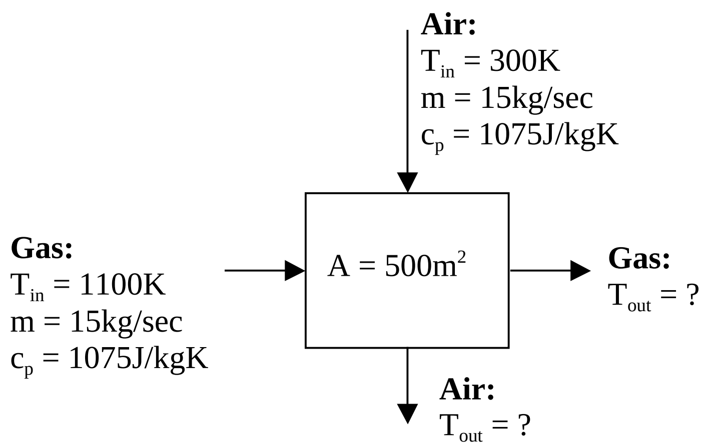
## Recitation 9

a.




$C_{min} = C_{max}$ = (15kg/sec)(1075J/kgK) = 16125W/K
R = 1

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(100\frac{W}{m^2K}\right)(500m^2)}{16125\frac{W}{K}} = 3.1$$

Fig. 11.18: ε = 0.69
Eqn. 11.33:

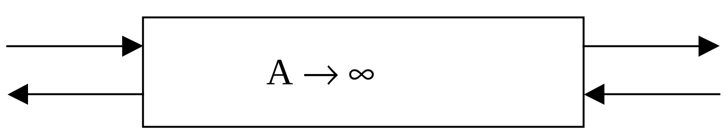
$$\varepsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(NTU)^{0.22}\left\{\exp\left[-C_r(NTU)^{0.78}\right]-1\right\}\right]$$

$$\varepsilon = 1 - \exp\left[(1)(3.1)^{0.22}\left\{\exp\left[-(1)(3.1)^{0.78}\right]-1\right\}\right] = 0.689$$

$$\varepsilon = \frac{T_{air,out} - T_{air,in}}{T_{gas,in} - T_{air,in}} = \frac{T_{air,out} - 300K}{1100K - 300K} = 0.689$$

$$\Rightarrow T_{air,out} = 852K$$

b.




Since $C_{max} = C_{min}$, the slopes of the lines for the two streams are the same (i.e. the lines are parallel). Therefore, the only way for a pinch point to occur is for the lines to completely coincide.

$\Rightarrow T_{air,out}$ = 1100K

c. Want $T_{air,out}$ = 1000K

Since $m_{air}$ will have to increase for this final temperature, $C_{gas} = C_{min}$
⇒ NTU = 3.1 still

$$\varepsilon = \frac{\left(\dot{m}\,c_p\right)_{air}\left(T_{air,out} - T_{air,in}\right)}{\left(\dot{m}\,c_p\right)_{air}\left(T_{gas,in} - T_{air,in}\right)} = \frac{1000K - 300K}{1100K - 300K} = 0.875$$

$$\Rightarrow R = 0.3 = \frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_{air}}{\dot{m}_{gas}}$$

$$\Rightarrow \dot{m}_{gas} \approx 50\frac{kg}{\sec}$$

d.i. This is the same solution technique as in part (a):
$C_{min} = C_{max}$ = (15kg/sec)(1075J/kgK) = 16125W/K
R = 1

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(100\frac{W}{m^2K}\right)\left(1000m^2\right)}{16125\frac{W}{K}} = 6.2$$

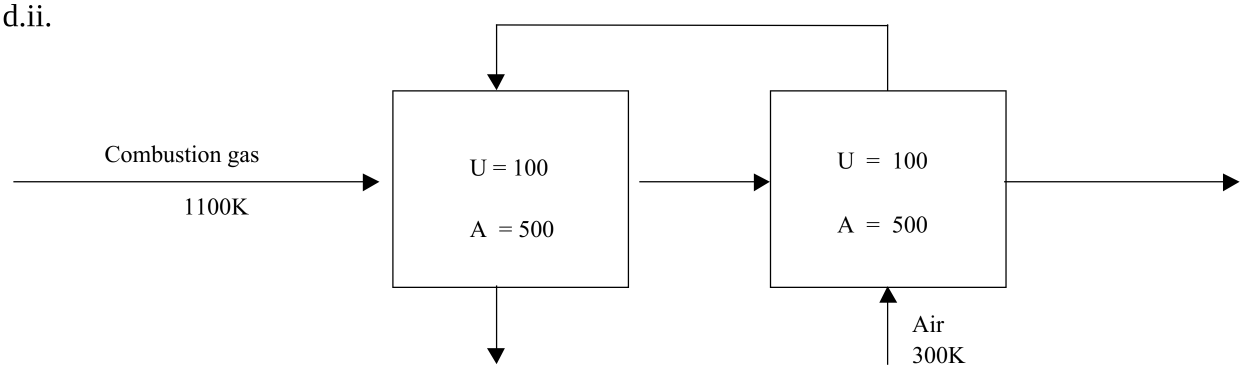
Fig. 11.18: off the chart!!!

Eqn. 11.33: $\varepsilon = 1 - \exp\left[(1)(6.2)^{0.22}\left\{\exp\left[-(1)(6.2)^{0.78}\right] - 1\right\}\right] = 0.77$

$$\varepsilon = \frac{T_{air,out} - T_{air,in}}{T_{gas,in} - T_{air,in}} = \frac{T_{air,out} - 300K}{1100K - 300K} = 0.77$$

$$\Rightarrow T_{air,out} = 916K$$

d.ii.



Since both exchangers have the same area and heat transfer coefficient as the heat exchangers in part (a), their efficiencies and NTU's will be the same:

$$NTU = \frac{UA}{C_{\min}} = \frac{\left(100\frac{W}{m^2K}\right)\left(500m^2\right)}{16125\frac{W}{K}} = 3.1$$

$$\varepsilon_1 = \varepsilon_2 = 1 - \exp\left[(1)(3.1)^{0.22}\left\{\exp\left[-(1)(3.1)^{0.78}\right] - 1\right\}\right] = 0.689$$

The following equations apply:

$$\varepsilon_1 = \frac{q_{act}}{q_{\max}} = \frac{C_h\left(T_{h,i} - T_{h,\text{int}}\right)}{C_{\min}\left(T_{h,i} - T_{c,\text{int}}\right)} = \frac{C_c\left(T_{c,o} - T_{c,\text{int}}\right)}{C_{\min}\left(T_{h,i} - T_{c,\text{int}}\right)}$$

$$\Rightarrow T_{h,\text{int}} = T_{h,i} - \varepsilon_1\left(T_{h,i} - T_{c,\text{int}}\right) = \left(1 - \varepsilon_1\right)T_{h,i} + \varepsilon_1 T_{c,\text{int}}$$

$$\varepsilon_2 = \frac{q_{act}}{q_{\max}} = \frac{C_h\left(T_{h,\text{int}} - T_{h,o}\right)}{C_{\min}\left(T_{h,\text{int}} - T_{c,i}\right)} = \frac{C_c\left(T_{c,\text{int}} - T_{c,i}\right)}{C_{\min}\left(T_{h,\text{int}} - T_{c,i}\right)}$$

$$\Rightarrow T_{c,\text{int}} = \varepsilon_2 C_r\left( \;_{h,\text{int}} - T_{c,i}\right) + T_{c,i} = \varepsilon_2 C_r T_{h,\text{int}} + \left(1 \quad \varepsilon_2 C_r\right) \;_{c,i}$$

$$C_h\left(T_{h,i} - T_{h,\text{int}}\right) = C_c\left( \;_{c,o} - T_{c,\text{int}}\right) \Rightarrow T_{c,o} = C_r\left( \;_{h,i} - T_{h,\text{int}}\right) + T_{c,\text{int}}$$

$$C_h\left(T_{h,\text{int}} - T_{h,o}\right) = C_c\left( \;_{c,\text{int}} - T_{c,i}\right) \Rightarrow T_{h,o} = T_{h,\text{int}} - \frac{1}{C_r}\left(T_{c,\text{int}} - T_{c,i}\right)$$

Putting the equations together:

$$T_{h,\text{int}} = \left(1 - \varepsilon_1\right)T_{h,i} + \varepsilon_1 T_{c,\text{int}} = \left(1 - \varepsilon_1\right)T_{h,i} + \varepsilon_1\varepsilon_2 C_r T_{h,\text{int}} + \varepsilon_1\left(1 - \varepsilon_2 C_r\right) \;_{c,i}$$

$$\Rightarrow T_{h,\text{int}} = \frac{\left(1 - \varepsilon_1\right)T_{h,i} + \varepsilon_1\left(1 - \varepsilon_2 C_r\right) \;_{c,i}}{1 - \varepsilon_1\varepsilon_2 C_r}$$

$\Rightarrow T_{h,int} = T_{gas,int} = 773.6K$ $\qquad$ $T_{c,int} = T_{air,int} = 626.4K$
$T_{h,o} = T_{gas,out} = 447.2K$ $\qquad$ $T_{c,o} = T_{air,out} = 952.8K$

Note: This solution can be checked using an overall energy balance on the system or on each heat exchanger individually:

$$q_{act} = C_h\left( \;_{h,i} - T_{h,o}\right) \overset{?}{=} C_c\left(T_{c,o} - T_{c,i}\right)$$

When numbers are plugged in, the solution works out correctly

$\Rightarrow$The configuration with the heat exchangers in series will allow for more efficient heat transfer compared to a single larger heat exchanger. This is because the heat exchangers in series are closer to a counter-current flow heat exchanger ($F \rightarrow 1$).