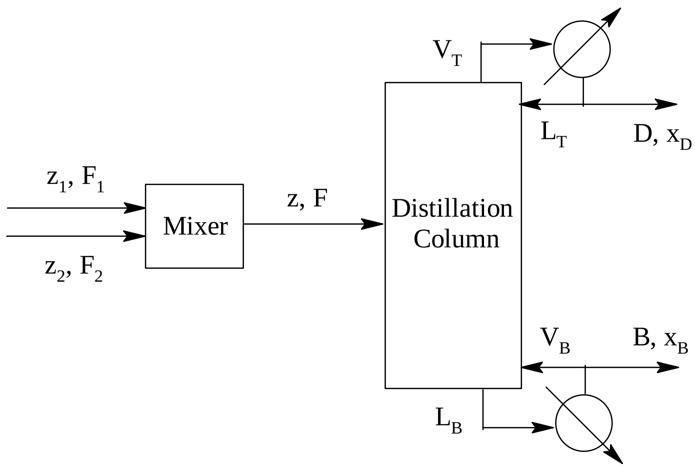
10.32 Answer to Problem Set #2, Problem 1.

Information from the problem statement:

- Two feed streams, each with a flow rate of 100 mol/hr
- F_1, is a saturated liquid feed stream containing 60 mole percent A
- F_2, is a saturated liquid feed stream containing 40 mole percent A
- Distillate should contain 99 mole percent of component A ($x_D = 0.99$)
- Bottom product should contain 1 mole percent A ($x_B = 0.01$)
- Relative volatility of A to B is 2.5 ($\alpha = 2.5$)

Case #1: The two streams, F_1 and F_2, are mixed together and introduced to the column at the optimal location. Assume that after mixing the feed to the column is a saturated liquid. The column has a partial reboiler and a total condenser.



Part A:

To begin, we must first perform material balances to determine the flow rate and composition of the feed stream feeding into the distillation column.

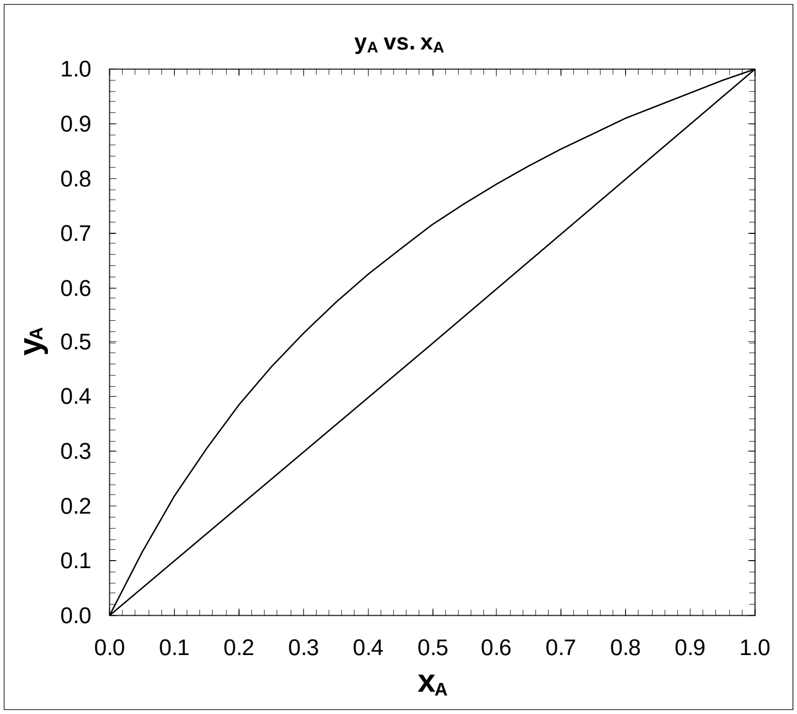
$$F_1 + F_2 = F$$
$$z_1 F_1 + z_2 F_2 = zF$$

From these material balances, we get F = 200 mol/hr and $z_A = 0.5$.

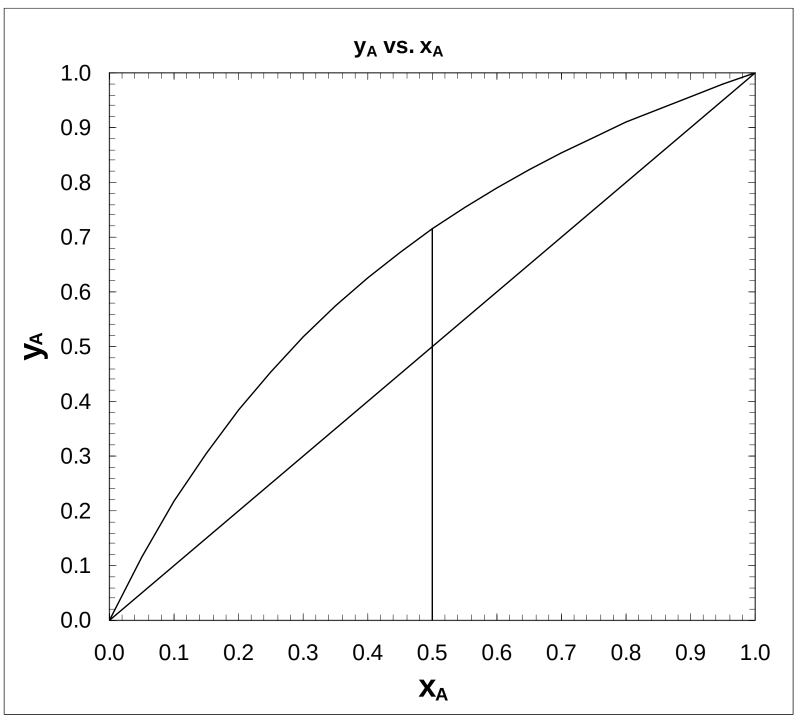
In order to find the minimum number of stages and the minimum reflux ratio, we use the McCabe-Thiele diagram. Using the relative volatility, we can construct the phase equilibrium line (PEQ) by using the following equation and choosing values for x_A between 0 and 1 and solving for y_A.

$$y_A = \frac{\alpha x_A}{1 + (\alpha - 1) x_A}$$

See next page for plot of PEQ with y = x line.



With the PEQ line drawn, we can now add in our feed line. Since we are told that the feed after mixing is a saturated liquid solution we know that the feed line on the graph will be a vertical line. From calculations, we found the composition of the feed stream to be $z_A = 0.5$, so we start from point (0.00, 0.50) and draw a vertical line up to the PEQ line.

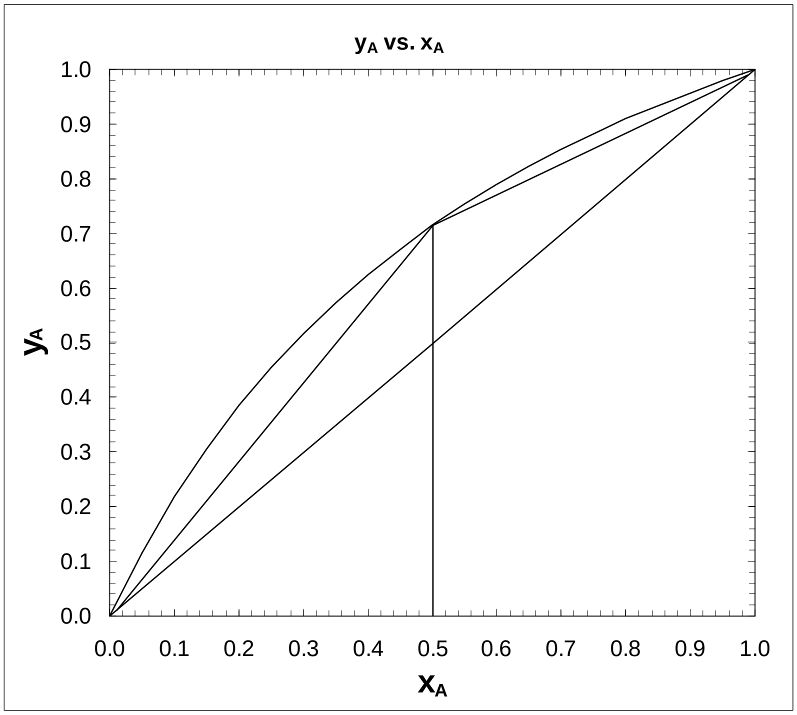


The next step is to determine where to draw the operating lines.

Let us first take the minimum reflux ratio case. In order to find the minimum reflux ratio, we consider the upper operating line.

$$\text{Slope of upper operating line} = \frac{R}{R+1}$$

From this relationship, we can see that as R decreases, the slope also decreases. Given that the distillate composition is $x_D = 0.99$, we start at the point (0.99, 0.99), draw a line until we intersect the feed line, and determine the slope. From various trials, you will see that **the minimum slope occurs when the upper operating line intersects the feed line at the PEQ line because this creates a pinch point**. Since both the upper and lower operating lines must intersect the feed line at the same spot, we can now also draw in the lower operating line.



By calculating the slope of the upper operating line, we can find R_{min}, the minimum reflux ratio.

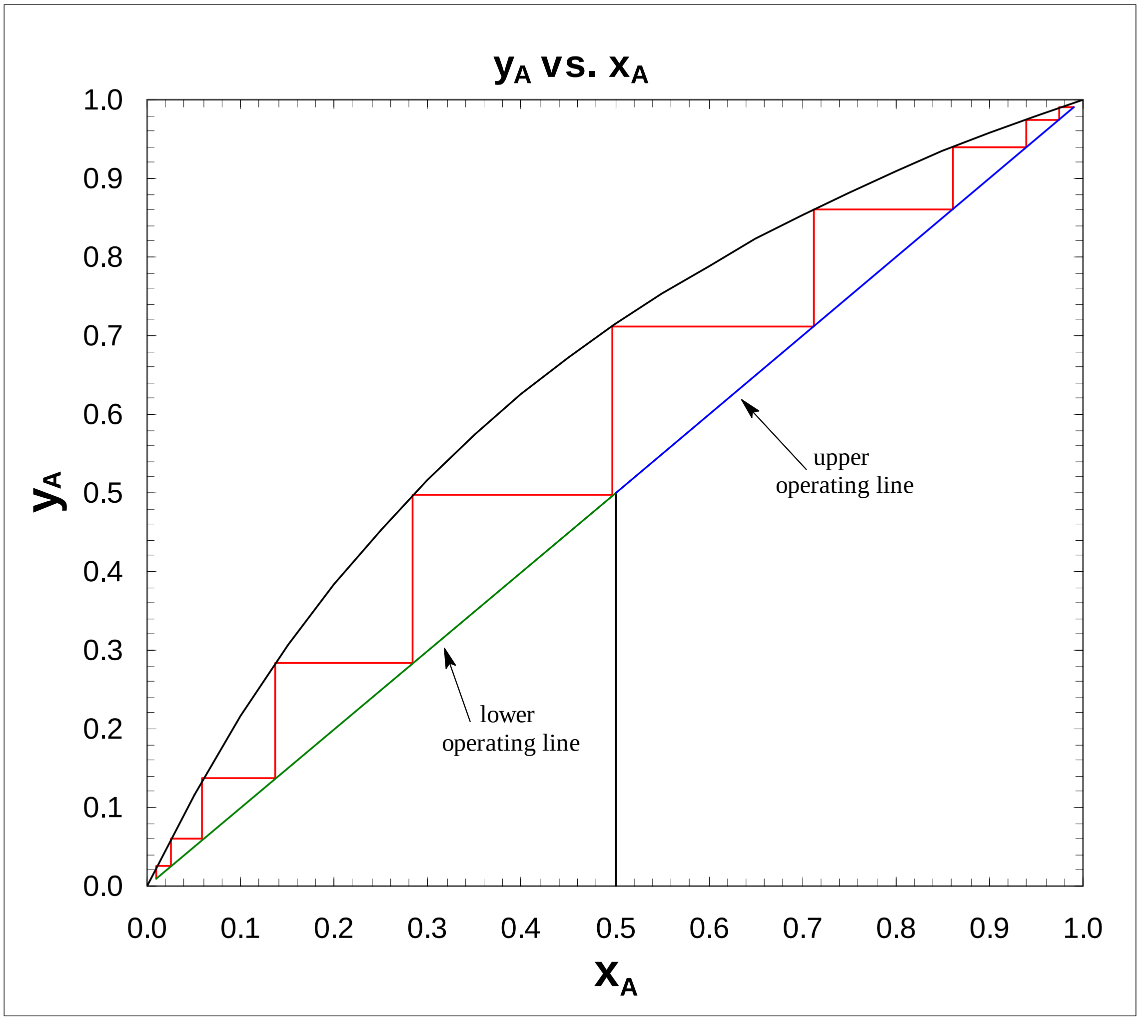
Slope = 0.57, R_{min} = 1.33

For the minimum number of stages case, we want to examine the relationship between the reflux ratio, R, and the number of stages needed to achieve the desired separation. At the minimum reflux ratio, we would need an infinite number of stages due to the pinch point created at $x_A = 0.5$ as seen in the figure above. As we increase R, we decrease the number of stages. Therefore, in order to find the minimum number of stages for this process, we use the above relationship for the slope of the upper operating line as a function of R and find the slope of the operating line as R approaches infinity.

$$\lim_{R\to\infty} \frac{R}{R+1} = 1$$

The slope of the upper operating line is 1 as R → ∞, which is the y = x line.

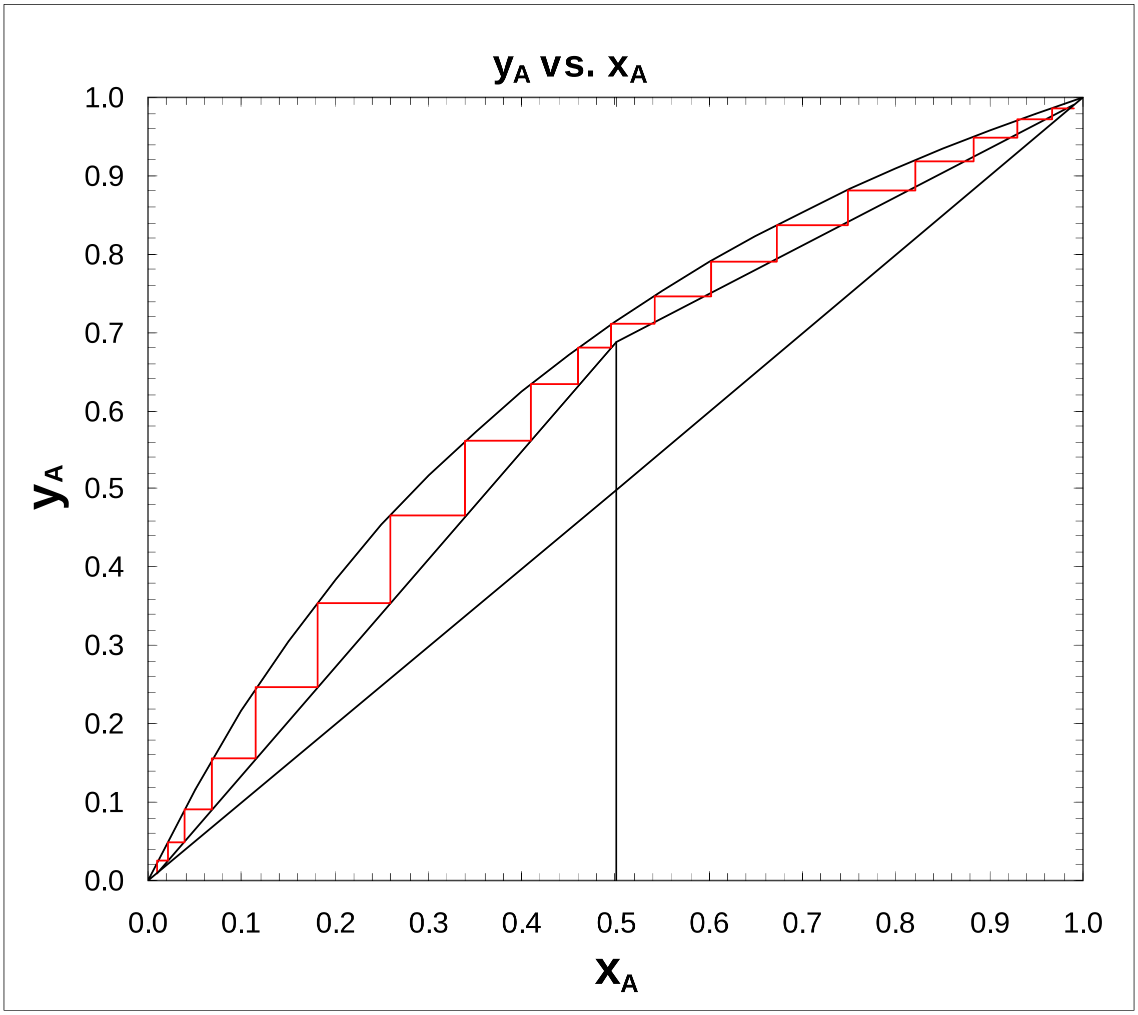
To find the minimum number of stages, we start at the bottoms composition, $x_B = 0.01$, which corresponds to the point (0.01, 0.01) and draw a vertical line up to the PEQ line. From there we draw a horizontal line to the lower operating line. We continue going first vertically to the PEQ line and then horizontally to the lower operating line until we reach the feed line. When the feed line is reached, we switch from drawing a horizontal line to the lower operating line to drawing a horizontal line to the upper operating line. We keep drawing in stages until the distillate composition is reached.



Each time the PEQ line is reached counts as one stage. Therefore, the minimum number of stages is 10 or 9 stages with the reboiler.

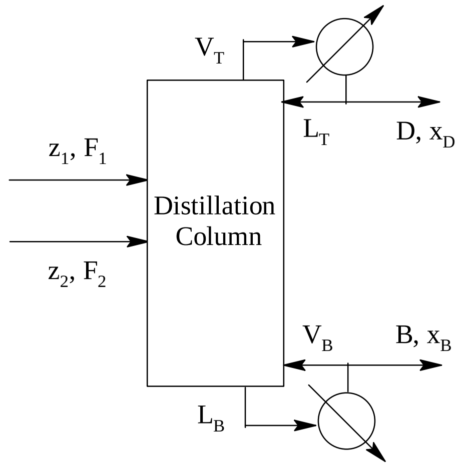
Part B:

We now want to run the column at a reflux ratio 1.2 times the minimum reflux ratio which is at **R = 1.60**. Knowing R, we can calculate the slope of the upper operating line. From the intersection of the upper operating line with the feed line, we can draw the lower operating line. Again we step vertically up to the PEQ line and horizontally to the operating lines to determine the number of stages in the column (See next page for McCabe-Thiele diagram and stages drawn in).

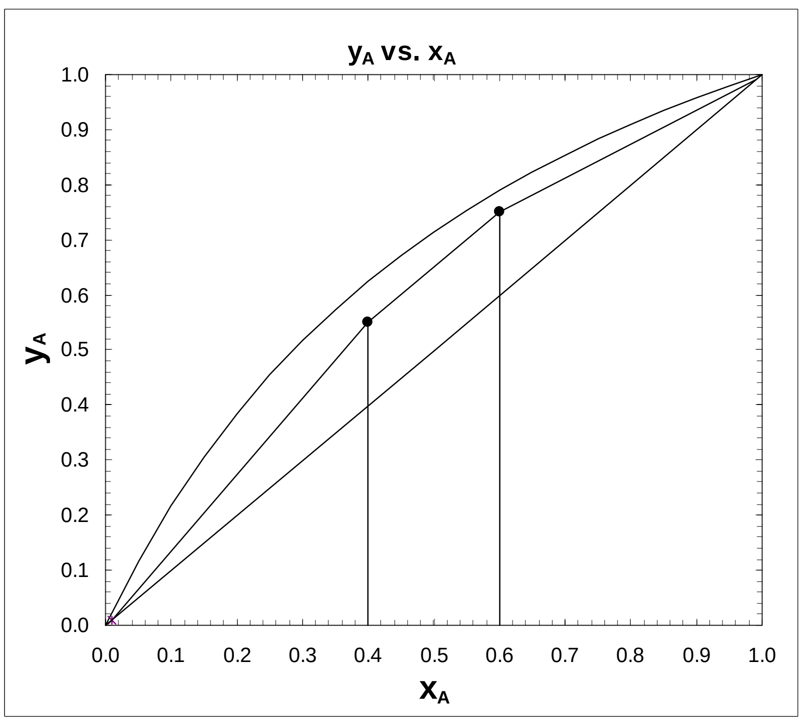


The total number of stages needed to reach the desired separation is 19 or 18 stages with reboiler. The feed should be added at the 10th stage above the reboiler.

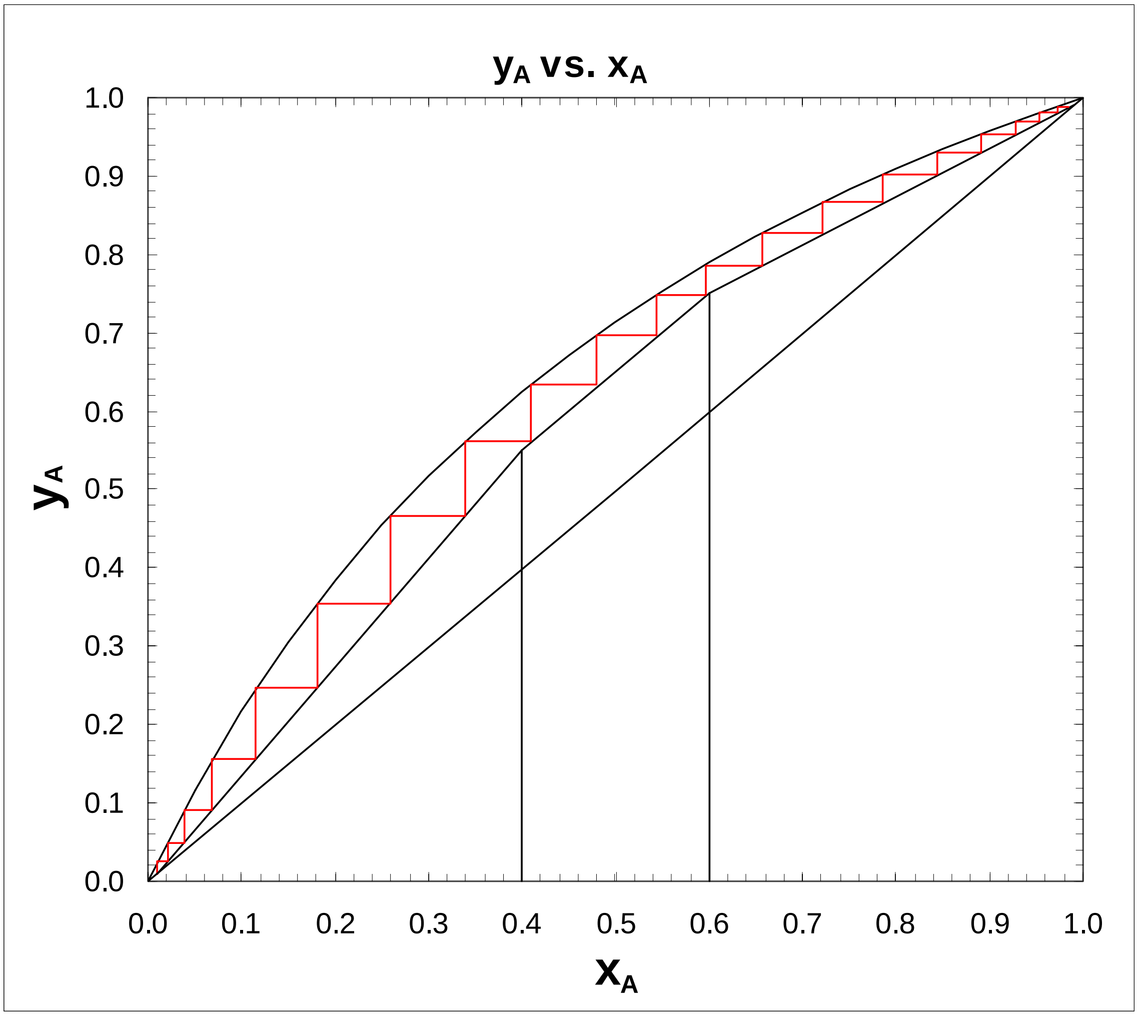
Case #2: The two streams are added to the column each at its optimal location.



In this case, we will need to adjust our McCabe-Thiele diagram so it includes two feed lines. Since both feeds are saturated liquids, we draw a vertical line at each corresponding composition starting from (0.40, 0.00) and (0.60, 0.00). In addition, since the reflux ratio is the same as in part B, the upper and lower operating lines have the same slopes as previously. In the case of two feeds though, we will need an additional operating line which corresponds to the region between the two feed lines. This line can be drawn by connecting the intersection of the upper operating line and the top feed stream with the intersection of the lower operating line and the bottom feed stream. The slope of this new operating line can be determined by taking the difference between the two intersection points.



To determine the number of stages, we use the same procedure as in parts A and B (See next page for diagram).



There are 20 stages in this column or 19 stages with the reboiler. F_2 should be added at the 7th stage above the reboiler and F_1 should be at the 11th stage above the reboiler.

10.32 Answer to Problem Set #2, Problem 2.

Given: VLE figure for ethanol and water at P=1atm
Problem statement has 96% recovery of ethanol in distillate.

Part A.

We are asked to find the maximum and minimum ethanol concentrations that can be obtained by continuous distillation of an ethanol and water mixture at P=1atm.

The presence of an azeotrope determines the maximum ethanol concentration.
From the graph, the equilibrium curve crosses the y=x line at approximately x_E=0.88 and, therefore, the **maximum ethanol concentration possible in the product stream is x_E=0.88.**
The minimum possible ethanol concentration in the product streams is approx. 0mol%. The slope of the equilibrium data approaches zero near x_E=0 and therefore complete separation of ethanol can be achieved.

Part B.

The liquid feed is 30mol% ethanol, the bottoms stream contains 2% ethanol and the distillate contains 96% of the ethanol fed to the column. We are asked to find the flow rates of the two product streams.

We perform an overall mass balance around the entire distillation unit:

$$F = D + B \qquad (1)$$
$$zF = x_D D + x_B B \qquad (2)$$

We know from the problem statement:

$$0.96zF = x_D D \qquad (3)$$

We are given the values z=0.30 and x_B=0.02 from the problem statement. Equations (1-3) can be solved in terms of F:

$$B = \frac{z(1-0.96)F}{x_B} = 0.6F \qquad (4)$$

$$D = F - B = 0.4F \qquad (5)$$

With a basis of F=100mol/h, ***B*=60 mol/hr and *D*=40 mol/hr.**

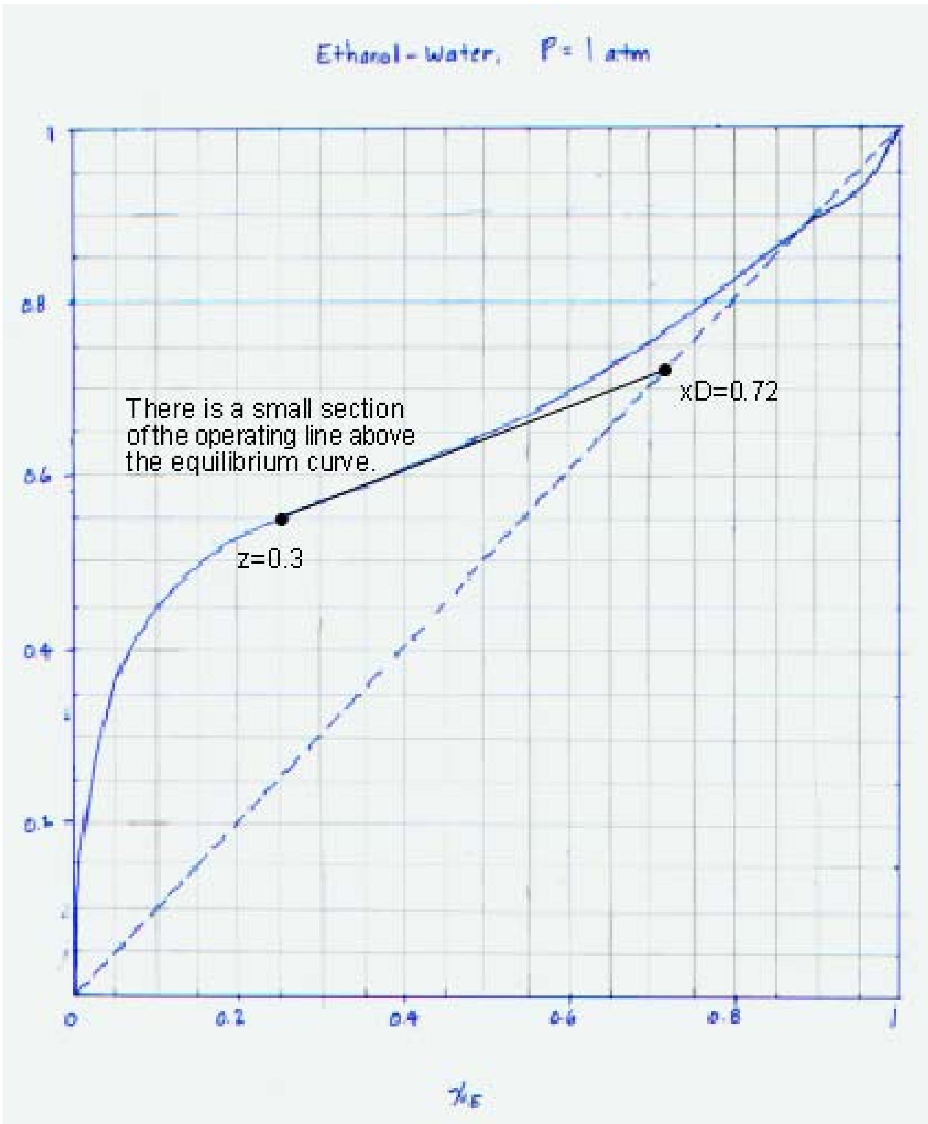
Part C.

We are asked to find the minimum reflux ratio, R_{min}, and the minimum number of stages, N_{min}, required to achieve this separation.

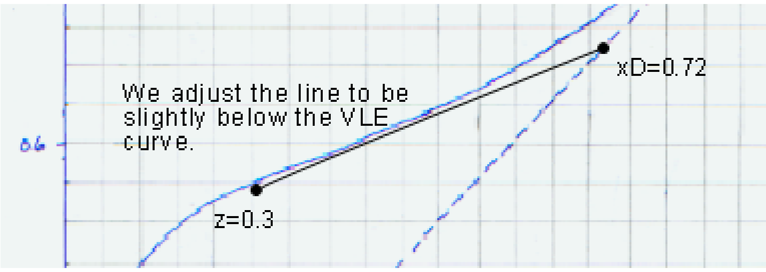
To find R_{min}, we first find the slope of the upper operating line given by the following equation:

$$Slope_{\min} = \frac{R_{\min}}{R_{\min} + 1} \qquad \text{(S\&H, 7-27)}$$

The upper operating line must intersect the VLE data at **x_D=0.72** (calculated from mass balance equation in part (b)) and *z*=0.3. We also know that this line must intersect the feed line at *z*=0.3. However, we see that such a line has a very small region above the VLE curve. Therefore we must move the operating line down slightly to be just below the data.

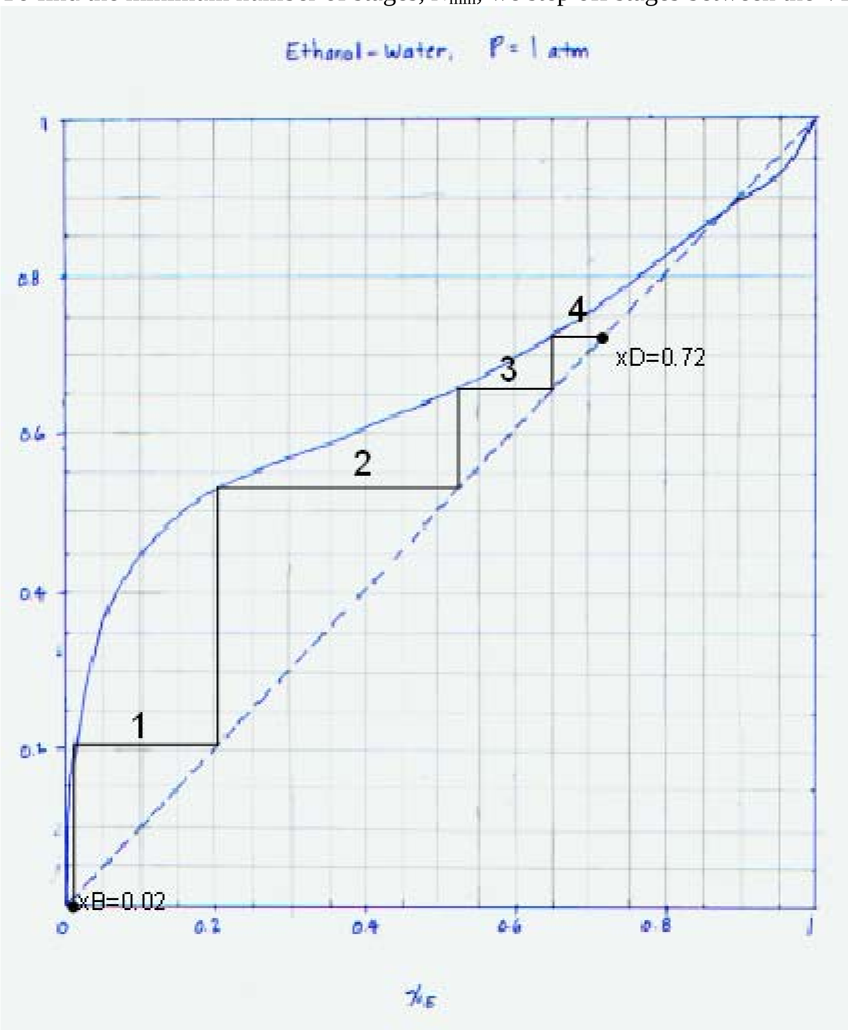


Therefore we must move the operating line down slightly to be just below the data:



The slope of the resulting line can be read from the graph and is ~ 0.35. **And $\underline{R_{min}}$ from equation 7-27 is 0.54 (approx.)**

To find the minimum number of stages, N_{min}, we step off stages between the VLE and operating line:



The minimum number of stages is $\underline{\boldsymbol{N_{min}=4}}$.

Part D.

We are asked to find the percent ethanol in the feed that we can recover if the composition of the distillate approaches the azeotrope. If the composition of the distillate approaches the azeotrope, x_D=0.88.

If P is the percent of ethanol in the distillate recovered from the feed, then P=0.96 from part (b), or

$$PzF = x_D D$$

Solving equations 3-5 in part (b) in terms of P, we obtain the following equation:

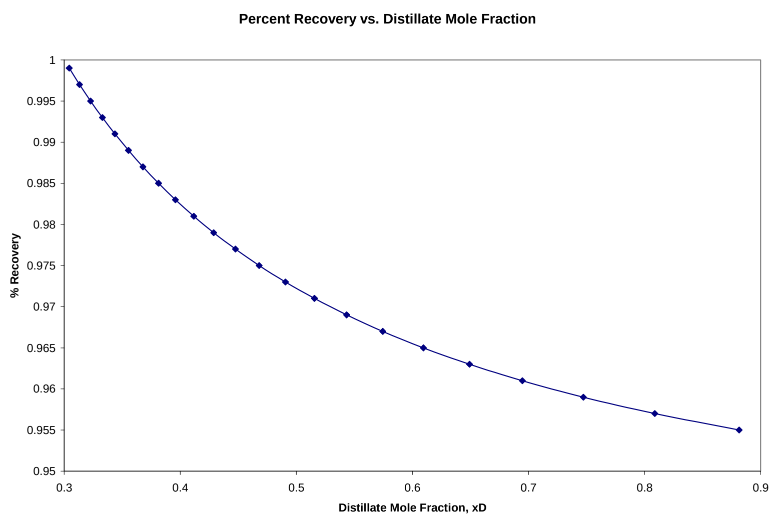
$$x_D = Pz\left(\frac{1}{1-\frac{z(1-P)}{x_B}}\right)$$

If z=0.3, x_B=0.02 and x_D=$x_{D,azeotrope}$=0.88, then ***P*=0.955.**

Finally, we are asked to comment on the sensitivity of the percentage of ethanol recovered to the mole fraction of ethanol in the distillate. By changing x_D=0.72 to x_D=0.88, P decreases from 0.96 to 0.955. Therefore the recovery decreases if we approach the azeotrope and have a more pure distillate product. If we continue to solve the column mass balances, we find that B=67.5 mol/hr and D=32.5 mol/hr and the amount of distillate decreases 19%.

We conclude that approaching the azeotrope does not increase our % recovery (an 18% increase in x_D corresponds to a less than 1% decrease in the overall recovery of ethanol). Therefore, the % recovery is not very sensitive to the distillate concentration. However, a small change in the concentration of the distillate makes a very large change in the overall mass balance and the *amount* of distillate that is recovered.

If we calculate P and x_D over a range of values (in graph below), we see that the sensitivity of the percentage of ethanol recovered to the distillate mole fraction decreases as we approach the azeotrope.



Point Allocation:
Part (a) 2 points (correct $x_{E,min}$, correct $x_{E,max}$)
Part (b) 4 points (correct D, B, and x_D)
Part (c) 4 points (correct R_{min} and $N_{stages,min}$)
Part (d) 2 points (correct % recovery at azeotrope, comment on sensitivity)