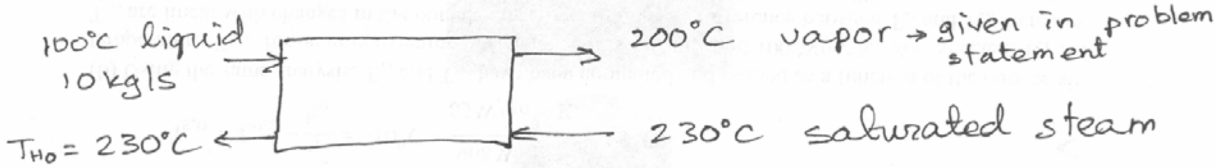
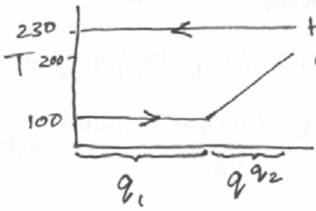
Exam 3 Solutions

Problem 1

(10) A. Countercurrent



$T_{Ho} = 230°C$ since no subcooling of condensate



Total energy required $= q = q_1 + q_2$

(2) $$= \dot{m}_c H_v + \dot{m}_c C_{p,v} (T_{co} - T_{c\,int.})$$

$T_{c,int} = 100°C$ → temperature of stream after it is vaporized

$$q = 10 \frac{kg}{s} \left(0.6 \times 10^6 \frac{J}{kg}\right) + 10 \frac{kg}{s} \left(3000 \frac{J}{kg\,K}\right)(200-100)°C$$

(2) $$= 6 \times 10^6 J + 3 \times 10^6 J$$

$$= 9 \times 10^6 J/s$$

Energy Balance:

(1) $$q = \dot{m}_H \Delta H_{v,steam}$$

$$9\times10^6\ J/s = \dot m_{SH}\,(1.8\times10^6\ \tfrac{J}{kg})$$

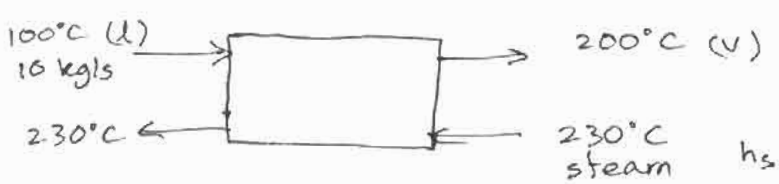
(1) $\boxed{\dot m_H = 5\ kg/s}$

(4) Cocurrent:

Boiler / Condenser requirements are independent of flow arrangement.

Can also repeat calculations to get $\boxed{\dot m_H = 5\ kg/s}$ again.

(20) B.

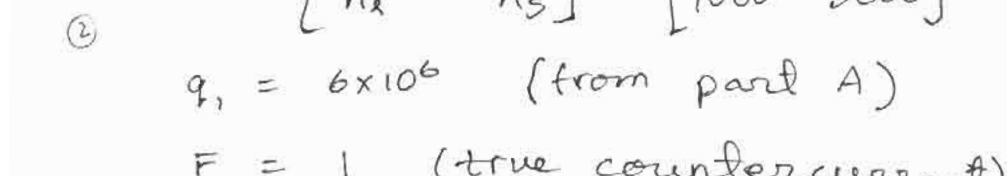


(2)

$h_l = 1000\ W/m^2K$
$h_v = 300\ W/m^2K$
$h_s = 5000\ W/m^2K$

- ignore wall resistance
- since walls are thin assume $A_i \cong A_o$

(5) - To determine area, treat as 2 exchangers



(1) (2) $$U_1 = \left[\frac{1}{h_l} + \frac{1}{h_s}\right]^{-1} = \left[\frac{1}{1000} + \frac{1}{5000}\right]^{-1} = 833\ \frac{W}{m^2K}$$

$q_1 = 6\times10^6$ (from part A)

$F = 1$ (true countercurrent)

(2) $\Delta T_1 = T_H - T_c$ → both constant ∴ don't need ΔT_{lm}

$= 230 - 100$

$= 130°C$

$q_1 = U_1 A_1 \Delta T_1$

(2) $6 \times 10^6 = 833\, A_1 (130)$

$A_1 = 55.4\ m^2$

(2) $U_2 = \left[\frac{1}{h_v} + \frac{1}{h_s}\right]^{-1} = \left[\frac{1}{300} + \frac{1}{5000}\right]^{-1} = 283\ \frac{W}{m^2K}$

(3) $q_2 = 3 \times 10^6\ J/s$

$$\Delta T_{lm} = \frac{(230-200) - (230-100)}{\ln \frac{230-200}{230-100}} = 68.2°C$$

$q_2 = U_2 A_2 \Delta T_{lm\,2}$

(2) $3 \times 10^6 = 283\, A_2\, 68.2$

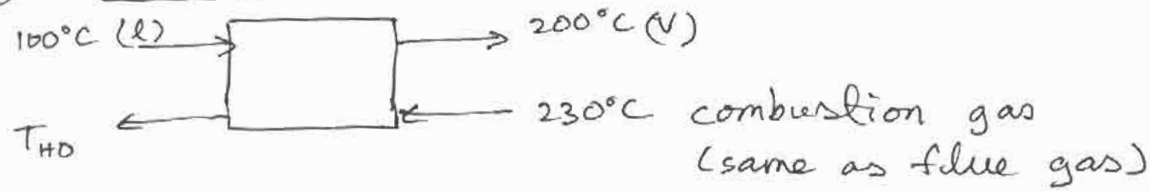
$A_2 = 155.4\ m^2$

$A = A_1 + A_2 = 55.4 + 155.4$

(2) $\boxed{A = 211\ m^2}$

(10) C. Parts C + D are almost exactly the same as A + B

(5) Counter:



T_{ci} + T_{co} are fixed

$\therefore$ for infinite area $T_{HO} = T_{ci} = 100°C$

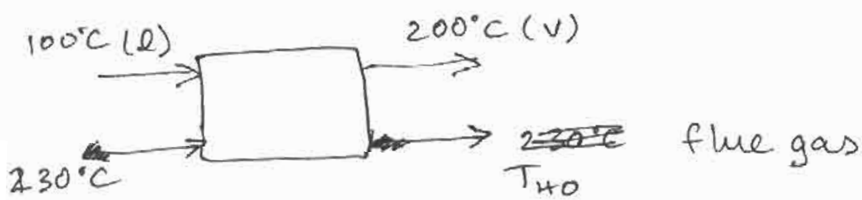
$q = 9 \times 10^6$ J/s (from part A)

$q = \dot{m}_H \, C_p \, \Delta T$

$9 \times 10^6 = \dot{m}_H (1000)(230 - 100)$

$\boxed{\dot{m}_H = 69.2 \text{ kg/s}}$

(5) Cocurrent:



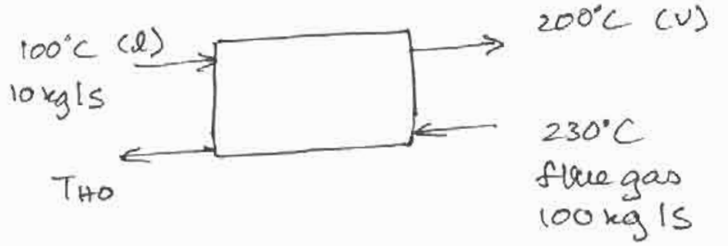
infinite area → $T_{HO} = 200°C$

$9 \times 10^6 = \dot{m}_H \, C_p \, \Delta T = \dot{m}_H (1000)(230 - 200)$

$\boxed{\dot{m}_H = 300 \text{ kg/s}}$

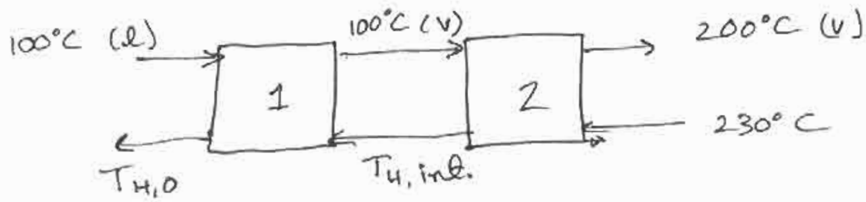
(20) D.

h_l, h_v same



① $h_F = 300 \ W/m^2K$

⑤ Break down into 2 exchangers



Determine temperatures:

Energy Balance

$$9 \times 10^6 = \dot{m}_H C_p \Delta T = 100 \tfrac{kg}{s}(1000)(230 - T_{HO})$$

②

$$T_{HO} = 140°C$$

$$q_2 = 3 \times 10^6 = \dot{m}_H C_p (T_{Hi} - T_{H,int})$$

$$= 100(1000)(230 - T_{H,int})$$

②

$$T_{H,int} = 200°C$$

① $$U_1 = \left[\frac{1}{h_l} + \frac{1}{h_F}\right]^{-1} = \left[\frac{1}{1000} + \frac{1}{300}\right]^{-1} = 231 \ W/m^2K$$

④ $$\Delta T_{lm,1} = \frac{(200-100) - (140-100)}{\ln \frac{200-100}{140-100}} = 65.5$$

$$A_1 = \frac{6 \times 10^6}{(231)(65.5} = 397 \ m^2$$

(2) $U_2 = \left[\frac{1}{h_V} + \frac{1}{h_F}\right]^{-1} = \left[\frac{1}{300} + \frac{1}{300}\right]^{-1} = 150 \ W/m^2K$

(4) $\Delta T_{lm} = \dfrac{30-100}{\ln \frac{30}{100}} = 58.1°C$

$A_2 = \dfrac{3 \times 10^6}{(150)(58.1)} = 344 \ m^2$

(2) $\boxed{A = 741 \ m^2}$

NTU Method for part D

(1) $C_{min} = C_H = (100)(1000) = 10^5$

$\cancel{NTU = \frac{UA}{C_{min}} = (230.8)}$

$q_{max} = C_{min}(T_{Hi} - T_{ci}) = 10^5(200-100) = 10^7$

(5) $\varepsilon = \dfrac{6 \times 10^6}{1 \times 10^7} = 0.6$

$NTU = -\ln(1-\varepsilon)$ for boiler

$= 0.916$

$A_1 = \dfrac{(NTU) C_{min}}{U} = \dfrac{0.916(10^5)}{231} = 397 \ m^2$

(2) $C_c = 10(3000) = 3 \times 10^4 = C_{min}$, $C_r = C_{min}/C_{max} = 0.3$

$q_{max} = 3 \times 10^4 (230-100) = 3.9 \times 10^6$

(5) $\varepsilon = \dfrac{3 \times 10^6}{3.9 \times 10^6} = 0.769$

$NTU = \dfrac{1}{C_r - 1} \ln \dfrac{\varepsilon - 1}{\varepsilon C_r - 1} = 1.72$

$A_2 = 344 \ m^2 \Rightarrow \boxed{A = 741 \ m^2}$ (2)

Prob 2

Assumptions

Constant Properties

Stagnant Fluid near bead

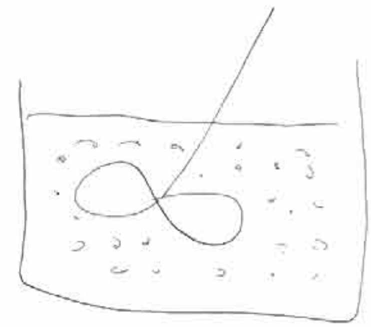
Well-mixed away from bead

$S_{b/w} = 100$

$D_b = 2\,mm = 0.2\,cm$

$\mathcal{D}_b = 10^{-7}\,cm^2/s$

$\mathcal{D}_w = 10^{-5}\,cm^2/s$



(A) 5pts

Mass Balance (2pts)

C_b - conc in bead

C_w = conc in water

C_i = initial conc

$$\underbrace{C_b V_b}_{\text{Bead}} + \underbrace{C_w V_w}_{\text{Water}} = \underbrace{C_i V_w}_{\text{Initial}}$$

At equilibrium (2pts)

$$C_b = S_{b/w} C_w$$

$$(S_{B/w} C_w) V_b + C_w V_w = C_i V_w$$

$$C_w = \frac{C_i V_w}{V_w + V_b S_{B/w}} = \frac{(1\ kg/m^3)(5m^3)}{5m^3 + 0.5m^3(100)}$$

$$C_b = \frac{S_{B/w}\, C_i V_w}{V_w + V_b S_{B/w}}$$

$C_w = 0.091\ kg/m^3$ (1pt) $C_b = 9.1\ kg/m^3$

(B) 10pts

$$Bi = \frac{h_m S_{ext/int} R}{\mathcal{D}_b} \quad (5pts)$$

for sphere in stagnant fluid

$$Sh = 2 = \frac{h_m D}{\mathcal{D}_w} \quad (7.59),\ Re = 0 \quad (3pts)$$

$$h_m = \frac{2\mathcal{D}_w}{D} = \frac{\cancel{2}\mathcal{D}_w}{\cancel{2}R} = \frac{\mathcal{D}_w}{R}$$

$$Bi = \frac{\left(\frac{\mathcal{D}_w}{R}\right) S_{ext/int} R}{\mathcal{D}_b} = \frac{\mathcal{D}_w}{\mathcal{D}_b \, S_{int/ext}} = \frac{\mathcal{D}_w}{\mathcal{D}_b \, S_{b/w}}$$

$$= \frac{10^{-5}\ cm^2/s}{10^{-7} cm^2/s\,(100)} = 1 \quad \text{(2pts)}$$

$Bi = 1$

$\therefore$ not lump capacitence

$\therefore$ diffusion into the bead is as important as diffusion to the bead

(c) from Fig D.9 (Could also use D.7)

10pts

$Bi = 1$

$F_o = 1$ $\quad Q/Q_o = .9$, less than this would not be using bead capacity well

$F_o = 2$ $\quad Q/Q_o = .99$, more than this would be taking excessive time

(Anywhere in this range was acceptable)

(I'll use $F_o = 1$)

$$F_o = 1 = \frac{\mathcal{D}_b t}{R^2}$$

$$t = \frac{R^2}{\mathcal{D}_b} = \frac{(0.1\ cm)^2}{10^{-7}\ cm^2/s} = 10^5\ s = 27.7\ hrs$$

* (5pts) for making a good judgement on how long
* (5pts) for using a Fourier analysis

(D) for $F_o = 1$, $Q/Q_o = 0.9$

10pts

for us $Q = m \cdot [Kg]$

$$Q_o = m_o = V_b C_{b,f} - V_b C_{b,i}$$

4pts

$$= (0.5 m^3)(9.1 \, Kg/m^3) - 0$$

$$m_o = 4.55 \, Kg$$ this is equilib. mass in beads

$$m = \left(\frac{m}{m_o}\right) m_o = (0.9)(4.55 \, Kg)$$ This is mass in the bead at $F_o = 1$

$$= 4.10 \, Kg$$

By mass balance, mass in water

$$C_w V_w = m_{w,i} - m_{b, F_o = 1}$$

3pts

$$= (1 \, Kg/m^3)(5 m^3) - 4.10 \, Kg$$

$$= 0.9 \, Kg$$

$$C_w = \frac{0.9 \, Kg}{5 m^3} = 0.18 \, Kg/m^3$$

This approximation is good to ~ +/- 50%

(E) The major assumption implicit in the F_o analysis (using Heisler charts, etc) is the external concentration is not constant. By mass balance the external conc drops 82%. This will diminish the driving force making our estimates inaccurate. (Particularly we won't acheive the final conc we expect.)