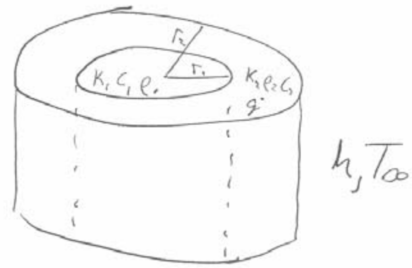
Test I

Prob 1



(a.) 1-D Diffusion, const. properties, unsteady

$$\frac{1}{r}\frac{\partial}{\partial r}\left(K_1 r \frac{\partial T}{\partial r}\right) = \rho_1 C_1 \frac{\partial T}{\partial r} \quad \boxed{2pts}$$

IC

$$T(r,0) = T_\infty \quad \text{for } 0 \le r \le r_1 \quad \boxed{1pt}$$

BC

$$\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \quad \text{Symmetry} \quad \boxed{1pt}$$

$$T_{in}\big|_{r_1} = T_{out}\big|_{r_1} \quad \text{Temp continuity} \quad \boxed{1pt}$$

* Note $-K_1 \frac{\partial T_{in}}{\partial r}\big|_{r=r_1} = -K_2 \frac{\partial T_{out}}{\partial r}\big|_{r=r_1}$ will not work because you need at least 1 temp BC to solve for both constants of integration.

b.) Same as (a) but with generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(K_2 r \frac{\partial T}{\partial r}\right) + \dot{q} = \rho_1 C_1 \frac{\partial T}{\partial r} \quad \boxed{2pts}$$

IC

$$T(r,0) = T_\infty \quad \text{for } r_1 \le r \le r_2 \quad \boxed{1pt}$$

BC

$$-K_1 \frac{\partial T_{in}}{\partial r}\bigg|_{r=r_1} = -K_2 \frac{\partial T_{out}}{\partial r}\bigg|_{r=r_1} \quad \text{finite flux} \quad \boxed{1pt}$$

$$T_{in}\big|_{r=r_1} = T_{out}\big|_{r=r_1} \quad \text{Temp continuity} \quad \boxed{1pt}$$

$$-K_2 \frac{\partial T}{\partial r}\bigg|_{r=r_2} = h\left(T\big|_{r=r_2} - T_\infty\right) \quad \text{Convection}$$

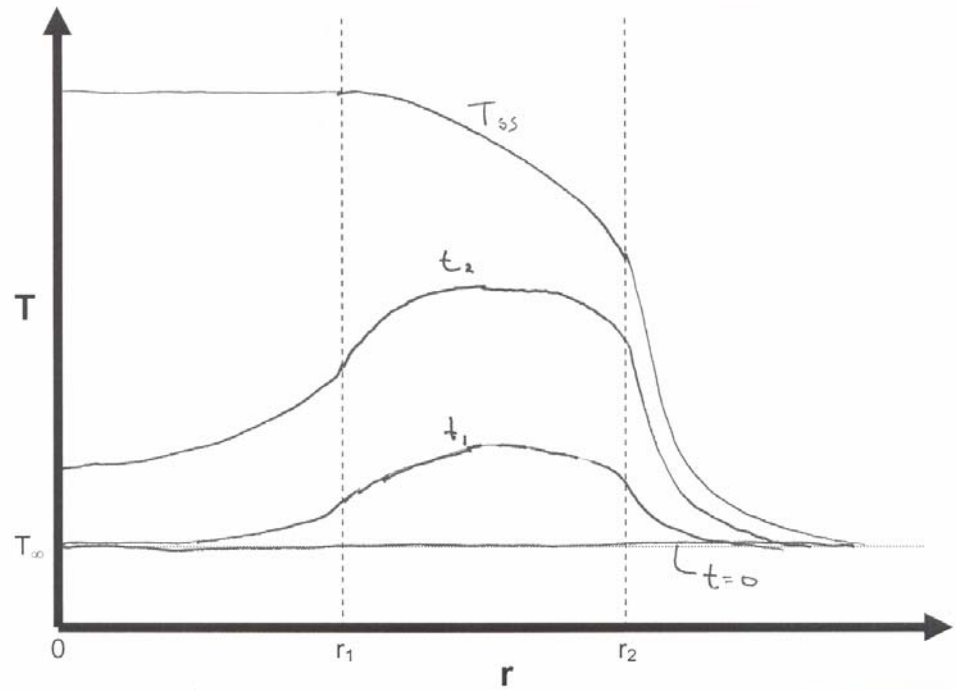
* Note: T_{s1}, T_{s2} were not defined in the problem as known parameter. An expression based on known parameters must be included.

* Note: Need convection BC to relate temp profile to T_∞.

TEMPLATE

Problem 1(c)

Name:____________________



$t=0 \quad T=T_\infty$

$t_1, t_2 \quad r=0 \quad \frac{\partial T}{\partial r}=0$ $\boxed{2\,pts/_{line}}$

$0<r<r_1$ hyperbolic

$r_1<r<r_2$ parabolic with T_{max} in it $\boxed{2\,pts/_{line}}$

$r_2<r \rightarrow$ hyperbolic decay to T_∞

$T_{ss} \quad 0<r<r_1$ no flux $\frac{\partial T}{\partial r}=0$ $\boxed{3\,pt}$

$r_1<r<r_2$ parabolic with $T_{max}|_{r=r_1}$ $\boxed{3\,pt}$

$r_2<r$ hyperbolic decay to T_∞ $\boxed{1\,pt}$

(d.) Inner cyl

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_1 r \frac{\partial T}{\partial r}\right) = \rho_1 C_1 \cancel{\frac{\partial T}{\partial t}}^{\,0 - ss}$$

$$r\frac{\partial T}{\partial r} = c_1$$

Integration $\boxed{1pt}$

$\boxed{1pt}$ Apply BC $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

$$(0)(0) = c_1$$

$$r\frac{\partial T}{\partial r} = 0$$

$$T = c_2$$

$\boxed{1pt}$ Apply BC $T_{in}|_{r=r_1} = T_{out}|_{r=r_1}$

$$T = T_{out}|_{r=r_1}$$

Outer Cyl

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_2 r \frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

$$r\frac{\partial T}{\partial r} = -\frac{\dot{q}}{2k_2}r^2 + C_1$$

$$T = -\frac{\dot{q}}{4k_2}r^2 + c_1 \ln r + C_2 \quad \boxed{3pts}$$

Apply BC $\left.-k_1\frac{\partial T_{in}}{\partial r}\right|_{r=r_1} = \left.-k_2\frac{\partial T_{out}}{\partial r}\right|_{r=r_1} = 0$ (Known from Inner Cyl solution)

$$r_1\left.\frac{\partial T}{\partial r}\right|_{r=r_1} = r_1(0) = -\frac{\dot{q}}{2k_2}r_1^2 + c_1$$

$$c_1 = \frac{\dot{q}r_1^2}{2k_2} \quad \boxed{1pt}$$

Apply BC $\left.-k_2\frac{\partial T_{out}}{\partial r}\right|_{r=r_2} = h\left(T|_{r=r_2} - T_\infty\right)$

$$-K_2\left(-\frac{\dot{q} r_2}{2K_2}+\frac{c_1}{r_2}\right)=h\left(-\frac{\dot{q}}{4K}r_2^2+c_1\ln r_2+c_2-T_\infty\right)$$

$$-K_2\left(-\frac{\dot{q} r_2}{2K_2}+\frac{\dot{q} r_1^2}{2K_2 r_2}\right)=h\left(-\frac{\dot{q} r_2^2}{4K}+\frac{\dot{q} r_1^2}{2K}\ln r_2+c_2-T_\infty\right)$$

$$\frac{\dot{q}}{2h}\left(r_2-\frac{r_1^2}{r_2}\right)=-\frac{\dot{q} r_2^2}{4K}+\frac{\dot{q} r_1^2}{2K}\ln r_2+c_2-T_\infty$$

$$\boxed{1pt}\quad c_2=\frac{\dot{q}}{2hr_2}\left(r_2^2-r_1^2\right)+\frac{\dot{q} r_1^2}{4K_2}-\frac{\dot{q} r_1^2}{2K_2}\ln r_2+T_\infty$$

Outer

$$\boxed{2pts}\quad T=-\frac{\dot{q}}{4K_2}\left(r^2-r_2^2\right)+\frac{\dot{q} r_1^2}{2K_2}\ln(r/r_2)+\frac{\dot{q}}{2hr_2}\left(r_2^2-r_1^2\right)+T_\infty$$

* Note inner cyl $T=T_{out}\big|_{r=r_1}$

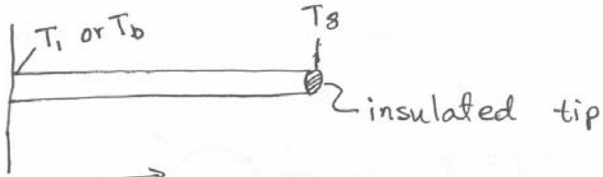
Inner

$$T=-\frac{\dot{q}}{4K_2}\left(r_1^2-r_2^2\right)+\frac{\dot{q} r_1^2}{2K_2}\ln(r_1/r_2)+\frac{\dot{q}}{2hr_2}\left(r_2^2-r_1^2\right)+T_\infty$$

e.) $T_{centerline}=T_{inner}(r=0)=T_{inner}=$ ________________

$\boxed{5pts}$

Problem 2



$T_\infty = 20°C$

$h = 17 \frac{W}{m^2 K}$

at steady state $T_1 = 60°C$

Dimensions:

$L = 0.35\ m$

$D = 0.01\ m$

Properties:

$\rho = 8500 \frac{kg}{m^3}$

$c = 380\ J/kg\,K$

$k = 121 \frac{W}{m\,K}$

Assumptions:

1. steady state
2. Adiabatic tip
3. Constant material properties
4. $Bi = \frac{hL}{k} = \frac{(17)(0.005)}{121} = 7.02 \times 10^{-4} < 0.1$

   $\therefore$ 1D heat transfer, use fin equation
5. Heat loss due to radiation negligible

Part a

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

Find: θ at $x=L$

$$\theta_b = T_b - T_\infty = 60-20 = 40°C$$

$$m^2 = \frac{hP}{kA_c} = \frac{h\cancel{\pi}\cancel{D}}{k\cancel{\pi}D^{\cancel{2}}/4} = \frac{4h}{kD} = \frac{4(17)}{(121)(0.01)} = 56.2$$

$$m = 7.5$$

$$mL = 7.5(0.35) = 2.62$$

$$\cosh 2.62 = 7.1917$$
$$\cosh 0 = 1$$

$$\theta = \frac{40}{7.1917} = 5.56°C = T(L) - T_\infty$$

$$T(L) = T_{\infty} = 20 + 5.56$$

$$\boxed{T_\infty = 25.6°C} = 298.6\ K$$

Part b

$$q_f = M \tanh mL$$

$$M = \theta_b\sqrt{hPkA_c} = \theta_b\sqrt{h\pi D k \frac{\pi D^2}{4}}$$

$$= 40\sqrt{(17)(\pi)^2(0.01)^3(121)/4} = 2.85$$

$$\tanh mL = \tanh 2.62 = 0.99022$$

$$q_f = 2.85(0.99022) = 2.82\ W$$

$$\boxed{q_f = 2.82\ W}$$

Part c

What is time to reach steady state (t_{ss}) if length is 0.70m. t_{ss} for $L = 0.35\,m$ is 100 min

Equation given in test is of the form:

$$\theta(x,t) = f(x) + g(t) + g'(x,t)$$

Note that $g(t) = g'(x,t)$ for $n=0$

At greater time exponential terms in g & g' will dominate.

Compare the exponential terms:

$g: \quad \alpha m^2 t = t_1^*$

$g': \quad [(n\pi)^2 + (mL)^2]\frac{\alpha t}{L^2} = t_2^*$

with increasing n, $t_2^* >> t_1^*$

$\therefore \; g' << g$

We can approximate θ as:

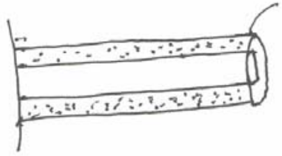
$$\theta(x,t) = f(x) + g(t)$$

t_1^* is independent of L

$\therefore$ Doubling the length will not significantly change t_{ss}.

$$\therefore \; \boxed{t_{ss} \approx 100 \text{ min}}$$

Part d



$D_i = 0.02\,m$
$k = 0.030 \frac{W}{m\,K}$

Calculate Bi for rod

There are two resistances to heat transfer

Rod $\bullet$—R_{ins}—R_{conv}—$\bullet$ T_∞

$$R_{conv} = \frac{1}{hA} = \frac{1}{h\pi D_i L}$$

$$R_{ins} = \frac{\ln r_i/r_R}{2\pi L k_i}$$ Table for cylinder

sum of resistances gives an overall heat transfer coefficient, UA

$$\frac{1}{UA_R} = R_{Tot} = \frac{1}{h\cancel{\pi} D_i \cancel{L}} + \frac{\ln r_i/r_R}{2\cancel{\pi}\cancel{L}\,k_i} = \frac{1}{U\cancel{\pi} D_R \cancel{L}}$$

$$\frac{1}{UD_R} = \frac{1}{hD_i} + \frac{\ln r_i/r_R}{2k_i}$$

$$\frac{1}{U(0.01)} = \frac{1}{(17)(0.02)} + \frac{\ln 2/1}{2(0.03)}$$

$$= 2.94 + 11.55$$

$$U = 6.9 \frac{W}{m^2K}$$

$$Bi = \frac{UL}{k} \qquad L = r_R$$

$$= \frac{\left(6.9\frac{W}{m^2K}\right)(0.005m)}{121\ \frac{W}{mK}}$$

$$Bi = 2.85 \times 10^{-4} < 0.1$$

$\therefore$ Fin analysis is still valid

Part e

What is T_1 (or T_b) at steady state?

same q_f supplied to rod as part (b)

$$q_f = 2.82\,W = M \tanh mL$$

$$m^2 = \frac{4U}{kD} = \frac{4(6.9)}{(121)(0.01)} = 22.8$$

$$m = 4.78$$

$$mL = (4.78)(0.35) = 1.67$$

$$\tanh 1.67 = 0.9313$$

$$M = \theta_b \sqrt{U\pi^2 D^3 k/4} = \theta_b \sqrt{(6.9)\pi^2(0.01)^3(121)/4} = 4.5 \times 10^{-2}$$

$$2.82 = \theta_b (4.5 \times 10^{-2})(\cancel{1.67}\ 0.9318)$$

$$\theta_b = 66.8^\circ C$$

$$T_b = \boxed{T_1 = 86.8^\circ C} = 359.8K$$

Part f

$T_8 = ?$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$mL = 1.67$

$\cosh mL = 2.753$

$$\theta(L) = \frac{66.8}{2.753} = 24.2$$

$$T(L) = \boxed{T_8 = 44.2\,^\circ C} = 317.2\,K$$

Part g

- We established in part (c) that $n \geq 1$ terms in $\theta(x,t)$ are negligible
- Again, exponential term will dominate
- Since h is different, m^2 will be different

$$m^2_{old} = 56.2 = \frac{4h}{kD}$$

$$m^2_{new} = 22.8 = \frac{4U}{kD}$$

$$\cancel{\alpha}\, m^2_{old}\, t_{old} = \alpha\, m^2_{new}\, t_{new}$$

$$t_{new} = \frac{h}{U}\, t_{old} = \frac{17}{6.9}\,(100\,\text{min})$$

$$\boxed{t_{new} = 246\ \text{min}}$$