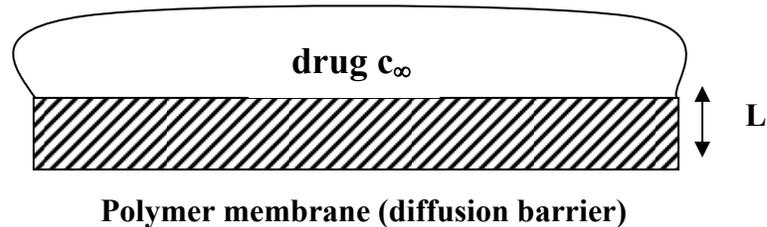


**10.302**  
**Fall 2004**  
**EXAMPLE PROBLEMS**

Problem 1

Transdermal drug delivery devices (such as the nicotine patch) typically consist of a reservoir of drug sealed onto the surface of a polymer membrane as shown in the figure. The device is manufactured by injecting a drug solution into the reservoir (labeled “drug” in the figure) and sealing the reservoir shut. The polymer membrane does not contain any of the drug initially. During use, the bottom of the polymer membrane is attached to the surface of the skin and drug diffuses from the reservoir down through the membrane into the skin. The drug essentially remains at a constant concentration  $c_\infty$  in the reservoir during use.



- a) How long after the device is manufactured will it take for drug to penetrate to the exterior (bottom) surface of the polymer membrane?
- b) How long after manufacture before the membrane is saturated with drug? You may assume saturation has been obtained if the concentration in the membrane has reached 99% of its final value.

Data:

- L = 1 mm  
c<sub>∞</sub> = 100 mg/ml  
D = 5 x 10<sup>-8</sup> cm<sup>2</sup>/s (diffusion coefficient of drug in membrane)  
S = 0.01 (concentration of drug in membrane/concentration of drug in reservoir fluid)

## Problem 2

Mammalian tissue is to be grown in the annular space between a tube of (poly) dimethyl siloxane (PDMS) and an impermeable outer shell (see figure). Oxygen will flow through the interior of the tubing. In order to meet the metabolic demands of the tissue, the oxygen will diffuse through the tubing wall and then into the tissue. For this problem, it may be assumed that any resistance to mass transport associated with the tubing may be neglected. Thus, the appropriate boundary condition at  $r = R_1$  is

$$C_{O_2} = C_{O_2}^{\circ} @ r = R_1$$

It is assumed that the  $O_2$  reacts with the tissue in accordance with

$$\dot{N}_{O_2} = \text{constant} = -7 \times 10^{-5} \text{ mol/m}^3 \cdot \text{s}$$

Furthermore, the diffusivity of  $O_2$  in tissue is known to be about  $2 \times 10^{-5} \text{ cm}^2/\text{s}$ . In addition, the value of  $C_{O_2}^{\circ}$  is about  $1.4 \text{ mol/m}^3$ . Convective transport of  $O_2$  may be neglected.

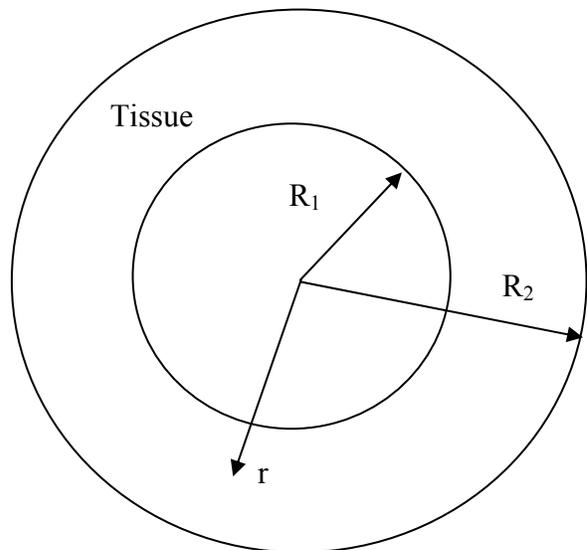
- i. What is the expression for  $C_{O_2}(r)$ ?
- ii. There is a critical value of  $R_2$  at which  $C_{O_2} = 0$  at  $R_2$ . What is the expression for this critical radius? For the parameters given above, show that  $R_{\text{crit}} < 4R_1$  if  $R_1 = 0.25 \text{ cm}$ .

## DATA

In cylindrical coordinates

$$\nabla \cdot \underline{F} \equiv \frac{1}{r} \frac{\partial}{\partial r} (rF_r) + \frac{1}{r} \frac{\partial F_{\theta}}{\partial \theta} + \frac{\partial F_z}{\partial z}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$



### Problem 3

A wetted wall column is to be used to study the absorption of CO<sub>2</sub> into water. The column is supplied with solute-free water at the top and air containing 5 mole percent CO<sub>2</sub> is supplied at the bottom. The air flow rate is such that the change in gas composition may be neglected. The gas phase resistance to mass transfer is negligible. The situation is as depicted in Sketch A.

The velocity profile in the draining film is given by:

$$u = \frac{1}{2} \frac{\rho g d^2}{\mu} \left[ 1 - \frac{y^2}{d^2} \right]$$

(See Sketch B)

It has been proposed that the absorption be analyzed by using “penetration” theory and setting

$$u \cong \frac{1}{2} \frac{\rho g d^2}{\mu} \quad \text{for all values of } y \text{ which lie within the penetration}$$

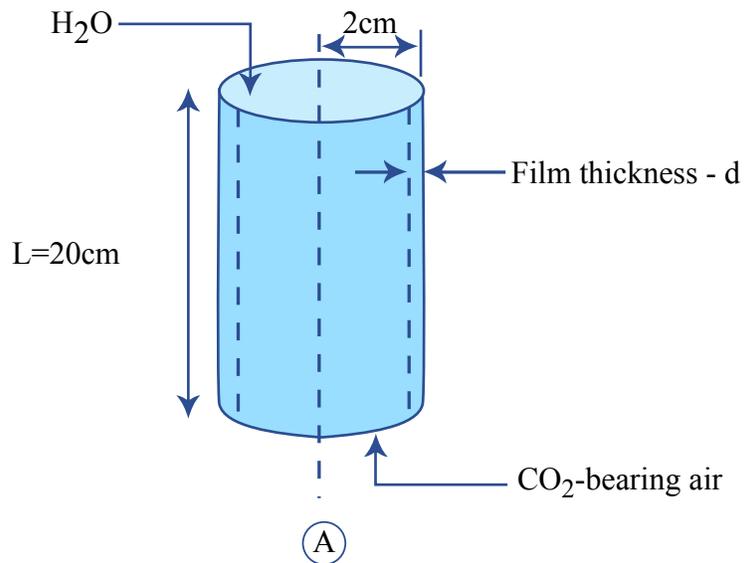
depth.

The anticipated range of  $d$  is 0.1 mm to 0.3 mm.

- A. Is the penetration theory valid for this range of  $d$ ? Justify.
- B. It is permissible to neglect axial diffusion in this problem. Please justify this conclusion.
- C. Assuming the penetration theory concept and the above approximation for the velocity profile to be valid, develop an expression for the mass transfer coefficient, as a function of  $x$  and other appropriate parameters.
- D. What is the CO<sub>2</sub> flux (gm/cm<sup>2</sup> sec) at  $x = 10$  cm if  $d = 0.3$  mm?

See Sketches A & B next page.

SKETCHES FOR PROBLEM 3



$$x = \frac{1}{2} \frac{\rho g d^2}{\mu} \left[ 1 - \frac{\gamma^2}{d^2} \right]$$

Properties etc:

$$\rho_{\text{H}_2\text{O}} = 1\text{gm/cm}^3$$

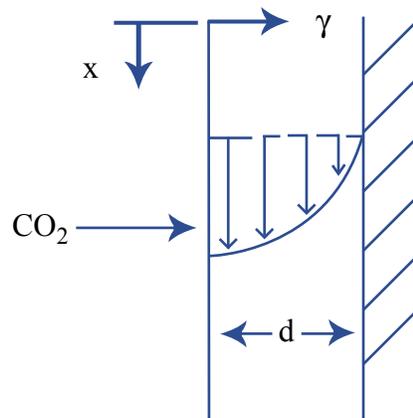
$$\nu_{\text{H}_2\text{O}} = 0.01\text{cm}^2/\text{sec}$$

$$g = 980\text{cm/sec}^2$$

$$D_{\text{CO}_2/\text{H}_2\text{O}} = 2 \times 10^{-3}\text{cm}^2/\text{sec}$$

$$0.1\text{mm} < d < 0.3\text{mm}$$

$\text{CO}_2$  solubility =  $0.15\text{ gm CO}_2/100\text{ gm H}_2\text{O}$   
at a  $\text{CO}_2$  partial pressure of  $2\text{ atm}$ .



(B)

Figure by MIT OCW.

#### Problem 4

A liquid/liquid extractor is being studied to determine the values of the “film coefficients” which characterize its performance. In the extractor, an aqueous solution forms the continuous phase and toluene drops which are dispersed by agitation constitute the organic phase. The volume ratio of the aqueous and organic phases is 2:1 ( $V_{aq}/V_{org} = 2$ ). It is also known that the interfacial area for transport,  $a$ , is  $20 \text{ cm}^2/\text{cm}^3$  of toluene. Furthermore, a “lumped” analysis may be employed in order to describe the device, i.e., it may be assumed that concentrations are uniform within each phase and all of the resistance to transport is associated with the interface.

Two separate experiments using quite different solutes, X and Y, have been performed in which the solute is dissolved in the aqueous phase and then extracted into the organic phase. In each case, the initial concentration in the aqueous phase was 0.01M.

The results are as follows:

<u>Time (seconds)</u>	<u>Concentration of X in organic phase (mol/l)</u>	<u>Concentration of Y in organic phase (mol/l)</u>
0	0	0
10	$3.2 \times 10^{-4}$	$7.9 \times 10^{-5}$
50	$1.5 \times 10^{-3}$	$3.8 \times 10^{-4}$
200	$5.3 \times 10^{-3}$	$1.3 \times 10^{-3}$
1000	$1.4 \times 10^{-2}$	$3.5 \times 10^{-3}$

It is believed that solutes X and Y have identical diffusivities in water. The diffusivities in toluene are different than in water, but still the same for X and Y. However, the partition coefficient for X and Y is quite different. It is defined here as:

$$S_{o/Aq} = \frac{\text{equilibrium concentration in toluene}}{\text{equilibrium concentration in water}}$$

and it is known that  $S_{o/Aq} = 8$  for X and  $S_{o/Aq} = 0.5$  for Y.

- What is the overall transport coefficient for solute X based on an overall organic phase driving force? Based on an overall aqueous phase driving force?
- What is the overall transport coefficient for solute Y based on an overall organic phase driving force? Based on an overall aqueous phase driving force?
- What are the values of the individual coefficients for the two phases?
- Does one or the other phase pose a dominant resistance for the transport of solute X? If so, which phase? Repeat for solute Y.

Notes: In the above, the individual coefficients are defined by:

$$N'' = h_{m,aq}(C_{aq} - C_{aq,i}) = h_{m,o}(C_{o,i} - C_o)$$

Where “i” refers to the interface

“o” refers to the organic phase

and “aq” refers to the aqueous phase

Overall coefficients are defined by: 
$$N'' = U_{m,o}(S_{o/Aq} C_{aq} - C_o) = U_{m,aq} \left( C_{aq} - \frac{C_o}{S_{o/Aq}} \right)$$

### Problem 5

Consider a single spherical ice crystal growing in a stagnant sea of subcooled salt water. The fluid is water at  $-4^{\circ}\text{C}$  containing  $0.04 \text{ gm NaCl/cm}^3$ ; and all NaCl is excluded from the interior of the growing ice particle.

- (a) If the diffusivity of sodium chloride in water were (for the moment) assumed to be infinite, how long would it take for the ice crystal to grow from infinitesimal size to a radius of  $0.1 \text{ cm}$ ? You may assume the temperature field around the ice particle to be quasi-steady.
- (b) Demonstrate whether the assumption of quasi-steady behavior is satisfactory.
- (c) In truth, the diffusivity of sodium chloride in water is about  $1.2 \times 10^{-5} \text{ cm}^2/\text{sec}$ . Please develop a better estimate of the time for a crystal to grow to a radius of  $0.1 \text{ cm}$ . You may assume that both the temperature and the concentration fields are quasi-steady. Since salt is totally excluded from the growing crystal, note that the flux of salt at the surface of the crystal is simply  $C_i \frac{dR}{dt}$  where  $C_i$  is the salt concentration at the surface of the crystal and  $R$  is the radius of the sphere. This flux due to rejection must equal the quasi-steady rate at which salt diffuses away.

### **Data**

- The freezing point depression is given by:  $\Delta T_{\text{FPD}} = 60 C_{\text{NaCl}}$   
where  $\Delta T_{\text{FPD}}$  is in degrees Kelvin and  $C_{\text{NaCl}}$  is in  $\text{gms/cm}^3$ .
- The heat of fusion for water is about  $250\text{J/gm}$
- The density of water and ice may be assumed equal at the value of  $1 \text{ gm/cm}^3$ .
- Other physical properties of liquid water include the following:

thermal conductivity	=	$5.7 \times 10^{-3} \text{ W/cm K}$
kinematic viscosity	=	$0.018 \text{ cm}^2/\text{sec}$
thermal diffusivity	=	$1.3 \times 10^{-3} \text{ cm}^2/\text{sec}$
specific heat	=	$4.2 \text{ J/gm K}$

## Problem 6

Consider a droplet of water with an initial diameter of 1mm. The droplet is placed in slightly humid, stagnant air. You may assume that the water concentration is dilute, i.e. diffusion-induced convection in the air may be neglected.

- a. If the air temperature is 25°C and the humidity of the air at large distances from the drop is 0.006 kg of water/kg of air, how much time will be required to totally evaporate the drop?
- b. If the temperature of the air is sufficiently high, the dilute assumption fails and diffusion-induced convection must be included in the analysis. Will this convection tend to increase or decrease the heat flux? Will it tend to increase or decrease the mass flux? Will the temperature of the evaporating drop increase or decrease relative to the no convection prediction? Justify your choices.
- c. In accordance with item (c), under the Data below, the process is quasi-steady, i.e. the drop temperature is independent of time. Please develop an expression which gives a rough estimate of the time required to establish this condition. Note that there are two time scales which may be much shorter than the evaporation time. One is the time required to establish the temperature profile in the gas phase. The other is the time required to cool (or heat) the droplet from its initial temperature to the evaporation temperature.

### Data

(a) Water:

$$\begin{aligned}\rho &= 1000 \text{ kg/m}^3 \\ c_p &= 4200 \text{ J/kg}\cdot\text{K} \\ \lambda_{fg} &= 2.4 \times 10^6 \text{ J/kg} \\ k &= 0.67 \text{ W/m}\cdot\text{k}\end{aligned}$$

(b) Air-Water:

$$\begin{aligned}D &= 0.26 \times 10^{-4} \text{ m}^2/\text{s} \\ k &= 0.027 \text{ W/m}\cdot\text{K} \\ c_p &= 1000 \text{ J/kg}\cdot\text{K} \\ \rho &= 1.04 \text{ kg/m}^3\end{aligned}$$

(c) Miscellaneous:

For Part (a) of the problem statement, sensible heat effects may be neglected relative to latent heat effects.

### Problem 7

A pharmaceutical product is expressed by yeast which are grown in a large aerated fermentor. The desired product, along with a lot of undesired products, is found in the aqueous phase. As a first step toward purification, current procedure is to filter the broth and then extract the product (and some of the undesired products) into an organic phase in a batch-operated direct-contact extractor. However, sales have been increasing, and the plant must be expanded. It has been suggested that it might be possible to eliminate the filtration step and use a continuous hollow-fiber exchanger in order to move the product from the broth into an organic phase. The general arrangement would be as shown in the sketch.

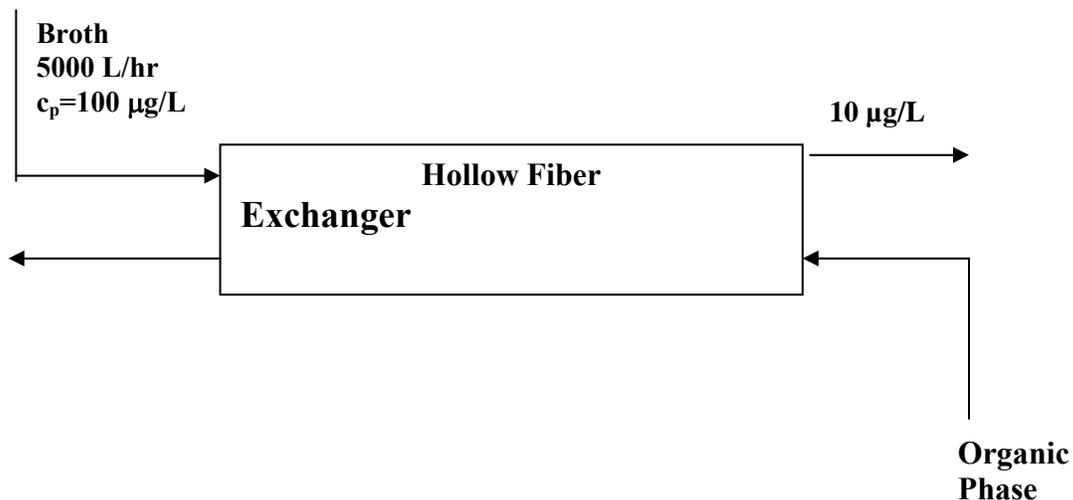
The broth will pass through the tubes, which are 1 cm in diameter, and the organic fluid will be the shell-side fluid. It is thought that the inside coefficient,  $h_{m, aq}$ , will have a value of  $1 \times 10^{-3}$  cm/sec. It is estimated that the outside coefficient,  $h_{m, org}$ , will have a value of  $1 \times 10^{-4}$  cm/sec. The tube wall will be highly porous and oil-filled. Its thickness will be 0.5 mm. The diffusivity of the pharmaceutical product in oil is  $0.5 \times 10^{-5}$  cm<sup>2</sup>/s.

The distribution coefficient,  $S$ , has a value of 8.5, i.e.

$$S = \frac{\text{conc. in organic phase}}{\text{conc. in aqueous phase}} = 8.5$$

The concentration of the pharmaceutical product,  $C_p$ , must be reduced from an inlet value of 100  $\mu\text{g/L}$  to a maximum value in the outlet of 10  $\mu\text{g/L}$ .

- (a) If the exchanger is infinitely large, what is the required flow rate of the organic phase?
- (b) What is the required exchanger area if the flow rate of the organic phase is 1000 L/hr?



### Problem 8

An artificial kidney (hemodialyzer) consists of a bundle of hollow fiber membranes through which blood flows (see Figure 1). The fibers, which are potted in epoxy sheets and contained within a plastic shell, are bathed on the outside by countercurrent flow of dialysate, which is an osmotically balanced aqueous solution that contains no biological wastes. During passage through the device, waste products of metabolism permeate from the bloodstream across the membrane to the dialysate phase. The device design parameters are as follows:

Active hollow fiber length	L	= 15 cm
Hollow fiber internal diameter	d	= 200 $\mu\text{m}$
Hollow fiber wall thickness (wet)	t	= 20 $\mu\text{m}$
Number of hollow fibers	n	= 10,000
Active surface area (based on d)	A	= 10,000 $\text{cm}^2$
Shell volume outside hollow fibers	$V_{\text{shell}}$	= 100 ml

You have carried out a laboratory experiment with the blood replaced by water to test the effectiveness for the removal of urea. Although there are only aqueous solutions for both streams, they will be referred to as blood and dialysate streams to denote the fluid flowing inside and outside of the hollow fibers, respectively. Operating conditions were as follows:

Blood flow rate	$Q_b$	= 100 ml/min
Dialysate flow rate	$Q_d$	= 500 ml/min
Inlet blood urea concentration	$C_{b,\text{in}}$	= 1 mg/ml

Temperatures of all streams were maintained at 37°C. A laboratory analysis states that urea was removed from the blood-side fluid at a rate of 75 mg/min.

Notes:

Diffusivity of urea in water at 37°C =  $1.8 \times 10^{-5} \text{cm}^2/\text{s}$ .  
Viscosity of water at 37°C =  $695 \times 10^{-6} \text{Ns/m}^2$

- What was the urea concentration in the blood outlet and in the dialysate outlet streams?
- Calculate the average overall mass transfer coefficient for urea in your experiments.
- Within the hollow fibers, what was the Reynolds number? Was the hydrodynamic boundary layer fully developed? Was the concentration boundary layer fully developed? Estimate the average mass transfer coefficient for urea in the blood-side fluid within the hollow fibers.

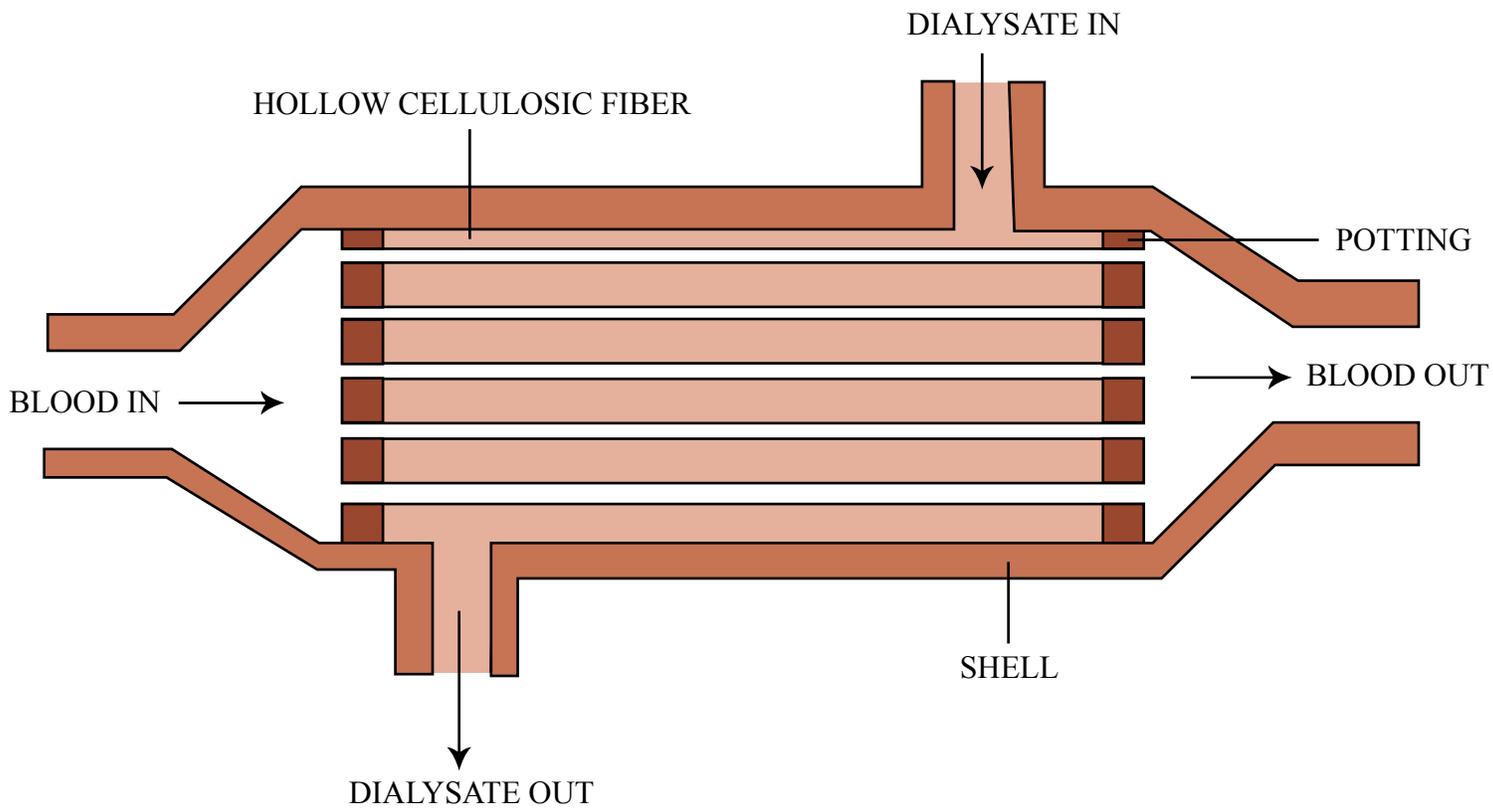
The rate of mass transfer per unit area across the membrane may be expressed as:

$$N'' = \frac{D_m K_{P\Box}}{t\Box} (C_{1,w\Box} - C_{2,w\Box})$$

where  $D_m$  is the solute diffusion coefficient in the membrane,  $K_p$  is the partition coefficient, defined as the ratio of equilibrium concentrations in the membrane and in water,  $C_m/C_w$ , and  $C_{1,w}$  and  $C_{2,w}$  are the concentration of solute in the water in contact with the internal (bloodside) and external (dialysate side) membrane surfaces, respectively. The group  $D_m K_p/t$  is often referred to as the membrane permeability,  $P_m$ . For the membrane in your experiments,  $D_m = 0.25 \times 10^{-5} \text{ cm}^2/\text{s}$ ,  $K_p=0.4$ , and  $P_m = 5 \times 10^{-4} \text{ cm/s}$  at  $37^\circ\text{C}$ .

- D. From your results in parts B and C and the membrane permeability information, estimate the average mass transfer coefficient in the dialysate.
- E. If the dialysate mass transfer coefficient varies with dialysate flow rate in proportion to  $Q_d^{0.8}$ , what would the overall mass transfer coefficient be if  $Q_d$  was increased to 1000 ml/min?
- F. If your experiment had been carried out in co-current flow (instead of counter-current flow), what rate of urea removal would you expect to have occurred?
- G. Assume that the entire device is initially at equilibrium with a urea concentration of zero. At  $t = 0$ , the inside surface of the hollow fiber membranes undergoes a step change to some non-zero value, while the outside surface is fixed at zero. Approximately how long will it take for the concentration throughout the membranes to reach about 90% of its steady-state value?
- H. Now assume that the step change occurs at the inlet blood concentration, and the inlet dialysate concentration remains at zero. Does your answer from part G provide a good estimate of the time scale required for the device to come to steady state? Describe all of the phenomena that would affect the device transient response, and provide an order-of-magnitude estimate for the time scale associated with each phenomenon.

SKETCH FOR PROBLEM 8



SCHEMATIC DIAGRAM OF HOLLOW FIBER MEMBRANE HEMODIALYZER

NOT DRAWN TO SCALE. Assume flow arrangement is: PURE COUNTERCURRENT FLOW

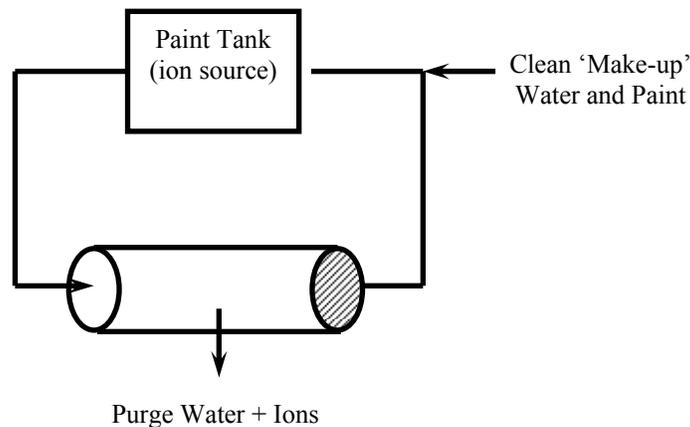
Figure by MIT OCW.

### Problem 9

In the painting of automobiles, the primer coat (which is the coat applied to the bare steel for corrosion protection) is electro-deposited. More specifically, the polymer which comprises the paint is ionic in character and therefore soluble in water at the right pH. In electro-deposition, two things happen:

- (1) an applied electrical field drives the ionic polymer toward the car body, and
- (2) the passage of electrical current produces an electrode reaction at the car body which shifts the local pH and leads to deposition of the polymer. The deposited polymer is later cured in an oven.

One of the problems with this process is that it requires that the aqueous solution be relatively free of any ions except the polymer and its counterions. Excess ions of any sort simply waste current. But extraneous ions are always present on the surface of the steel, so the procedure requires use of a process to remove these extraneous ions. One way to do this is to use ultrafiltration membranes which retain the polymer, but allow passage of the ions and water. A schematic showing how the process works is as follows:



The design group which is responsible for constructing the ultrafilter has developed a schism in which one subgroup prefers small diameter ultrafiltration tubes (because they offer large area per unit volume) and another subgroup prefers large diameter ultrafiltration tubes (because they tend to offer low pressure drop).

You have been asked to help with one aspect of the comparison. Specifically, ultrafiltration tends to concentrate the polymer molecules near the surface of the membrane, and this is deleterious to performance. Consequently, it is important to know the values of the mass transfer coefficients for the transport of polymer in the various designs.

- A. What value of  $h_m$  would you predict for polymer transport if:
- the tubes are 3m long and the I.D. of each tube is 1mm? The average velocity of the aqueous solution is 0.30m/s.
  - the tubes are 3m long and the I.D. of each tube is 1cm? The average velocity of the aqueous solution is 3m/s.

## DATA

$$\begin{aligned}v &= 1 \times 10^{-6} \text{ m}^2/\text{s} \\D &= 5 \times 10^{-11} \text{ m}^2/\text{s} \text{ for polymer in water} \\ \alpha &= 1 \times 10^{-7} \text{ m}^2/\text{s} \\k &= 0.6 \text{ W/m}\cdot\text{k} \\ \rho &= 1000\text{kg/m}^3 \\ c_p &= 4000 \text{ J/kg}\cdot\text{K}\end{aligned}$$

- B. The bulk concentration of the polymer is  $50\text{kg/m}^3$  and the trans-membrane flux of water is  $1.0 \times 10^{-3} \text{ kg/m}^2\cdot\text{s}$ . The membrane does not permit the passage of any polymer across it. For each of the design cases, what is the polymer concentration at the surface of the membrane?
- C. Solely from a mass transfer perspective, which tube would you recommend?
- D. For design (a), what is the (approximate) thickness of the concentration layer?
- E. On the basis of some vague notion about “disrupting the boundary layer”, it has been suggested that the tubes be fabricated with annular bumps on the tube wall. These would be located every 50cm along the length of the tube. Would this concept substantially enhance the value of  $h_m$  for the 1mm tube? Why or why not? For the 1cm tube? Why or why not?

### Problem 10

It has been suggested that the diffusivity of a sparingly soluble gas in a liquid (e.g. CO in water) may be measured as follows:

1. A small bubble of pure gas is injected into the liquid and held in place on the end of a small rod.
2. The diameter of the bubble is observed vs time; and the diffusivity is inferred from application of an appropriate theory.
  - A. Please derive the appropriate theory for the bubble diameter as a function of its initial diameter ( $D_i$ ), the density of the bubble phase ( $\rho_B$ ), the mass concentration of the dissolved gas in equilibrium with the bubble ( $\rho_{DG}$ ), etc. You may assume that the bubble is spherical, that the presence of the stabilizing rod may be ignored, that convection is absent in both phases, that the pressure in the bubble is constant, and that the process is quasi-steady. In addition, it may be assumed that the initial concentration of dissolved gas in the liquid is zero.
  - B. What criterion must be satisfied if the quasi-steady assumption is to be valid?
  - C. Very small velocities in the liquid phase due to natural convection are almost inevitable. What criterion must be satisfied if these velocities are to have no perceptible effect on the rate of dissolution, i.e. how small a velocity would be permissible?
  - D. For NO in water, it is thought that  $\mathcal{D} \cong 2 \times 10^{-5}$  cm/s and  $\rho_{DG} \cong 0.05$  gm/L. The value of  $\rho_B$  would be approximately 1 gm/L. How long would it take for the bubble to dissolve? Assume that the initial diameter is 1 mm. Would this be a sensible technique to use as a means of measuring diffusivity?

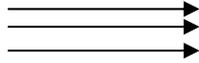
### Problem 11

Western Mountain Sports (WMS) is a small company which caters to the needs of hikers and campers who enjoy the high desert areas of the American West. The company is now considering production of a device which would serve as a passive cooler in order to provide relatively cool drinking water for thirsty campers. The idea would be to use microporous polymer in order to manufacture a bag which would be essentially spherical when filled with water. The porosity of the bag would be carefully selected to permit the flow of just enough water to keep the exterior of the bag constantly wet. So, the campers would fill the ECO-CHILLER® with warm water, hang it in the shade, and allow evaporative cooling to proceed. It is believed that desert conditions will typically correspond to a temperature of 37°C (310K), a relative humidity of 20%, and a wind of 5 m/s. The situation is shown in the sketch.

- A. What is the value of the heat transfer coefficient which would characterize exchange between the bag and the air?
- B. What is the value of the mass transfer coefficient which would characterize exchange between the bag and the air?
- C. What temperature will the contents of the ECO-CHILLER® eventually reach? Assume the contents to be isothermal at all times.
- D. WMS has consulted with two old prospectors, Pyrite Pete and Calcite Cal, who tell WMS that they had used this trick often with a simple leather bag and that the contents would cool down “right quick”. Please give your estimate of the time constant for the cool-down process.
- E. If it is not possible to hang the ECO-CHILLER® in the shade, it is expected that it will intercept 50W of solar energy. Will this significantly compromise the performance of the device?
- F. Will evaporation claim a significant fraction of the contents of the device during the time required for cool-down?

Sketch (Problem 11)

$$V = 5 \text{ m/s}$$



$$T = 37^\circ\text{C}$$
$$(310\text{K})$$

$$\text{RH} = 20\%$$

$$\text{Volume of Contents} = 8.2\text{L}$$

PROPERTIES

Air/Water Vapor @ 310K

$$\rho = 1.13 \text{ kg/m}^3$$

$$c_p = 1008 \text{ J/kg}\cdot\text{K}$$

$$\mu = 190 \times 10^{-7} \text{ N}\cdot\text{s/m}^2$$

$$\nu = 16.8 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.027 \text{ W/m}\cdot\text{K}$$

$$D = \alpha = 23.7 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.71$$

$$\text{Sc} = 0.71$$

Liquid Water @ 310K

$$\rho = 993 \text{ kg/m}^3$$

$$c_p = 4200 \text{ J/kg}\cdot\text{K}$$

$$\mu = 690 \times 10^{-6} \text{ N}\cdot\text{s/m}^2$$

$$k = 630 \times 10^{-3} \text{ W/m}\cdot\text{K}$$

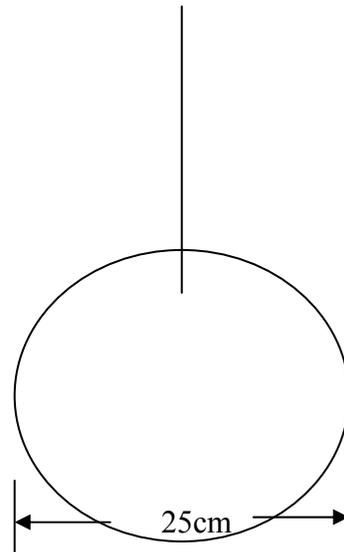
$$\nu = 6.9 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\alpha = 1.5 \times 10^{-7} \text{ m}^2/\text{s}$$

$$\text{Pr} = 4.6$$

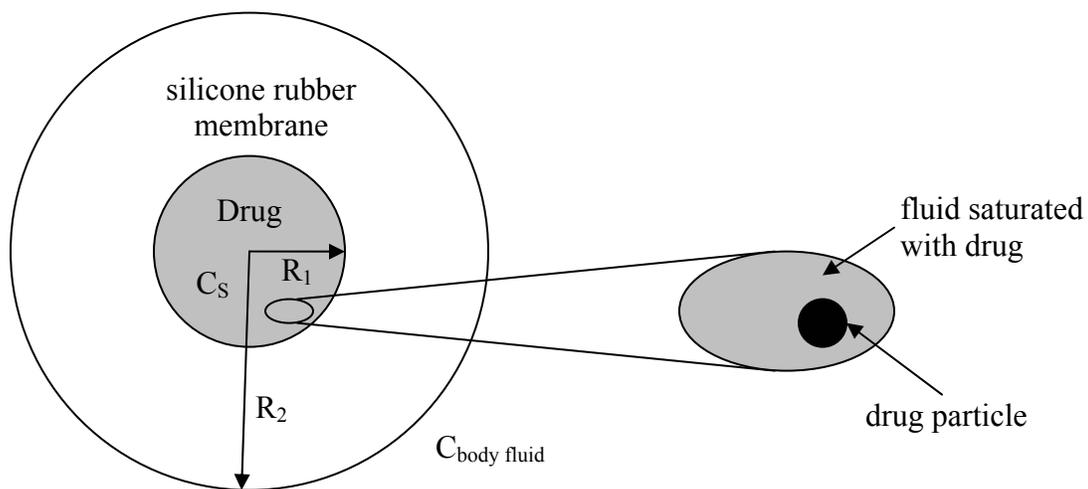
$$\lambda_{fg} = 2.4 \times 10^6 \text{ J/kg (latent heat of vaporization)}$$

$$P_{VP} = 6.22 \times 10^3 \text{ Pa}$$



## Problem 12

The bioavailability of drugs delivered orally is often low. Following absorption from the digestive tract into the blood, many drugs are quickly converted by the liver to an inactive form before they can reach the rest of the circulation. One way to get around this problem is to implant a device which will release the drug at a constant rate into the systemic circulation. This type of device can also improve the efficacy of the drug because patients often forget to take drugs at the right time. Norplant™ is an implantable device which releases the steroid hormone Norgestrel into the blood when implanted under the skin of the arm. It consists of a silicone rubber cylinder surrounding a core matrix containing the drug, as shown below. The silicone rubber serves as a diffusion barrier to control the release rate of the steroid.



- Derive an equation for the rate of steroid release from the device in terms of:
  - $c_b$  = concentration of drug in the body fluid at the surface of the device (assume constant)
  - $c_s$  = drug concentration in the core (assume constant)
  - $R_1, R_2$  = inner and outer radii of the membrane
  - $D_m$  = diffusion coefficient of the steroid in the membrane
  - $S = c_{\text{membrane}}/c_{\text{body fluid}}$  (partition coefficient)
- It is desired to deliver  $76\mu\text{g}$  of the drug per day to the patient using device that is 2 cm long with an inside diameter of 1.57mm. The desired steady-state concentration in the plasma of the patient is 1 ng/ml. What should the wall thickness be to give the correct dosage?
 

Data:  $S = 70$ ;  $C_s = 0.2 \text{ mg/ml}$ ;  $D_{\text{steroid-silicone}} = 2 \times 10^{-7} \text{ cm}^2/\text{s}$   
 The partition coefficient between the core fluid and the silicone for this steroid is unity.
- In the clinical trials of the device, it is noted that the release rate is much slower in some patients than in others. This is attributed to the formation of scar tissue around the device, which adds a resistance to transport from the device to the

blood. In one patient, a capsule of scar tissue 0.75mm thick was found surrounding the device. If the solubility of the drug in the scar capsule is the same as that in body fluids, and the diffusion coefficient in tissue is  $6 \times 10^{-7} \text{ cm}^2/\text{s}$ , what would be the expected rate of drug delivery in this patient? Assume that the scar capsule acts as a 0.75 mm thick coating on the device, and that the surface of the scar is in contact with body fluid.

Graph removed due to copyright reasons.

(Dry-bulb temperature [°C] vs Humidity ratio [kg moisture per kg dry air] vs  
Enthalpy [kJ/kg of dry air] vs Saturation temperature [°C] vs Specific volume  
[m<sup>3</sup> per kg dry air] vs Wet-bulb temperature [°C] vs Relative humidity)