10.40 Thermodynamics **Fall 2003**
Problem Set 1

Problem 3.9 Text

Solution:

(a) The pressurization of the gas volume in the storage tank from 1 atm (1.013 bar) to 150 psia (10.34 bar) with air may be modeled in several ways.

One model assumes that the operation is done rapidly with negligible heat transfer from the gas to the RP-4 or walls. Also, when pressurizing, the gas entering is well-mixed with the gas already present. In this case, Eq. (3-74) is applicable. $C_v = 20.93$ J/mol K, R = 8.314 J/mol K, $\kappa = 1.397$, $P_1 = 1.013$ bar, $P_2 = 10.34$ bar, T_{in} = 294 K (≈70°F), $T_i = T_1 = T_{in}$. Then, $T_2 = 395.3$ K. This final gas temperature is too low to lead to ignition at the air-RP-4 interface.

Another model might involve the assumption that the entering air does not mix with the air initially present but compresses it as though a piston divided the two air masses. Choosing the original air as the system, if this were compressed adiabatically from P_1 to P_2,

$$dU = C_v\,dT = -PdV$$

with the ideal gas law to eliminate the specific volume V,

$$(T_2/T_1) = (P_2/P_1)^{(\kappa - 1)/\kappa}$$

$$T_2 = 568.9 \text{ K}$$

In this model, the air at the interface is heated sufficiently high that ignition may be possible. Clearly, this model leads to an unsafe condition.

Other models may be selected and evaluated. The important point is that thermodynamics can only tell you what may happen after one specifies the fluid mechanics and heat transfer applicable to the problem. (It is also interesting to calculate the final air temperature of the pressurizing air in the second model.)

(b) The method of solution is identical with P_2 = 1000 psia = 69 bar.

T_2 (well-mixed) = 408.3 K

T_2 (lower layer is pressurized adiabatically) = 975.7 K

Clearly, in this case ignition is probable.

(c) During liquid transfer from the tank, the gas pressure remains constant at 10.34 bars as additional gas flows into the tank. The enthalpy of this gas is constant and equal to H_{in}. Choosing the gas in the tank as the system,

$$d\underline{U} = \delta Q + \delta W + H_{in}\,\delta n_{in} = NdU + UdN$$

where n_{in} represents the moles of gas into the tank and N is the moles in the tank at any time. Thus, $\delta n_{in} = dN$ and $\delta Q = 0$ (adiabatic):

$$-\delta W = Pd\underline{V} = PNdV + PVdN$$

combining, with the definition of enthalpy, $H \equiv U + PV$:

$$\frac{dH}{H_{in} - H} = \frac{dN}{N}$$

and integrating

$$\frac{H_{in} - H_2}{H_{in} - H_1} = \frac{N_1}{N_2} = \frac{(P\underline{V}_1/RT_1)}{(P\underline{V}_2/RT_2)} = \frac{\underline{V}_1}{\underline{V}_2}\frac{T_2}{T_1} = \frac{C_p(T_{in} - T_2)}{C_p(T_{in} - T_1)}$$

or,

$$T_2 = \frac{T_{in}}{\frac{\underline{V}_1}{\underline{V}_2}\left(\frac{T_{in}}{T_1} - 1\right) + 1}$$

where T_1 is the gas temperature initially when the volume is $\underline{V}_1$; T_{in} is the temperature of the pressurizing gas, and T_2 is the temperature when the volume is $\underline{V}_2$. $\underline{V}_1$ and $\underline{V}_2$ are related to time t (minutes) by

$$\underline{V}_2 = \underline{V}_1 + (t/18)(0.9\,\underline{V}_T - 0.1\,\underline{V}_T)$$

but, with $\underline{V}_1 = 0.1\,\underline{V}_T$

$$(\underline{V}_1/\underline{V}_2) = 1/[1 + (4/9)t]$$

Substituting in the expression for T_2, one obtains a relation between T_2, t, T_{in}, and T_{in} is 294.3 K and T_1 is obtained from (a), e.g., for the well-mixed case, T_1 = 395.8 K. Note that t is in minutes.