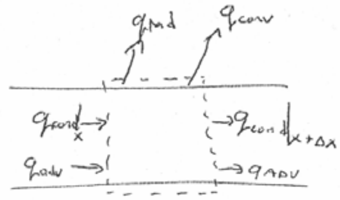
Quiz 3 Solutions 11/2/04 Recitation

(1)



$$\rho V_e A_c c_p T - \rho V_e A_c C_p (T + dT) - dq_{conv} - dq_{rad} + q_{cond}|_x - q_{cond}|_{x+dx} = 0$$

$$-\rho V_c \left(\frac{\pi D^2}{4}\right) C_p\, dT - \pi D\, dx \left[h (T - T_\infty) + \varepsilon \sigma (T^4 - T_{sur}^4)\right]$$

$$- K A_c \frac{\partial T}{\partial x} + \left(+ K A_c \frac{\partial T}{\partial x} - K \frac{\partial}{\partial x}\left(A_c \frac{\partial T}{\partial x}\right) dx = 0\right.$$

$$+ \frac{1}{4} \rho C_p V_c D \frac{\partial T}{\partial x} + h (T - T_\infty) + \varepsilon \sigma (T^4 - T_{sur}^4) - \frac{1}{4} D K \frac{\partial^2 T}{\partial x^2} = 0$$

(2) Evaluate Conduction Terms in Eqn

Axial Conduction

$\Delta T \sim T_i - T_\infty$ from earlier soln

$\hat{x} \sim L = 5m$

$$D K \frac{\partial^2 T}{\partial x^2} \sim D K \frac{\Delta T}{\hat{x}^2}$$

$$\cong \frac{(.005\,m)(400\,\frac{W}{mK})(600 - 25)}{(5m)^2}$$

$$\cong \boxed{46\ \frac{W}{m^2}}$$

$\frac{Kg}{m^3}$ $\frac{J}{KgK}$ $\frac{W}{mK}$ m

Convection

$$h(T-T_\infty) \sim h\Delta T$$

$$\approx 100 \frac{W}{m^2K}(600-25K)$$

$$\approx \boxed{57500 \frac{W}{m^2}}$$

$h(T-T_\infty) >> DK \frac{\partial^2 T}{\partial x^2}$, so we can neglect axial diffusion.

* Can also compare it to advection term $\left(\frac{1}{4}\rho C_p V_e D \frac{\partial T}{\partial x}\right)$

(3) Here we need to compare radial conduction to either

(a) convection
(b) axial conduction

either comparison is acceptable

(a) Radial conduction to convection is the Bi #

$$Bi = \frac{hr}{K}$$

$$= \frac{(100 \frac{W}{m^2K})(.005m)}{400 \frac{W}{mK}} = .00125 << 0.1$$

(b) By showing radial is less significant than axial, we can assume radial is not significant because in (2) we proved axial was insignificant.

$$R_x = \frac{L}{KA_c} = \frac{L}{K\left(\frac{\pi D^2}{4}\right)} \sim \frac{L}{KD^2}$$

$$R_r = \frac{r}{K(\pi D L)} \sim \frac{1}{KL}$$

$$\frac{R_x}{R_r} = \frac{\frac{L}{KD^2}}{\frac{1}{KL}} = \frac{L^2}{D^2} = \left(\frac{5m}{.005m}\right)^2 = 10^6$$

so resistance in axial direction is much larger than in radial direction,

so radial resistance is negligible