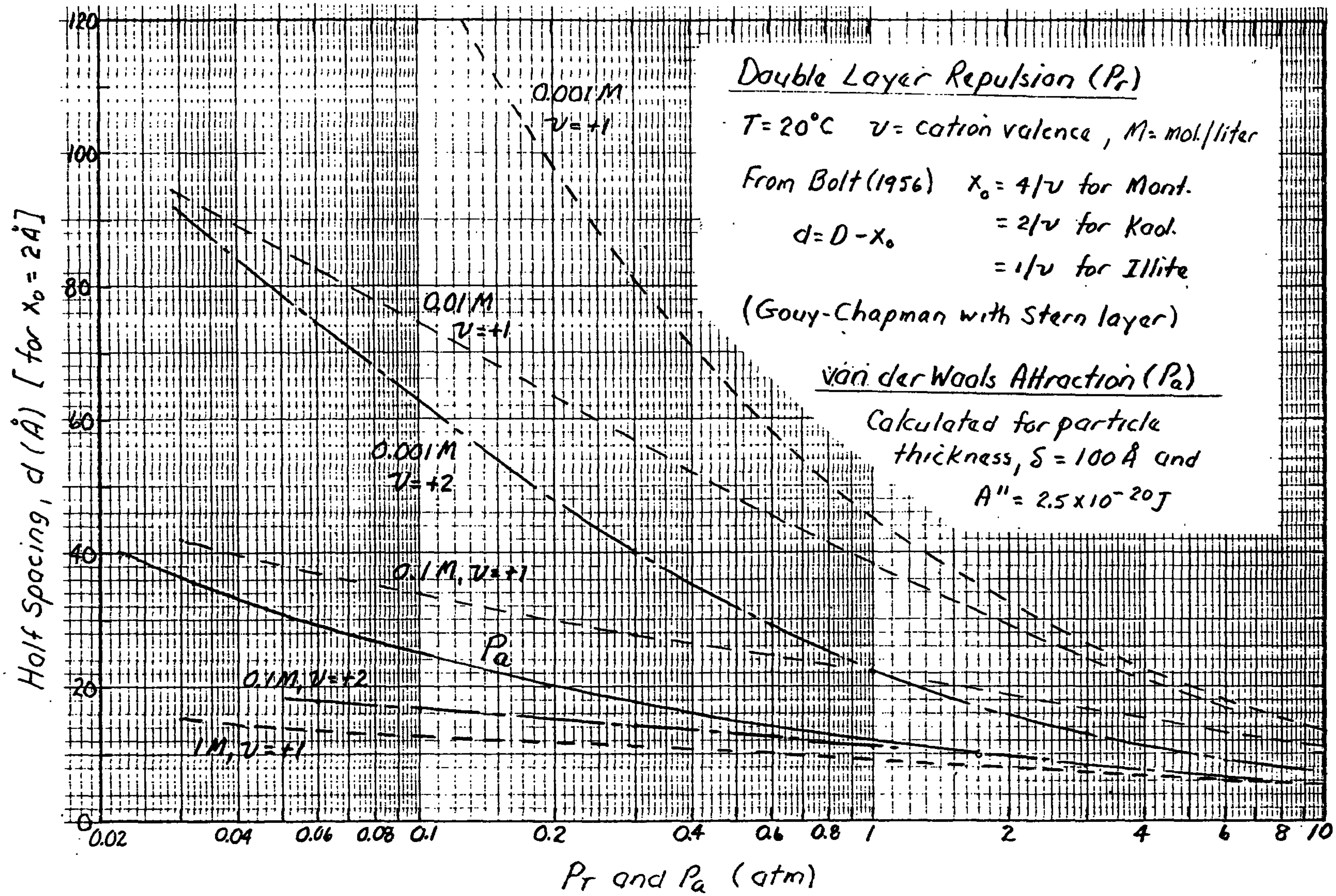

Fig. II2-1 Double Layer (Osmotic) Repulsion and van der Waals Attraction vs. Half Spacing for Parallel Infinite Plates (after Ladd, 1961 ScD Thesis)

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1.322 Part A-III

Adapted from (Ladd & Kinner 1967)

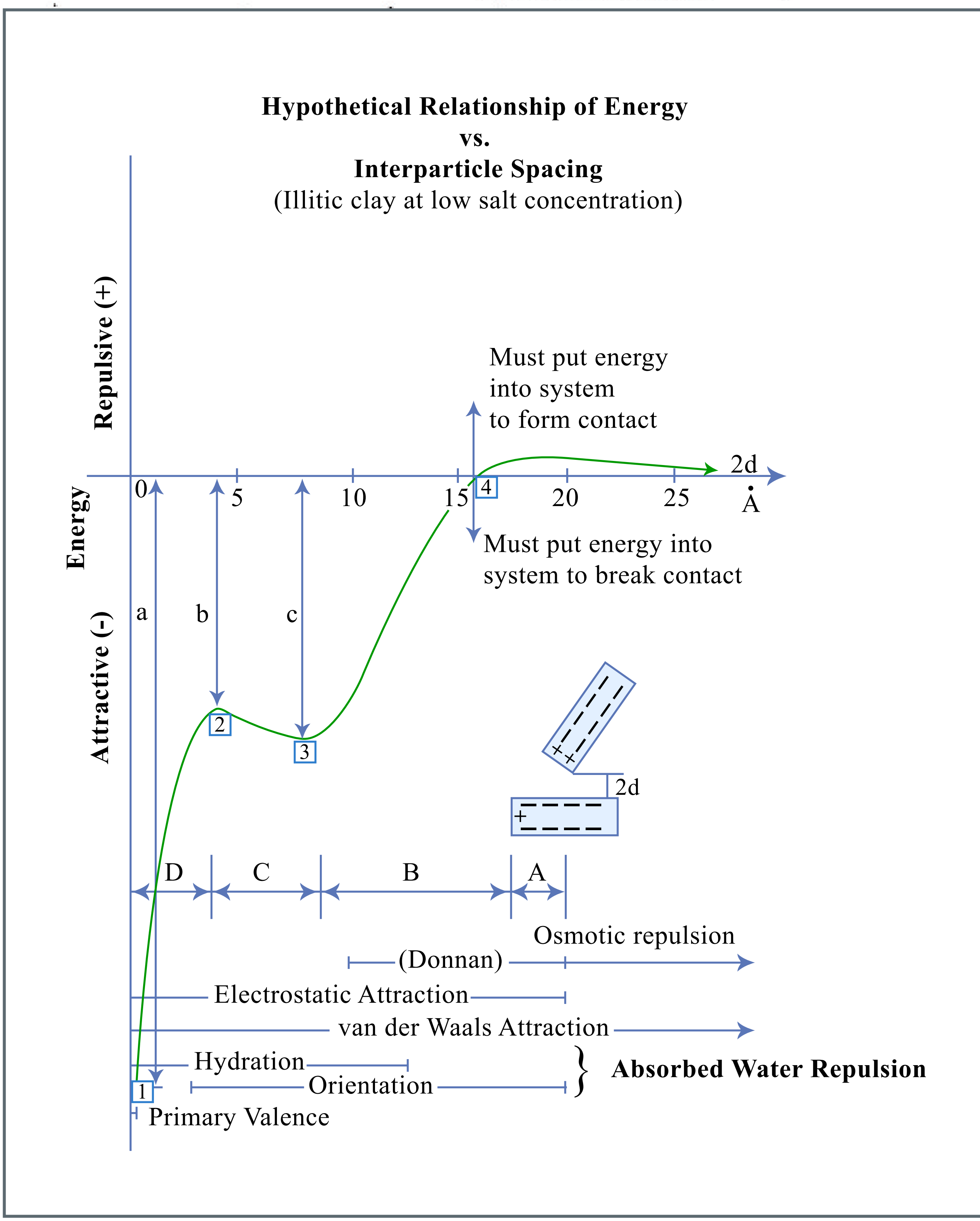

Figure by MIT OCW.

$$\sigma' = (\bar{\sigma}_r - \bar{\sigma}_a) a_c + (R - A)$$

A.W.
Born

vdw
electrostatic
primary valence.

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