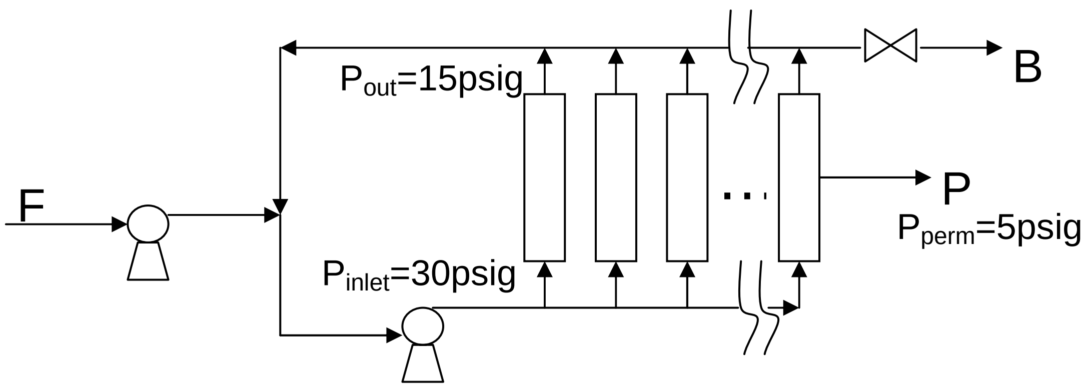
**10.32 Spring 2005**

## **Problem Set 4 Solution**

We revisit the ultrafiltration process from lecture that we used to produce a protein product from a waste whey stream from a cheese manufacturing process. We are given the feed rate of the whey stream, its market value, the rejection coefficient of the membranes in the cartridges, the flux equation, as well as the basic parameters for the plant configuration and operation.

(a) We are asked if it is possible to concentrate the protein solution to 80% and, if not, what the maximum protein concentration is.

To solve this problem, we consider the mass balance equations for the system illustrated in Figure 1:




As was derived in lecture, we can relate the concentration of species in the bleed stream, $C_{Bi}$ to the concentration in the feed stream, $C_{Fi}$, the membrane rejection coefficient for the species, $R_i$, and the feed to bleed ratio, f, by the following equation:

$$C_{Bi} = \frac{fC_{Fi}}{f + R_i(1-f)}.$$

We can solve the mass balance using the Solver in Excel. A spreadsheet can be set up to calculate $C_{Bi}$, the percentage of protein (both true protein and NPN) and the percentage of solids in the bleed stream in terms of the feed to bleed ratio, f, from the above equation. The cell defining the percentage of protein references the $C_{B,i}$ values as:

$$\%\,\text{Protein} = \frac{C_{B,TP} + C_{B,NPN}}{\Sigma C_{Bi}} \text{x}100$$

$$\%\text{Solids, total} = \sum C_{Bi} \text{x}100$$

The flux, N, is computed from the equation given in the problem statement:

$$N'' = 5.4 - 2.1 \ln C_c$$

where $C_c$ is the mass fraction of solutes in the concentrate and is equal to the total percentage of solids (divided by 100). We can then calculate the total area of mass transfer required from the following equation:

$$A_T = \frac{P_1\left[\frac{\text{lb}}{\text{day}}\right]\left[\frac{1\text{day}}{20\text{hr}}\right]}{N''}$$

where the day-to-hour conversion takes into account that the plant is operated only 20 hours per day. We can then calculate the total number of cartridges from the given parameter of 26.5ft$^2$ mass transfer area per cartridge:

$$N_{\text{cartidges}} = \frac{A_T}{\frac{26.5\text{ft}^2}{\text{cartridge}}}.$$

If we use Solver to maximize the value in the cell with the Total % Protein value by varying f, we find that **the maximum protein concentration is 59%. So, an 80% protein product is unattainable.** The following spreadsheet shows the results of these calculations.

| Solute | $C_i$ | R | $C_{B,i}$ | $C_{P,i}$ |
|---|---|---|---|---|
| TP | 0.6 | 0.96 | 13.0398 | 0.5216 |
| NPN | 0.3 | 0.40 | 0.4979 | 0.2988 |
| Lactose | 4.9 | 0.09 | 5.3813 | 4.8970 |
| Ash | 0.8 | 0.15 | 0.9401 | 0.7991 |
| Butterfat | 0.05 | 0.99 | 3.0862 | 0.0309 |

| Variables | Value | Units |
|---|---|---|
| F | 1000000.00 | lbs/day |
| B | 6263.68 | lbs/day |
| P | 993736.32 | lbs/day |
| f = F/B | 159.65 | |
| Total % Protein | 0.5900 | in bleed |
| Total % Solids | 0.2295 | in bleed |
| N | 8.4913 | lb/hr-ft$^2$ |
| Total Area | 5851.4844 | ft$^2$ |
| No. of cartridges | 221 | |

(b) The membrane solids concentration cannot exceed 15% or the membranes will foul and diafiltration can be used to eliminate this problem. We are asked if diafiltration is an economically-sound method of making a 50% protein product.

Before proceeding to analyze the mass balances for diafiltration, we can return to our original configuration from part (a) and solve for the feed-to-bleed ratio, f, by constraining the protein concentration to be 50%:

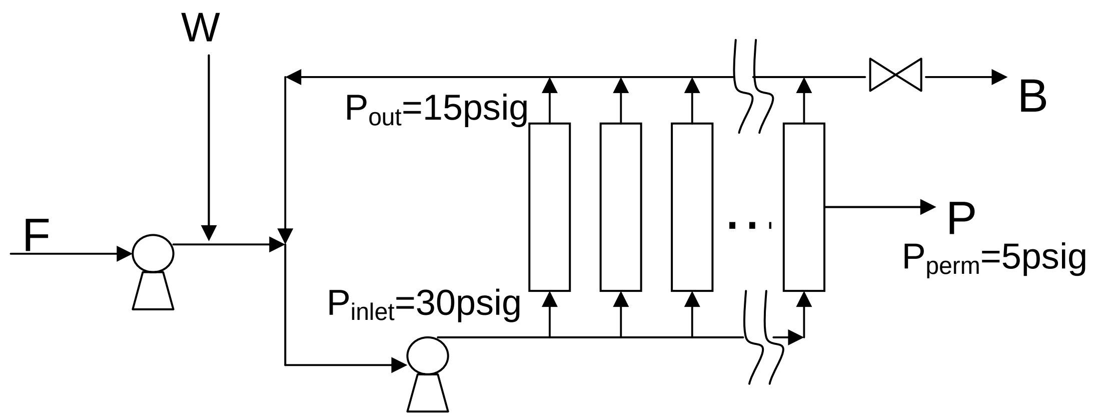
$$\frac{C_{B,TP} + C_{B,NPN}}{\Sigma C_{Bi}} = 0.50 .$$

We find that the bleed-to-feed ratio, f=18.9, and the solids concentration is 14.2%. **Therefore, we can conclude that diafiltration is not necessary.**
The following spreadsheet shows the results of these calculations:

| Solute | $C_i$ | R | $C_{B,i}$ | $C_{P,i}$ |
|---|---|---|---|---|
| TP | 0.6 | 0.96 | 6.6074 | 0.2643 |
| NPN | 0.3 | 0.40 | 0.4830 | 0.2898 |
| Lactose | 4.9 | 0.09 | 5.3566 | 4.8745 |
| Ash | 0.8 | 0.15 | 0.9325 | 0.7926 |
| Butterfat | 0.05 | 0.99 | 0.8013 | 0.0080 |

| Variables | Value | Units |
|---|---|---|
| F | 1000000.00 | lbs/day |
| B | 52924.06 | lbs/day |
| P | 947075.94 | lbs/day |
| f = F/B | 18.89 | |
| Total % Protein | 0.5000 | in bleed |
| Total % Solids | 0.1418 | in bleed |
| N" | 9.5019 | lb/hr-ft$^2$ |
| Total Area | 4983.6158 | ft$^2$ |
| No. of cartridges | 188.06 | |
| No. of stages | 8.00 | |

If we were to proceed to explore a diafiltration system, the mass balances change due to the presence of the water stream right before the feed enters the recycle loop. The following figure illustrates this configuration:



Similarly to part (a), we can write both total and component mass balances:

$$F + W = B + P$$

$$FC_{Fi} = BC_{Bi} + PC_{Pi}$$

Note that since the water stream contains no concentrate, it is not present in the species mass balance. We can now solve this algebraic equation for $C_{Bi}$:

$$C_{Bi} = \frac{C_{Fi}\left(\frac{F}{B}\right)}{1 + (1 - R_i)\left[\left(\frac{F}{B}\right) + \left(\frac{W}{F}\right)\left(\frac{F}{B}\right) - 1\right]}.$$

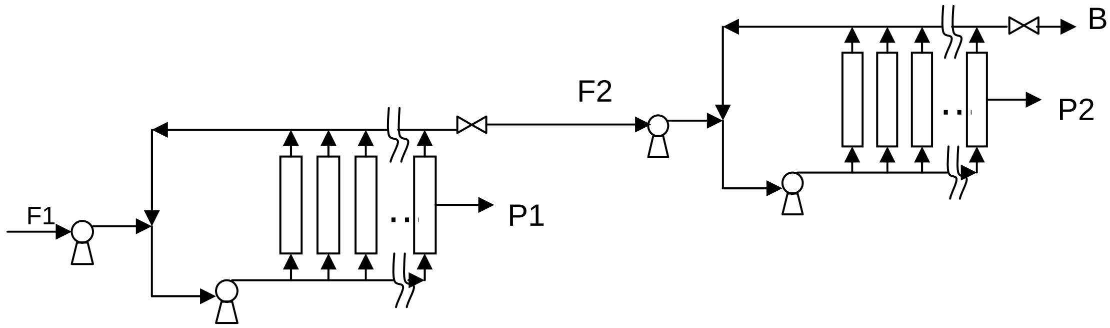
If we use a similar approach in Excel to part (a) and we generate a spreadsheet that calculates $C_{Bi}$ by varying W and B, we can optimize the water stream flow rate to achieve a certain protein concentration.

To assess the economics of the diafiltration versus the standard one-unit configuration, we need to consider the following additional costs:

1. DI water material costs for W stream
2. the costs for additional piping and fittings for this additional inlet stream
3. the cost of evaporating the excess water in the permeate and bleed streams

However, we would save money on cartridge replacement by avoiding plugging the membranes.

(c) Finally, we consider a 2-unit system that may save cartridges and money. Here the bleed stream from the first unit is the feed to the second unit and we are asked to evaluate the advantages and disadvantages of using a 2-unit system to make a 35% protein product.




If we write total and species mass balances around the first unit, we find that

$F_1 = F_2 + P_1$ and

$F_1 C_{F1,i} = F_2 C_{F2,i} + P_1 C_{P1,i}$.

Where $F_2 = B_1$

If we define $f_1$ as the ratio of the feed to the bleed from the first unit:

$$C_{F2,i} = \frac{f_1 C_{F1,i}}{1 + (f_1 - 1)(1 - R_i)}.$$

We can also write mass balances for the second unit:

$F_2 = B + P_2$

$F_2 C_{F2,i} = B C_{B,i} + P_2 C_{P2,i}$

and solving for $C_{Bi}$ we get:

$$C_{B,i} = \frac{f_2 C_{F2,i}}{1 + (f_2 - 1)(1 - R_i)}$$

where $f_2 = F_2/B$.

In Excel, we solve this system by varying both B and $F_2$ to maximize the percentage of protein, defined as

$$\%\,\mathrm{Protein} = \frac{C_{B,TP} + C_{B,NPN}}{\Sigma C_{Bi}}.$$

We can calculate the area of mass transfer required as well as the number of cartridges, similar to part (a). The simplest case is to assume that the areas of mass transfer are approximately equal for the 2 units. This is the most basic case and is adequate for this economic analysis.

As the following spreadsheet shows, the flow rate of the streams B and $F_2$ can be varied subject to the constraint that the mass transfer areas of each unit are equal and the final flow rates and compositions can be computed using Excel:

**Unit #1:**

| Solute | $C_i$ | R | $C_{B,i}$ | $C_{P,i}$ |
|---|---|---|---|---|
| TP | 0.6 | 0.96 | 1.0172 | 0.0407 |
| NPN | 0.3 | 0.40 | 0.3618 | 0.2171 |
| Lactose | 4.9 | 0.09 | 5.0959 | 4.6373 |
| Ash | 0.8 | 0.15 | 0.8548 | 0.7266 |
| Butterfat | 0.05 | 0.99 | 0.0866 | 0.0009 |

| Variables | Value | Units |
|---|---|---|
| $F_1$ | 1000000.00 | lbs/day |
| $B_1$ | 572786.59 | lbs/day |
| $P_1$ | 427213.41 | lbs/day |
| $f_1 = F_1/B_1$ | 1.75 | |
| Total % Protein | 0.1859 | in bleed |
| Total % Solids | 0.0742 | in bleed |
| N | 10.8631 | lb/hr-ft$^2$ |
| Total Area | 1966.3487 | ft$^2$ |
| No. of cartridges | 74 | |

**Unit #2:**

| Solute | $C_i$ | R | $C_{B,i}$ | $C_{P,i}$ |
|---|---|---|---|---|
| TP | 1.0172 | 0.96 | 3.0915 | 0.1237 |
| NPN | 0.3618 | 0.40 | 0.5022 | 0.3013 |
| Lactose | 5.0959 | 0.09 | 5.4380 | 4.9486 |
| Ash | 0.8548 | 0.15 | 0.9549 | 0.8117 |
| Butterfat | 0.0866 | 0.99 | 0.2813 | 0.0028 |

| Variables | Value | Units |
|---|---|---|
| $F_2$ | 572786.59 | lbs/day |
| $B_2$ | 172442.02 | lbs/day |
| $P_2$ | 400344.58 | lbs/day |
| $f_2 = F_2/B_2$ | 3.32 | |
| Total % Protein | 0.3500 | in bleed |
| Total % Solids | 0.1027 | in bleed |
| N | 10.1799 | lb/hr-ft$^2$ |
| Total Area | 1966.3487 | ft$^2$ |
| No. of cartridges | 74 | |

For part (c), we are also asked to compare the cost-effectiveness of the 2-unit case to the 1-unit system.

The following economic parameters were given in the problem:

| | | |
|---|---|---|
| **Drying System:** | | |
| Performance of evaporator | 2.5 | lb-water/lb-steam |
| Performance of spray dryer | 0.5 | lb-water/lb-steam |
| Cost of Steam | 5 | $/106 Btu |
| Heat of vaporization of water | 900 | Btu/lb |
| **Cartridge Characteristics:** | | |
| Total transfer area/cartridge | 26.5 | ft2 |
| Inlet Pressure | 30 | psig |
| Pressure drop on retentate side | 15 | psig |
| Pressure drop across membrane | 25 | psi |
| Flow rate of concentrate | 25 | gpm |
| **Cartridge Cost and Lifetime:** | | |
| Cost of cartridge | 225 | $ |
| Cartridge Lifetime | 1 | year |
| **Cost for Stages and Automation:** | | |
| Cost of a stage | 1250 | $/cartridge |
| Cost/stage for multiple stages | 10000 | $ |
| Cost to automate | 8000 | $/stage |
| Control panel | 50000 | $ |
| Cleaning system | 65000 | $ |
| **Other costs:** | | |
| Deionized water | 15 | $/1000 gal |
| Electricity | 0.09 | $/kw-hr |

Number of cartridges

Again, we can compute the number of cartridges from the equations described in part (a). To determine the total cost of the system, we must first determine the number of stages in our process. The maximum number of cartridges per stage was computed in class as 23.5 by taking the ratio of the maximum flow rate (based on the maximum velocity of 15ft/s) to the minimum cartridge flow rate. In our 2 stage operation, we have 74 cartridges per unit. So if we divide the total number of cartridges by this number, we see that 3.15 stages are required per unit. Since we must have a integral number of stages and all stages must have the same number of cartridges, we therefore select 4 stages X 19 cartridges.

Capital Costs

Now that we know the number of stages and cartridges, we can calculate our capital costs. The cartridge capital cost is simply

$$\mathrm{cost}_{\mathrm{cartridges}} = N_{\mathrm{cart}}\left[\left(\frac{\$225}{\mathrm{cartridge}}\right)+\left(\frac{\$1250}{\mathrm{cartridge}}\right)\right].$$

And the cost per stage for additional stages is

$$\mathrm{cost}_{\mathrm{Xtrastage}} = \left(\frac{\$10000}{\mathrm{stage}}\right)\left(N_{\mathrm{stage}}-1\right).$$

The cartridge replacement cost per day can be calculated based on the estimated 1-year life of a cartridge:

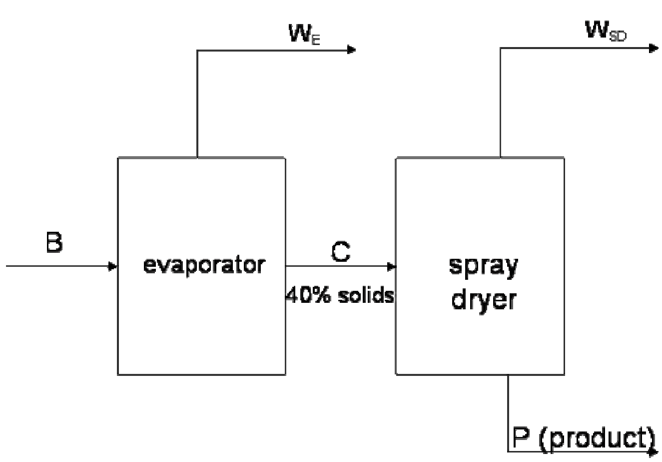
$$\mathrm{cost}_{\mathrm{cartridgereplace}} = \left(\frac{\$1250}{\mathrm{cartridge}\times 1\mathrm{year}}\right)\left(N_{\mathrm{cart}}\right)\left(\frac{1\mathrm{year}}{365\mathrm{days}}\right)$$

Then the remaining capital costs are the automation, control panel and cleaning costs:

$$\mathrm{cost}_{\mathrm{capital,add'l}} = \frac{\$8000}{\mathrm{stage}}N_{\mathrm{stages}} + \$50,000 + \$65,000$$

Operating costs

In order to sell our protein product, we must first evaporate the bleed stream. The standard drying configuration discussed in lecture has 2 units, an evaporator that concentrates the solids to 40wt% followed by a spray dryer that completely dries the system:



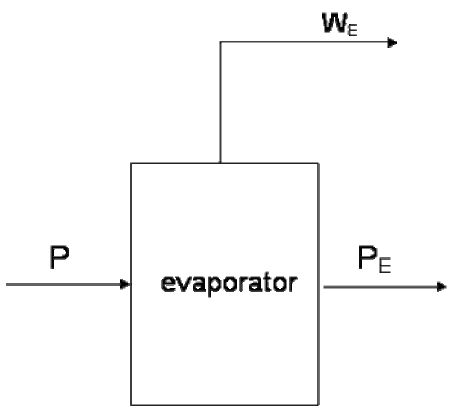

We can write a mass balance on the water in the bleed stream to determine the flow rate of the C stream:

$(1-C_C)B = W_E + C(0.6)$ and we find that

$$\frac{C_C B}{0.4} = C.$$

Then we can determine the flow rate of the steam streams from simple mass balances on the evaporator and spray dryer, respectively where $W_E$=B-C, P=(total amount of solids)(B) and $W_{SD}$=P-C.

The permeate stream must also be evaporated to 20% solids by a single evaporator unit before we can discard it:




$P_E$ is the flow rate of the permeate after evaporation. We can then calculate the cost of the steam by taking into account the steam efficiencies given in the problem:

$$\text{Steam cost}_{\text{Bleed}} = \left[(W_E)\left(\frac{\text{lbsteam}}{2.5\text{lbH}_2\text{Oevap}}\right) + (W_{SD})\left(\frac{\text{lbsteam}}{2\text{lbH}_2\text{Oevap}}\right)\right]\left(900\frac{\text{BTU}}{\text{lb}}\right)\left(\frac{\$5}{10^6\text{BTU}}\right)$$

Similarly, we can calculate the steam costs for evaporating the permeate streams:

$$\text{Steam cost}_{\text{Permeate}} = (W_E)\left(\frac{\text{lbsteam}}{2.5\text{lbH}_2\text{Oevap}}\right)\left(900\frac{\text{BTU}}{\text{lb}}\right)\left(\frac{\$5}{10^6\text{BTU}}\right).$$

It is important to note that in the 2-unit system, we have 2 permeate streams to evaporate.

The additional operating costs are for the electric power. From lecture, we learned that each stage requires approximately 3400W of power so that the total cost of electric power can be computed as follows:

$$\text{cost}_{\text{electric,pumping}} = \left(\frac{3400\text{W}}{\text{stage}}\right)(N_{\text{stages}})\left(\frac{\$0.09}{\text{kWhr}}\right)\left(\frac{1\text{W}}{10^3\text{kW}}\right)\left(\frac{20\text{hr}}{\text{day}}\right).$$

Finally, the profit can be computed from the final amount of dried protein product and the market cost:

$$\text{profit} = m_{\text{prod}}\left(\frac{\$0.65}{\text{lb}}\right)$$

where $m_{prod}$ is the mass of the product leaving the spray dryer.

To find the net profit, we subtract the operating and capital costs from the profit:

$$\text{profit}_{\text{net}} = \text{profit} - \cos t_{\text{capital,total}} - \cos t_{\text{operating,total}}$$

The following spreadsheet summarizes the economic analysis for the single and 2 unit systems:

| Single Unit | | | Two Units in Series | | |
|---|---|---|---|---|---|
| **Capital Cost:** | | | **Capital Cost:** | | |
| No. of cartridges/stage | 160 | | No. of cartridges | 152 | |
| No. of stages | 8 | | No. of stages | 8 | |
| Cost of stages | $70,000.00 | | Cost of stages | $70,000.00 | |
| Cost of cartridges | $236,000.00 | | Cost of cartridges | $224,200.00 | |
| Cost of automation | $64,000.00 | | Cost of automation | $64,000.00 | |
| Control Panel | $50,000.00 | | Control Panel | $50,000.00 | |
| Cleaning System | $65,000.00 | | Cleaning System | $65,000.00 | |
| Cartridge Replacement | $98.63 | /day | Cartridge Replacement | $93.70 | /day |
| **Total Capital** | **$231.51** | **/day** | **Total Capital** | **$223.34** | **/day** |
| **Operating Costs:** | | | **Operating Costs:** | | |
| **Electric Power** | **$48.96** | **/day** | **Electric Power** | **$48.96** | **/day** |
| **Cost of bleed Evaporation/Spray-drying process** | | | **Cost of bleed Evaporation/Spray-drying process** | | |
| Evaporator | $220.14 | /day | Evaporator | $230.72 | /day |
| Spray Dryer | $220.39 | /day | Spray Dryer | $239.03 | /day |
| Total | $440.54 | /day | Total | $469.75 | /day |
| Human Product | $10,611.56 | /day | Human Product | $11,509.08 | /day |
| **Profit** | **$10,171.02** | **/day** | **Profit** | **$11,039.33** | **/day** |
| **Cost of Permeate Evaporation:** | | | **Cost of Permeate Evaporation:** | | |
| **Evaporator** | **$1,054.82** | **/day** | **Evaporator** | **$1,050.46** | **/day** |
| **Net Profit** | **$8,835.73** | **/day** | **Net Profit** | **$9,716.57** | **/day** |

**Since the Total Profit for the 2-unit system is $880 more per day, we conclude that the 2-unit system is more profitable.**

**<u>Point Breakdown</u>**:

10 points total

**Part a 3 points**
**Part b 2 points**
**Part c 5 points (2 for mass balances, 2 for cost analysis, 1 for correct final answer that 2 units are cheaper if the areas are equal).**