IV-6 SLOPE STABILITY (Drained Case)

Slope stability analyses using the method of slices (Sections 3 & 4) have been moved from 1.361–1.366 to 1.364

See pci for brief overview of important practical guidelines/comments on material contained in Sec. 3 & 4

A. Comments on Common Methods of Slices (Section 3)

1) Fellenius circular arc: F much too low
2) Bishop " " : reasonable F, but check output when $\theta \approx \phi' - 90°$ since computed σ' can be ± very large number

Force & moment equilibrium → $F = \frac{\tau_{ff}}{\tau_m}$ GOOD

3) Janbu circular & non-circular: considers only force equilibrium and hence not recommended

4) Spencer circular & non-circular: considers both force and moment equilibrium → $F = \tau_{ff}/\tau_m$ GOOD

5) Morgenstern & Price: only method where one can evaluate influence on F of various side force assumptions.

NOTE: For $\phi = 0$, $c = s_u$, all methods (except Janbu) give same F since τ_{ff} does not depend on σ' distribution.

B. Comments on Computer Programs (Section 4)

1) For circular arc, most have adequate search routines to find the most critical surface (i.e., minimum F).

2) For non-circular failure surfaces (e.g., see p5b), there are major differences in capabilities that can lead to significant underestimates of minimum F (by → 10-15%)

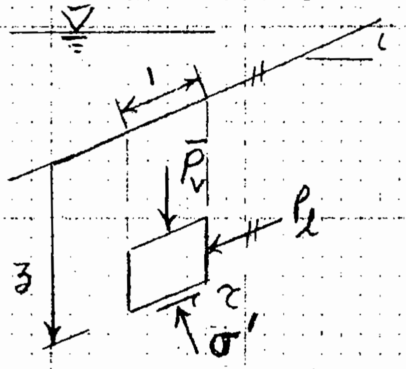
a) STABL (Purdue Univ.): relies on random search, generally with Janbu, and hence not recommended.

b) UTEXAS3 (Wright): more systematic and rational approach using Spencer, and hence recommended (but still need experience to get min. F for cases such as p5b)

Part IV-6 SLOPE STABILITY (Drained case) (p1/6)

1. INFINITE SLOPES (Side forces cancel out)

1.1 Submerged Slope (c', ϕ', γ_t)



- $\bar{P}_v = z\,\gamma_b \cos i \rightarrow \tau = \bar{P}_v \sin i = z\,\gamma_b \cos i \sin i$

  $\sigma' = \bar{P}_v \cos i = z\,\gamma_b \cos^2 i$

- $F = (\text{shear strength})/(\text{shear stress}) = \tau_{ff}/\tau$

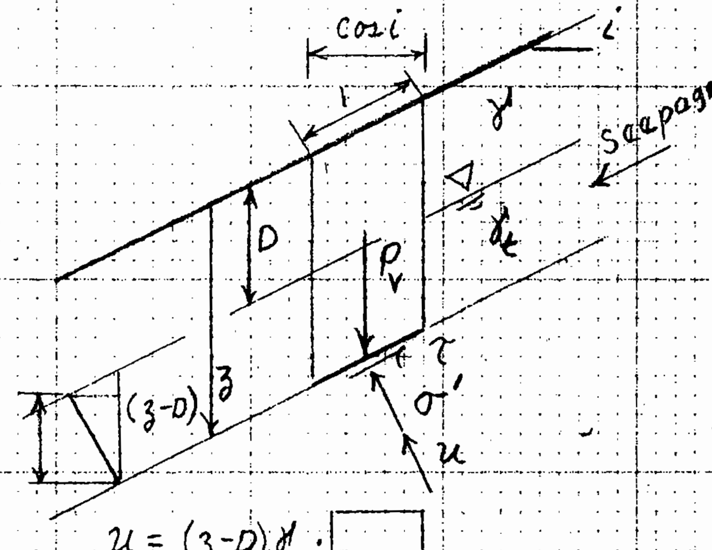
  $$= \frac{c' + z\,\gamma_b \cos^2 i \tan\phi'}{z\,\gamma_b \cos i \sin i} \qquad \boxed{= \frac{\tan\phi'}{\tan i} \text{ for } c'=0}$$

For $F=1$ (failure)

$$\frac{c'}{\gamma_b H_c} = \cos^2 i\,(\tan i - \tan\phi'), \qquad \text{For } c'=0 \quad \tan i = \tan\phi'$$

H_c ← critical height $\qquad$ $\boxed{\therefore \text{same as dry slope}}$

1.2 Seepage Parallel to Surface (Drawndown case for dams)



- $P_v = \gamma D \cos i + \gamma_t (z-D)\cos i$
- $\tau = [\gamma D + \gamma_t (z-D)] \cos i \sin i$
- $\sigma = [\gamma D + \gamma_t (z-D)] \cos^2 i$
- $\sigma' = \sigma - u \quad ; \quad u = (z-D)\gamma_w \cos^2 i$
- $\therefore \sigma' = [\gamma D + \underbrace{(\gamma_t - \gamma_w)}_{\gamma_b}(z-D)] \cos^2 i$

- $$F = \frac{c' + [\gamma D + \gamma_b (z-D)] \cos^2 i \tan\phi'}{[\gamma D + \gamma_t (z-D)] \cos i \sin i}$$

- For $D=0$, $F=1 \rightarrow \dfrac{c'}{\gamma_t H_c} = \cos^2 i \left(\tan i - \dfrac{\gamma_b}{\gamma_t} \tan\phi'\right)$ ← Less than submerged

$\therefore$ And for $c'=0$, $F = \dfrac{\gamma_b}{\gamma_t} \dfrac{\tan\phi'}{\tan i}$; If $F=1$, $\tan i = \dfrac{\gamma_b}{\gamma_t} \tan\phi'$

{ * Rapid drawndown submerged slope → large decrease in F }

2. SLOPE STABILITY ANALYSES: GENERAL

2.1 Background

- Critical mode of failure depends on geometry of problem and strength variations. But usually assume either circular arc or non-circular (e.g., wedge) failure surface
- Limiting equilibrium analyses with plane strain (∞ long) ~~rigid body motion (neglects stress-strain behavior)~~

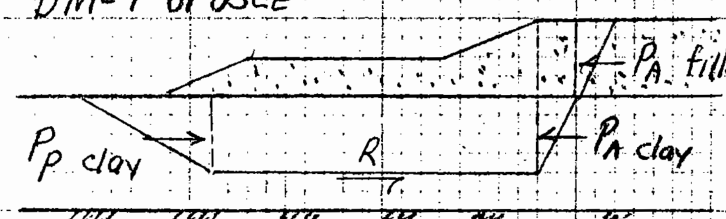
$\therefore F = \dfrac{\text{Available Resisting Moment / Force}}{\text{Driving Moment / Force}}$ Best if $F = \tau_{ff}/\tau_m$ * but not always true

- Trial & error → potential failure surface with lowest F

* τ_m = average mobilized shear stress = average τ required for equilibrium.

2.2 Example: Sliding Wedge: Embankment on Soft Clay (Shown for "total stress" c & ϕ)

DM-7 or USCE

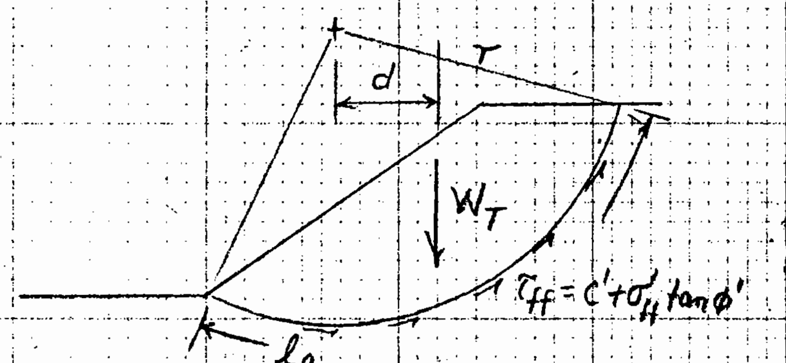


$\Sigma V = 0$ & $\Sigma H = 0$
(Neglects $\Sigma M = 0$)

$R = \Sigma(\underbrace{c + \sigma_v \tan\phi}_{\tau_{ff}})\Delta L$

- $F = (\Sigma \text{ Resisting Forces}) / (\Sigma \text{ Driving Forces}) \neq \tau_{ff}/\tau_m$
- F usually conservative compared to more correct (see 4.2)

2.3 Example: Circular Arc: Cut Slope (Shown for drained shear)



$\Sigma M = 0$ per unit length

$F = \dfrac{\tau_{ff}\, l_a\, r}{W_T\, d}$

Mobilized $\tau_m = W_T\, d / l_a\, r$

$\therefore F = \tau_{ff}/\tau_m$ (good!)

- Should use $\Sigma V, H, M = 0 \rightarrow \bar{\sigma}'$ distribution $\rightarrow \tau_{ff} = c' + \sigma'_{ff} \tan\phi'$
- But above is indetermination since 4 unknowns (L&W 24.4)

  i.e. $F + \bar{N}$ magnitude + direction + point of application

  $\bar{N} = \Sigma \bar{\sigma}'$ along arc

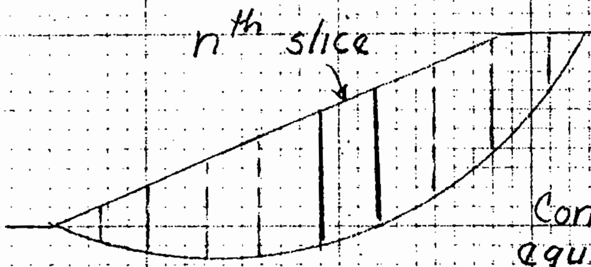
- Different Methods of Slices (Sect. 3) → different magn. & distr. of σ' → different F

3. METHOD OF SLICES : CIRCULAR ARC [Drained Shear: $\tau_{ff} = f(\sigma')$]

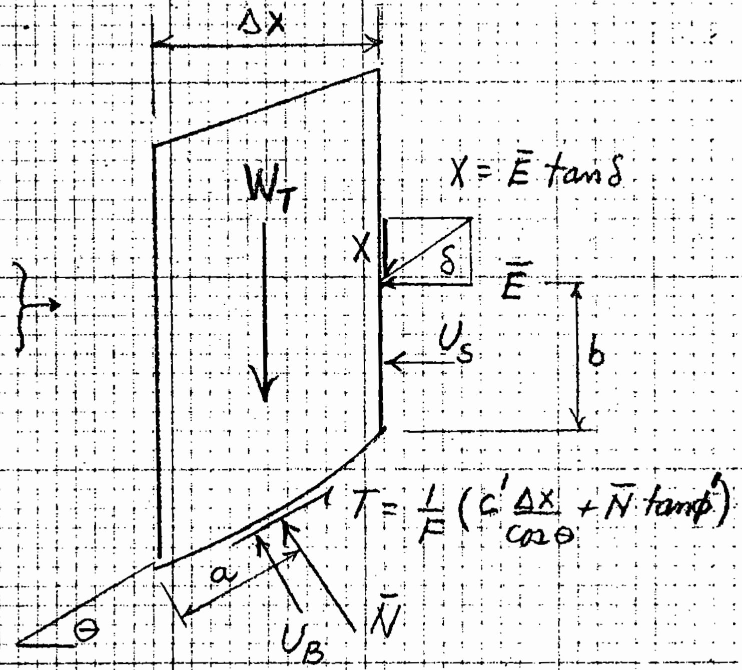
3.1 Background

(1) Purpose : Estimate magnitude & distribution σ' so can → τ_{ff}

(2) Number of unknowns



Consider equilibrium of one slice } →



For n slices

- Σ V, H, M = 0 → 3n eqn.
- No. of unknowns

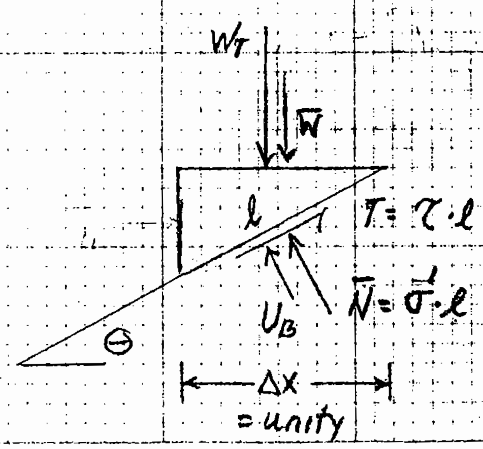
F	1
$\bar{N}$	n
$\bar{E}$	n-1
δ	n-1
a	n
b	n-1
	5n-2 vs. 3n eqn.

∴ Highly indeterminate

∴ Must make simplifying assumptions

∴ Various assumptions → different methods of slices

(3) Basis for comparing common methods, i.e. different ways of "neglecting" side forces



$\ell = 1/\cos\theta$

- $\sigma_v = W_T$ & $\sigma_v' = \bar{W}$
- $\sigma' = \frac{\bar{N}}{\ell} = \bar{N}\cos\theta$
- $\tau = \tau_{ff}/F$
- $\tau_{ff} = c' + \sigma'\tan\phi'$

Part IV-6 SLOPE STABILITY (p 4/6)

3.2 Common Methods of Slices (Circular Arc) (L&W 24.5)

$F = M_R/M_D$

(1) Fellenius ≡ Swedish ≡ Ordinary (1930's)

$\Sigma F_{TOTAL} \perp arc = 0 \quad N = \underbrace{\sigma_v \Delta x}_{W_T} \cos\theta \rightarrow \sigma' \doteq \overbrace{\sigma_v \cos^2\theta - u}^{\sigma}$

* F much too low

(2) Modified Fellenius

$\Sigma F_{EFFECTIVE} \perp arc = 0 \rightarrow \sigma' = \overbrace{(\sigma_v - u)}^{\sigma_v'} \cos^2\theta \rightarrow \sigma'/\sigma_v' = \cos^2\theta$

Better, but F still too low

(3) Simplified Bishop (1950's)

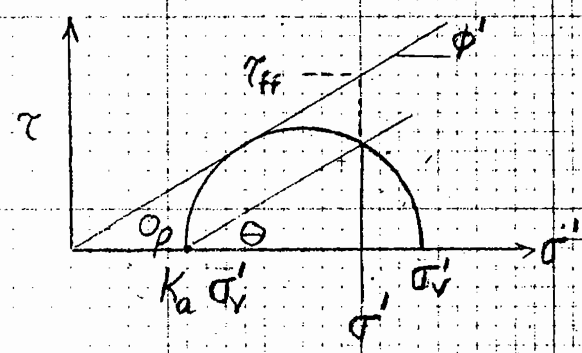
$$\Sigma F \text{ vertical} = 0 \rightarrow \sigma' = \frac{(\sigma_v - u) - \frac{c'}{F}\tan\theta}{1 + \frac{\tan\theta \tan\phi'}{F}}$$

$$\boxed{c' = 0 \rightarrow \quad \frac{\sigma'}{\sigma_v'} = \frac{1}{1 + \frac{\tan\theta \tan\phi'}{F}}}$$

Requires interation, but F more reasonable

(4) Maine State Highway Commission (MSHC)

- Assumed Rankine Active state of stress for all slices *



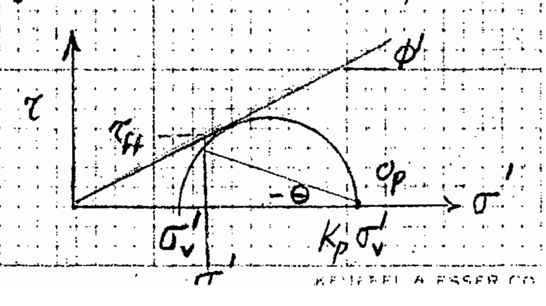
- $\sigma' = (\sigma_v - u)(\cos^2\theta + K_a \sin^2\theta)$
- Obvious Problems
  1) Computed τ_{ff} not on Mohr Circle
  2) For passive zone (neg. θ), computed τ_{ff} much too low.

* $\therefore \sigma_v' = \sigma_{ff}'$ for $\theta \lesseqgtr 0$

(5) CCL = MIT Method

- Assume Rankine passive for negative θ (toe of slope)

$\therefore \sigma' = (\sigma_v - u)(\cos^2\theta + K_p \sin^2\theta)$



NOTE: Mohr Circle $\sigma_\theta' = \sigma_1' \cos^2\theta + \sigma_3' \sin^2\theta$

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Part IV-6 SLOPE STABILITY (p 5/6)

3.3 Comparison of Above Methods

- See p.6 ($\alpha \equiv \Theta$) for comparison of $f(\Theta) = \sigma'/\sigma'_v$ vs Θ
- $\Theta = 0$, all the same
- $+\Theta$ Fellenius < $K_a \geq$ B (Bishop)
- $-\Theta$ Fellenius < K_a << B $\leq K_p$

NOTE: For "$\phi = 0$", All → same F

Note: With Bishop, σ' NO GOOD at $\Theta \approx \phi' - 90°$ since $\tan\Theta \tan\phi' \approx -1 \rightarrow 1/(1 - \frac{1}{F}) \rightarrow$ very high $\pm\sigma'$

{A good computer program should eliminate this BUT MOST DO NOT}

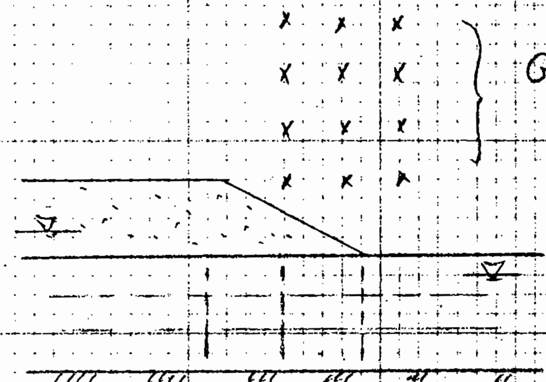
"Purdue" Program: Low cost + Search minimum for any shape } BUT uses Force Equilibrium

STABL w/ Janbu SHOULD NOT USE (F often much too low)

4. COMPUTER PROGRAMS

[Good reference = Fredlund & Krahn (1977) Can. Geot. J. V14 No.3]

4.1 Common Circular Arc



Grid or search using varying radii

- Input strength parameters by zone $c', \phi' \,\&\, \gamma_t$ or $c, \phi \,\&\, \gamma_t$
- Input u – PWE lines or $r_u = u/\sigma_v$
- 1st computer program = ICES LEASE developed by Whitman et al.

Methods: Mod. Fellenius
Simplified Bishop
MIT
Janbu (FORCES)

[Note: Make sure F = f(moments) and not just = f(forces)]

4.2 Generalized (Wright et al, 1973, JSMFD, V99, SM10, Oct)

- Spencer, Morgenstern-Price — See p5a & b (UTEXAS 3)
- Any shape failure surface
- Satisfy statics, but must assume δ distribution for M-P

4.3 STAB 3-D (Azzouz, Baligh & Ladd, 1983, ASCE JGE 109(5))

- Considers "end effects", i.e. F vs. F plane strain (∞ long)
- $F(3\text{-}D)/F(2\text{-}D) = \left(1 + 0.7\,\frac{DR}{L}\right)$ where DR = depth of failure surface (thickness of failed soil)
- For typical embankment failures, increases F by 10 ± 4% — L = total length of failure ($\perp$ to 2-D cross-section)

11/92

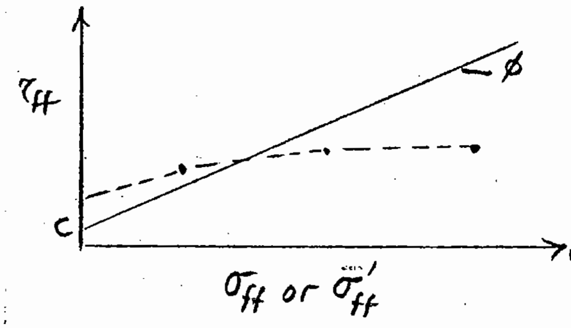
(p5a)

Some Features of UTEXAS3 Computer Program for Non-Circular Failure Surfaces: Spencer Method

A. Shear Strength Options: "c, ϕ" soils

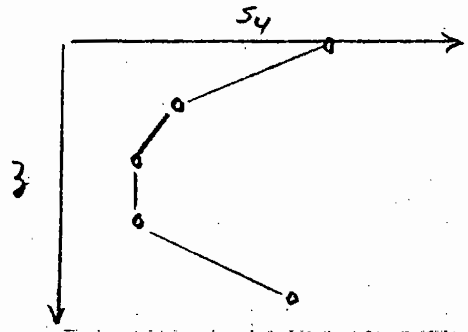
1) Linear or non-linear Mohr-Coulomb with isotropic strength

2) Linear Mohr-Coulomb with anisotropic strength: vary c & ϕ with θ à la B-2

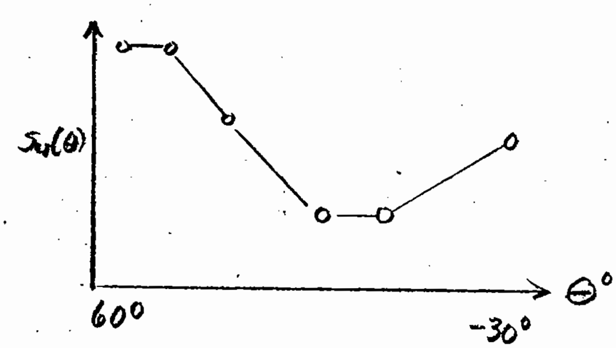


B. Shear Strength Options: "c = s_u, ϕ = 0" Soils

1) Isotropic strength



2) Anisotropic strength for given "soil"



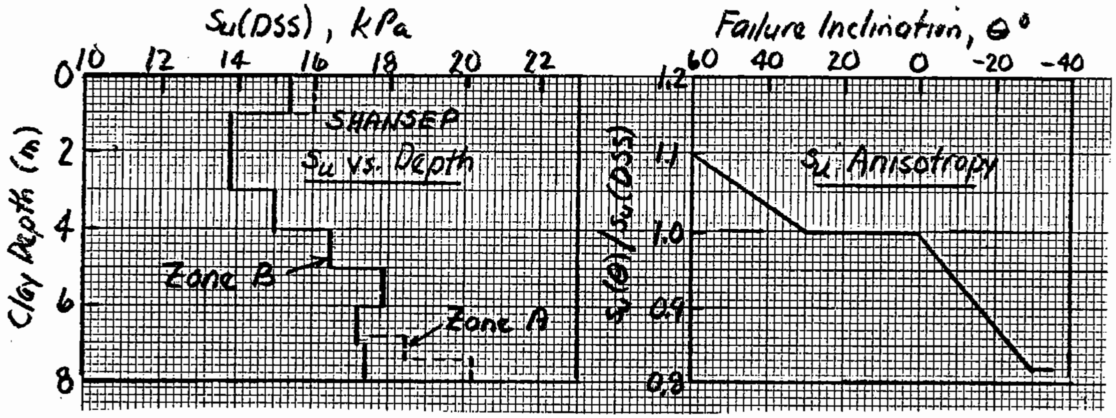
C. Pore Pressure Options (For each "soil")

1) u = constant 2) PWE = constant 3) $r_u = u/\gamma h$ = constant

4) Specify grid of u or r_u values

D. Search Routine (Much better than STABL-Purdue)

- Input initial surface defined by up to 30 points
- Program then shifts each point successively by specified distance approximated $\perp$ to shear surface
- Repeat above for new surfaces using progressively smaller distances → most critical (if reasonable starting surface)
- See p5b for example from back analysis of actual failure.



NOTE: Predicted 3-D FS = 0.87 × 1.1 = 0.96

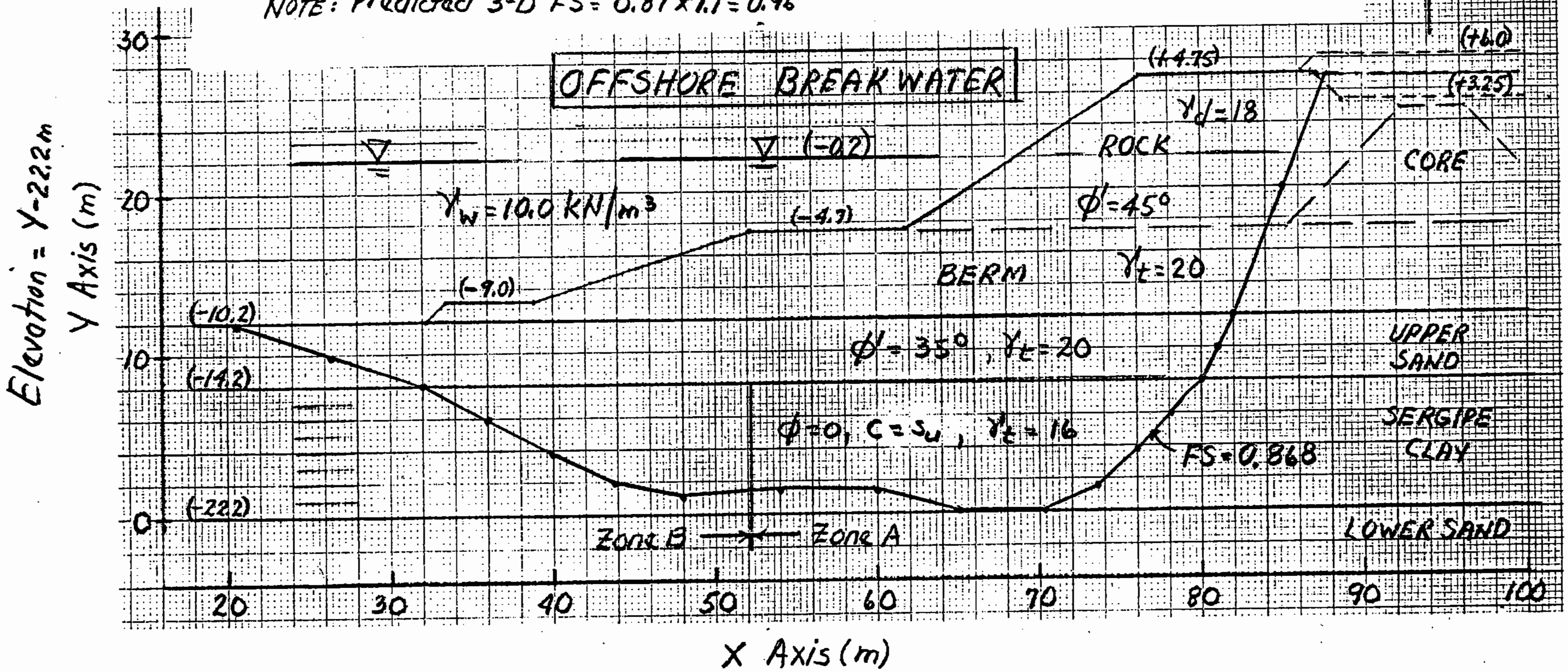


Fig. Backanalysis of Reinder Failure: El. +4.75/4.75, Sergipe, Brazil
Example of UTEXAS3 Critical Failure Surface From Search Routine

1.361-1.366 Part IV-6

(p6/6)

10/86 11/89 10/90 11/7/98

CCL 27/10/68 HSA 1640

CCL 11/70
1.361 & 1.366

Figure 1 Comparison of Methods of Slope Stability Analysis for Cohesionless Soil, $\phi' = 35°$