10.40 Thermodynamics **Fall 2003**
Problem Set 4

5.6 Text

For the purpose of this solution, we have numbered the equations appearing in the problem statement 1 (regarding the force on a particle) through 5 (regarding the momentum of a particle).

Solution:
(a)

The three functions F, L and –**H** all describe the same system and contain the same information. Thus, they can be related to each other through Legendre transformations. For this problem, we will obtain –**H** through a Legendre transform of L. The canonical coordinates of L, $\dot{x}$ and x, are given in equation (3) and the canonical coordinates of –**H**, p and x, are given in equation (4). It appears that –**H** is the first Legendre transform of L since one of their canonical coordinates is different while the other is the same. To determine the conjugate pairs in this system of Legendre transforms, we find the total differential of L in terms of x, $\dot{x}$, p and $\dot{p}$.

$$dL = \left(\frac{\partial L}{\partial \dot{x}}\right)_x d\dot{x} + \left(\frac{\partial L}{\partial x}\right)_{\dot{x}} dx \qquad (6)$$

Combining with equation (5) yields,

$$dL = p d\dot{x} + \left(\frac{\partial L}{\partial x}\right)_{\dot{x}} dx \qquad (7)$$

From equation (2),

$$\left(\frac{\partial L}{\partial x}\right)_{\dot{x}} = \left(\frac{\partial}{\partial t}\right)\left(\frac{\partial L}{\partial \dot{x}}\right)_x = \left(\frac{\partial p}{\partial t}\right) = \dot{p} \qquad (8)$$

Combining equations (7) and (8),

$$dL = p d\dot{x} + \dot{p} dx \qquad (9)$$

Equation (9) is the total derivative of L in terms of x, p, and their derivatives. By inspection of equation (9) we can determine the conjugate pairs for this system. The charts below show the conjugate pairs and the canonical variables for L and –**H**.

$$L \equiv y^{(0)}$$

	x_i	ξ_i
1	$\dot{x}$	p
2	x	$\dot{p}$

$$-\mathbf{H} \equiv y^{(1)}$$

	x_i	ξ_i
1	p	$\dot{x}$
2	x	$\dot{p}$

Applying equation (5-91) text gives,

$$\boxed{-\mathbf{H} = L - p\dot{x}} \tag{10}$$

(b)

The total differential form of **–H** can be obtained from equation (5-93) text.

$$d(-\mathbf{H}) = -\dot{x}dp + \dot{p}dx \tag{11}$$

From equation (11) the partial differentials can be evaluated.

$$\boxed{\left(\frac{\partial \mathbf{H}}{\partial p}\right)_x = \dot{x} \qquad \left(\frac{\partial \mathbf{H}}{\partial x}\right)_p = -\dot{p}} \tag{12}$$