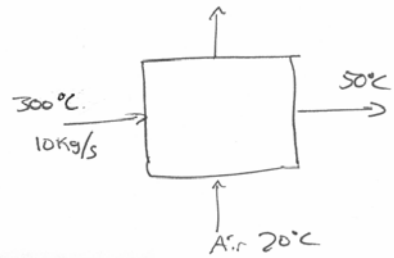
Test II

Problem 1

Assume ① SS
② Const Properties

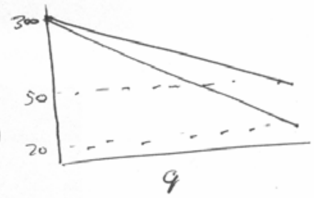


A.) Product

$q_p = \dot{m}_p C_{p,p} \Delta T_p$

$= (10\,\text{Kg/s})(2500\,\frac{J}{KgK})(300-50°C)$

$= 6.25 \times 10^6\,W$ (+1)



Air

$T_{Air,o} = 300\,°C$ (+2)

$q = \dot{m}_{Air}\, C_{p,Air}\, \Delta T_{Air,max}$

$\dot{m}_{Air} = \frac{q}{C_{p,Air}\,\Delta T_{Air}} = \frac{6.25 \times 10^6\,W}{1000\,\frac{J}{KgK}\,(300-20°C)} = 22.3\,\text{Kg/s}$ (+2)

minimum Flowrate

B.) $\dot{m}_{Air} = (1.1)(22.3\,\text{Kg/s}) = 24.55\,\text{Kg/s}$

$(T_{Air,out} - T_{Air,in}) = \frac{q}{\dot{m}_{Air}\,C_{p,Air}} = \frac{6.25 \times 10^6\,W}{(24.55\,\text{Kg/s})(1000\,J/KgK)} = 254°C$

$T_{AIR,out} = 275°C$ (+2)

LMTD

$$R = \frac{T_i - T_o}{t_o - t_i} = \frac{20-275}{50-300} = 1.02 \text{ (or } .98)$$

$$P = \frac{t_o - t_i}{T_i - t_i} = \frac{50-300}{20-300} = 0.89$$

Fig 11.12

F is off chart (+3)

This is not a reasonable HX. (+2.5)

NTU $\quad C_x = \dot{m}_x C_{p_x}$

$C_{air} = 24{,}600\ W/K \leftarrow \underline{C_{min}}$

$C_{prod} = 25{,}000\ W/K$

$C_{min}/C_{max} = .98$

$q_{max} = C_{min}(T_{hi} - T_{ci})$

$= 24600(300-20)$

$= 6.87\times10^6\ W$ (+1)

$$\varepsilon = \frac{q}{q_{max}} = \frac{6.25}{6.87} = .91 \text{ (+1)}$$

Fig 11.18

NTU ~ 49 (+1)

This is not reasonable (+2.5)

NTU ~ 2-3

C.) i.) h_i

$$Re_D = \frac{(1\ m/s)(0.0203\ m)}{2\times10^{-6}\ m^2/s} = 1.01\times10^4 \Rightarrow \text{turbulent (+2)}$$

$$\frac{L}{D} = \frac{3.05}{.02} = 152 > 10 \Longrightarrow \text{fully developed (+2)}$$

$\overline{Nu}_D = .023\,(Re_D)^{.8}\,Pr^{.3}$ $\quad$ Eq 8.60 (other correlations acceptable) (+4)

$\overline{Nu}_D = .023\,(1.01\times10^4)^{.8}(21)^{.3}$

$= 91.6$ (+1)

$$\overline{Nu}_D = \frac{\bar{h}_i D}{K} = 91.6 = \frac{\bar{h}_i(.0203\,m)}{(.25\ \frac{W}{mK})}$$

$\bar{h}_i = 1130\ \frac{W}{m^2K}$ (+1)

c ii) Outer h_o $Re = \frac{(4\frac{m}{s})(.0254\,m)}{21\times10^{-6}\,m^2/s} = 4840 \Rightarrow$ laminar (+2)

$Pr = .7$

$\overline{Nu} = 0.193\,Re^{.618}\,Pr^{1/3}$ Eq 7.55b $4000 < Re < 40{,}000$

(Other correlations acceptable) (+6)

$= 0.193\,(4840)^{.618}(.7)^{1/3}$

$= 32.4$ (+1)

$\overline{Nu} = \frac{\bar{h}_o D}{k} = 32.4 = \frac{\bar{h}_o\,(.0254\,m)}{0.03\,\frac{W}{mK}}$

$\bar{h}_o = 38.3\,\frac{W}{m^2K}$ (+1)

iii) $\frac{1}{UA_i} = \frac{1}{h_iA_i} + \frac{\ln(r_o/r_i)}{2\pi LK} + \frac{1}{h_oA_o}$ (+3)

* If you noted that you ignored conductance, or approx. it as planar, OK!

$\frac{1}{U} = \frac{1}{h_i} + \frac{A_i \ln(D_o/D_i)}{2\pi LK} + \frac{A_i}{A_o h_o}$

$= \frac{1}{h_i} + \frac{\pi D_i L \ln(D_o/D_i)}{2\pi L K} + \frac{\pi D_i L}{\pi D_o L\, h_o}$

$= \frac{1}{1130\frac{W}{m^2K}} + \frac{(.0203m)\ln(1/.8)}{2\quad 60\,W/mK} + \frac{.8}{1.0}\frac{1}{38.3\frac{W}{m^2K}}$

$= 8.85\times10^{-4}\,\frac{m^2K}{W} + 3.77\times10^{-5}\,\frac{m^2K}{W} + 0.0209\,\frac{m^2K}{W}$ limiting resistance

$U = 45.8\,\frac{W}{m^2K}$ (+2)

C iv.) $T_{Air,out} = \dfrac{q}{\dot{m}_{Air} C_{p_{Air}}} + T_{Air,in} = \dfrac{6.25 \times 10^6 \, W}{100 \, kg/s \;\; 1000 \, J/kgK} + 20°C$

$= 82.5°C$

<u>LMTD</u>

$\Delta T_{LMTD} = \dfrac{(300-82.5)-(50-20)}{\ln\left(\frac{300-82.5}{50-20}\right)} = 95°C$

$R = \dfrac{T_i - T_o}{t_o - t_i} = \dfrac{20-82.5}{50-300} = 0.25 \; (\text{or } 4)$

$P = \dfrac{t_o - t_i}{T_i - t_i} = \dfrac{50-300}{20-300} = 0.89 \; (\text{or } .223)$

<u>Fig. 11.12</u>

$F = .84$ +2

$q = U_i A_i F \Delta T_{LMTD}$ +1

$A_i = \dfrac{q}{U_i F \Delta T_{LMTD}}$

$= \dfrac{6.25 \times 10^6 \, W}{(45.8 \frac{W}{m^2K})(.84)(95°C)}$

$= 1717 \, m^2$ +1.5

<u>NTU</u>

$C_x = \dot{m}_x C_{p_x}$

$C_{prod} = 25{,}000 \, W/K = C_{min}$

$C_{Air} = 100{,}000 \, W/K$

$q_{max} = (25{,}000 \, W/K)(300-20)$

$= 7 \times 10^6 \, W$ +1

$\varepsilon = \dfrac{6.25 \times 10^6}{7.00 \times 10^6} = 0.89$ +1

$C_{min}/C_{max} = .25$

Fig 11.18

$NTU = 2.8$ +1

$A = \dfrac{NTU \, C_{min}}{U}$

$= \dfrac{(2.8)(25{,}000 \, W/K)}{45.8 \frac{W}{m^2K}}$

$= 1528 \, m^2$ +1.5

D) Fins on the exterior (+2)

The limiting resistance is between the tube and air. Increasing

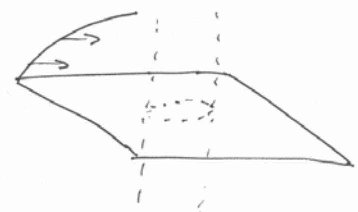
A_o will decrease $\frac{1}{h_o A_o}$. (+3)

Changing A_i will not significantly effect U_i

E.) Approx as a plane

$$Re_x = \frac{(4\,\frac{m}{s})(0.0762m)}{21\times10^{-6}\,m^2/s}$$

$= 1.45\times10^4$ Laminar (+2)



Assume boundary layers do not touch in adjoining squares

$$\overline{Nu}_x = 0.664\, Re_x^{1/2}\, Pr^{1/3} \quad (\text{Eq } 7.31)$$

$$= 0.664\,(1.45\times10^4)^{1/2}\,(0.7)^{1/3}$$

$= 71$ (+1)

$$\overline{Nu}_x = \frac{\bar{h}\,L}{K}$$

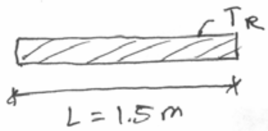
$$\bar{h} = \frac{\overline{Nu}\,K}{L} = \frac{(71)(.03\,\frac{W}{mK})}{(.0762m)}$$

$= 28\,\frac{W}{m^2K}$ (+1)

Problem 2

$T_\infty = 6°C = 279 K$

$T_{sur} = 230 K$



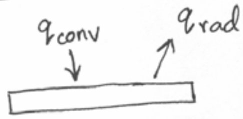
2 Assumptions:

1. Car roof is a flat plate
2. Ignore effect of windshield
3. Car roof is insulated, black → assume black body
4. Treat surroundings as black body
5. Properties of air are constant

a) $h = 5 W/m^2K$

$T_R = ?$

Energy balance on car roof:



2 $E_{in} - E_{out} = 0$

$q_{conv} - q_{rad} = 0$

$h\cancel{A}(T_\infty - T_R) = \sigma \cancel{A}(T_R^4 - T_{sur}^4)$

2 2

$$5(279 - T_R) = 5.67 \times 10^{-8} (T_R^4 - 230^4)$$

2 $$T_R^4 + 8.818 \times 10^7 T_R - 1554 = 0$$

Solve: $\boxed{T_R = 259\,K = -14°C}$

b) $T_R = 0°C = 273\,K$

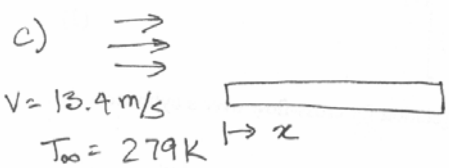
$h = ?$

3 $$h(T_\infty - T_R) = \sigma (T_R^4 - T_{sur}^4)$$

$$h(279 - 273) = 5.67 \times 10^{-8} (273^4 - 230^4)$$

2 $$\boxed{h = 26.0\ W/m^2K}$$

c)



at $x = 0.3\,m$, is $T_R \gtrsim 273\,K$?

Minimum h required for $T_R = 273\,K$ is $26.0 \frac{W}{m^2K}$

Restate question: @ $x = 0.3\,m$ is $h \gtrsim 26.0$?

Determine h using correlation for external flow over flat plate.

$$Re_x = \frac{Vx}{\nu} = \frac{13.4\,\tfrac{m}{s}\,x}{1.5\times10^{-5}\,m^2/s} = 8.93\times10^5\,x$$

(2) Determine Re_x at $x = 0.3$, $Re_{0.3} = 2.68\times10^5$

$Re_{x,c} = 5\times10^5$

(2) $\therefore$ flow is still laminar

To determine local heat transfer coefficient, use:

(2) $$Nu_x = 0.332\,Re_x^{1/2}\,Pr^{1/3} = \frac{h_x x}{k} \qquad \text{Eq 7.23}$$

$$0.332\,(2.68\times10^5)^{1/2}(0.71)^{1/3} = \frac{h_x(0.3\,m)}{2.5\times10^{-2}\,W/mK}$$

(2) $$\boxed{h_x = 12.8\,\frac{W}{m^2K}}$$

$h_x < 26.0$

(2) $\therefore$ Frost will <u>not melt</u>

d) (2) $Re_{1m} = 8.93\times10^5 > Re_{x,c}$

(2) $\therefore$ Flow is turbulent

(2) $$Nu_x = 0.0296\,Re_x^{4/5}\,Pr^{1/3} = \frac{h_x X}{k} \qquad \text{Eq. 7.37}$$

$$0.0296\,(8.93\times10^5)^{4/5}(0.71)^{1/3} = \frac{h_x(1\,m)}{2.5\times10^{-2}\,W/mK}$$

(2) $$\boxed{h_x = 38\,W/m^2K}$$

$h_x > 26.0$ $\therefore$ frost <u>will melt</u>

(2)

e) Determine x_c

$$Re_{x,c} = 5\times10^5 = 8.93\times10^5 x_c$$

1 $$x_c = 0.56\,m$$

$x < x_c$, flow is laminar → A

$x > x_c$, flow is turbulent → B

A

$$\frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

Determine maximum x for which $h \geq 26$

$$\frac{h_x x}{k} = 0.332\,(8.93\times10^5 x)^{1/2}\,(Pr)^{1/3}$$

$$\frac{26\, x^{1/2}}{0.025} = 0.332\,(8.93\times10^5)^{1/2}\,(0.71)^{1/3}$$

1 $$x = 0.072\,m$$

B Determine maximum x for which $h \geq 26$

$$\frac{h_x x}{k} = 0.0296\,(8.93\times10^5 x)^{4/5}\,(0.71)^{1/3}$$

$$\frac{26\, x^{1/5}}{0.025} = 0.0296\,(8.93\times10^5)^{4/5}\,(0.71)^{1/3}$$

1 $x = 6.7 >$ length of car roof

Frost-free: $0 \leq x \leq 0.072\,m$ and

2 $$x \geq 0.56\,m$$

Rough sketch of h vs. x