10.40 Thermodynamics **Fall 2003**
Problem Set 1

Problem 3.15 Text

Solution:
(a)
Balloon inflation is modeled by a chamber of air pushing against a spring.

The pressure in the gas space is given by

$$P - P_i = k(L - L_i)$$

where k = constant = 5 bar/m

Figure removed. Please see "Tester, J. W., and Michael Modell. *Thermodynamics and Its Applications*. Upper Saddle River, NJ: Prentice Hall PTR, 1997, p. 64. Fig. P3.15."

P_i = 1 bar; T_i = 300 K; L_i = 0.15 m; piston area, A = 0.02 m^2; C_v = 20.9 J/mol K, C_p = 29.2 J/mol K

What is the air temperature (T_f) when L = 0.6 m?

Subscript	System
i	initial air in cylinder
f	final air in cylinder
in	air entering cylinder

Assumptions:
Ideal gas
$C_p, C_v \neq f(T)$
Adiabatic process
Quasi-static process (air is well mixed)

Solution:
Choose air in cylinder as system. It is a simple system since it has no inertial or body forces, so $\underline{E} \rightarrow \underline{U}$ It is an open system due to the influx of air. The air in is from a constant pressure and temperature reservoir, so it has a constant enthalpy (H_{in}).

First Law for an Open System

$$d\underline{U} = \underbrace{\delta Q}_{=0} + \delta W + H_{in}\delta n_{in} - \underbrace{H_{out}\delta n_{out}}_{=0}, \quad \delta n_{in} = dN \qquad (1)$$

$\delta W = -Pd\underline{V}$ work done by the system on the environment

$$= -\left[P_i + k\left(L - L_i\right)\right] AdL$$

$$\int_{\underline{U}_i}^{\underline{U}_f} d\underline{U} = \int_{L_i}^{L_f} -\left[P_i + k\left(L - L_i\right)\right] AdL + \int_{N_i}^{N_f} H_{in} dN$$

$$\underline{U}_f - \underline{U}_i = -A\left[P_i\left(L_f - L_i\right) + \frac{k}{2}\left(L_f - L_i\right)^2\right] + H_{in}\left(N_f - N_i\right)$$

$$\underline{U} = NU$$

$$A\left[P_i\left(L_f - L_i\right) + \frac{k}{2}\left(L_f - L_i\right)^2\right] = N_f\left(H_{in} - U_f\right) - N_i\left(H_{in} - U_i\right) \quad (2)$$

From Table 3.2

$$H_{in} = \int_{T_0}^{T_{in}} C_p dT + H_0 = C_p\left(T_{in} - T_0\right) + H_0$$
$$U_f = \int_{T_0}^{T_f} C_v dT + U_0 = C_v\left(T_f - T_0\right) + U_0 \quad (3)$$

Inserting equation (3) into the RHS terms of equation (2) and simplifying,

$$N_f\left(H_{in} - U_f\right) = N_f\left(C_p T_{in} - C_v T_f\right) + N_f [\underbrace{\underbrace{\left(C_v - C_p\right)T_0}_{-RT_0} + \underbrace{H_0 - U_0}_{RT_0}}_{0}] \quad (4)$$

Similarly, $H_{in} - U_i = N_i\left(C_p T_{in} - C_v T_i\right)$

$$T_{in} = T_i$$

$$N_i\left(C_p T_{in} - C_v T_i\right) = N_i R T_i = P_i \underline{V}_i = P_i A L_i$$
$$N_f = P_f A L_f / R T_f \quad (5)$$

Inserting equations (4) and (5) into (2) and canceling terms,

$$\cancel{A}\left[P_i\left(L_f - \cancel{L_i}\right) + \frac{k}{2}\left(L_f - L_i\right)^2\right] = \frac{P_f \cancel{A} L_f}{R T_f}\left(C_p T_{in} - C_v T_f\right) - \cancel{P_i A L_i} \quad (6)$$

$$T_f = \frac{T_{in} P_f L_f \left(C_p \big/ R \right)}{P_i L_f + \frac{k}{2}\left(L_f - L_i\right)^2 + \frac{P_f L_f C_v}{R}}$$

plugging in $P_f = P_i + k\left(L_f - L_i\right) = 3.25$ bar (7)

yields $T_f = 345$ K