Homework problems on Fluid Dynamics

(1.63J/2.21J)

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Hele-Shaw analogy between lubrication and seepage flows

Isothermal flows

Two dimensional flows in a homogeneous porous medium can be simulated by slow flows in a thin gap. The principle is the subject of this exercise.

Consider the slow flow in a vertical thin gap of width b. See Figure 1. Let the x, z axes

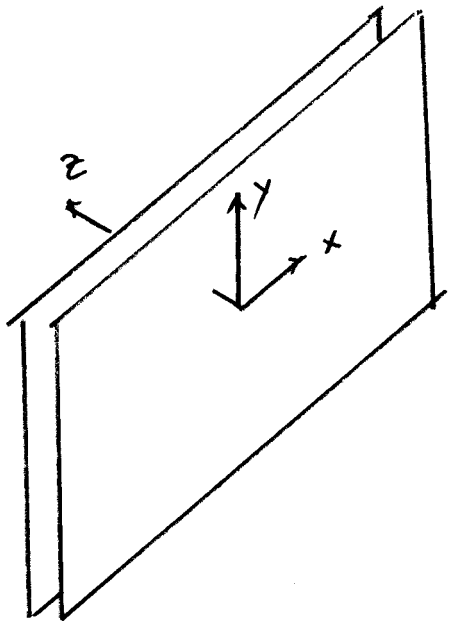

Figure 1: A thin gap between two plates

be horizontal and y vertical. The Stokes equations are

$$u_x + v_y + w_z = 0 \tag{1}$$

and

$$\mu u_{zz} = p_x, \quad \mu v_{zz} = p_y + \rho g \tag{2}$$

where x, z are in the horizontal plane. The boundary conditions are

$$u = v = w = 0, \quad z = 0, b. \tag{3}$$

There may be a material surface in the flow normal to the gap.

First integrate the momentum equations to derive the velocity (u, v) as functions of z. Then show that the gap average of the two momentum equations are the same as those governing the seepage flow in a gravitational field. What is the equivalent permeability?

What do you get by taking the gap average of the continuity equation?

What are the boundary conditions on a rigid cylindrical obatacle across the gap?

Summarize the equivalence with the 2-D seepage flow.

Can there be an analogy in non-isothermal flows?

Please examine the possibility of extending the Hele-shaw analogy to a flow with heat transfer, with a view to simulating flow induced by a heated cylinder burried in a porous soil. Can there be an analogy if (a) the two plates are perfectly insulated, and (b) if the temperature of the plates is prescribed (such as linear function of y).