10.40 Thermodynamics **Fall 2003**
Problem Set 3

Problem 14.12 Text

Solution:
(a)

First, evaluate the maximum possible work that could be produced from a fully reversible process. Here we divide the Carnot heat transfer into three distinct parts:

(i) sensible heat from liquid phase from 1730°C to 1000°C (melting point)

(ii) latent heat during solidification at 1000°C (T_f)

(iii) sensible heat from solid phase from 1000°C to T_o

An infinite set of Carnot heat engines are available to carry out the 3-step process, each with an efficiency $(T - T_o)/T = \eta_{carnot} = \delta W / \delta Q_H$

$$W_{\max} = W_{\max,(i)} + W_{\max,(ii)} + W_{\max,(iii)} \tag{1}$$

(i) *liquid cooling*

$$W_{max,(i)} = \int_{T_i}^{T_f} \frac{T - T_o}{T} \delta Q_H \approx \int_{T_i}^{T_f} C_p \left[\frac{T - T_o}{T}\right] dT \text{ (per kg basis)} \tag{2}$$

assuming C_p is constant and that the heat exchange process is isobaric:

$$W_{\max,(i)} = C_{p,L}[T_f - T_i] - T_o C_{p,L} \ln \frac{T_f}{T_1}$$

$$W_{\max,(i)} = 1000[1000 - 1730] - (273)(1000) \ln\left(\frac{1273}{2003}\right) = -606{,}257 \text{ J/kg} \tag{3}$$

(ii) *solidification*

$$W_{max,(ii)} = -\left(\frac{T_f - T_o}{T_f}\right) \Delta H_{fusion} = \frac{-1000}{1273} (100{,}000)$$

$$W_{max,(ii)} = -78550 \text{ J/kg} \tag{4}$$

(iii) *solid cooling*

$$W_{max,(iii)} = \int_{T_f}^{T_o} C_{p,s}\left[\frac{T - T_o}{T}\right] dT = C_{p,s}[T_o - T_f] - T_o C_{p,s} \ln \frac{T_o}{T_f}$$

$$W_{max,(iii)} = 1000\left[1000 - 273 \ln\left(\frac{273}{1273}\right)\right] = -579{,}967 \text{ J/kg} \tag{5}$$

Therefore

$$W_{\max} = \sum_{i=1}^{3} W_{max,i} = -606{,}257 - 78{,}550 - 579{,}967$$

$$W_{\max} = -1.26 \times 10^6 \text{ J} \tag{6}$$

Therefore, CTI's claim that USER produces 0.34 kWh (1.2×10^6 J) doesn't violate the 2nd law limit.

Alternatively the availability change could have be used to evaluate $W_{\max}$. Again the 3-step process makes the calculation easier

$$W_{\max} = \Delta B \Big|_{T_i}^{T_o} = \Delta B_{(i)} + \Delta B_{(ii)} + \Delta B_{(iii)} \tag{7}$$

$$\Delta B_{(i)} = \Delta H - T_o \Delta S \Big|_{T_i}^{T_f} = C_p[T_f - T_i] - T_o C_p \ln \frac{T_f}{T_i}$$

The maximum work in part (ii) occurs when the phase transition happens reversibly. Thus,

$$\Delta \underline{S} = \int \left(\frac{\partial Q}{T}\right)_{rev}$$

From a first law balance, $\Delta H_f = \Delta Q \quad \Rightarrow \quad \Delta S_f = \Delta H_f / T_f$

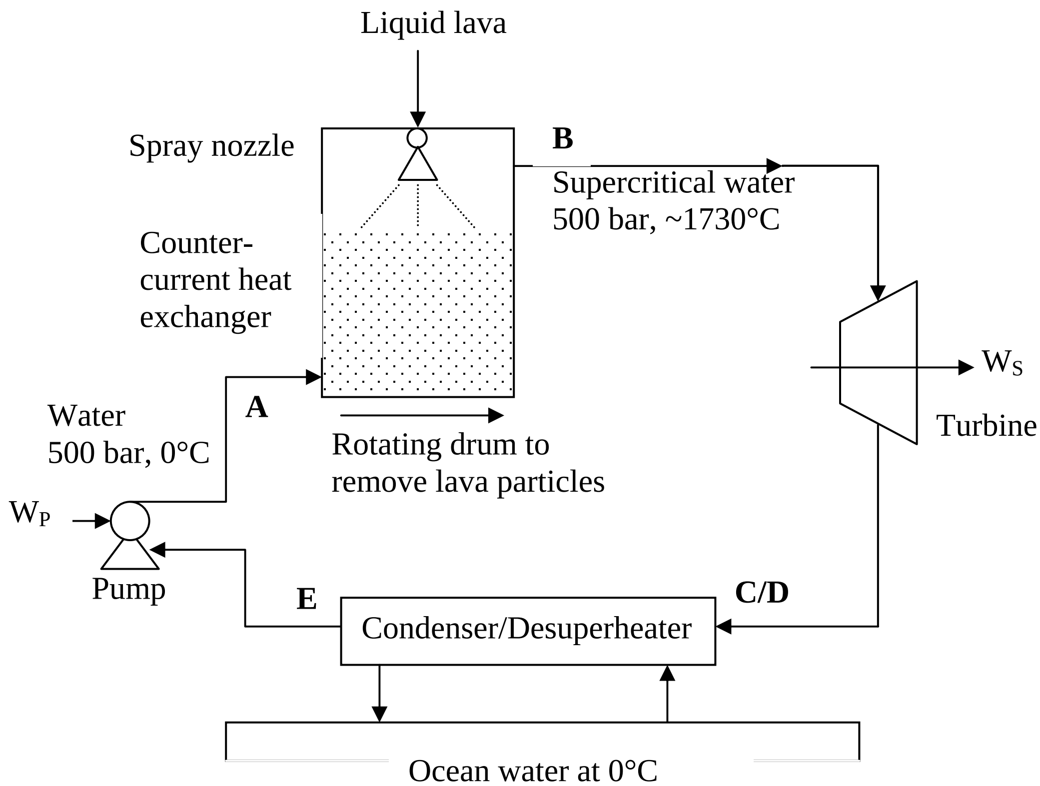
$$\Delta B_{(ii)} = \Delta H_f - T_o \Delta S_f = \Delta H_f - T_o \frac{\Delta H_f}{T_f}$$

$$\Delta B_{(iii)} = \Delta H - T_o \Delta S \Big|_{T_f}^{T_o} = C_p[T_o - T_f] - T_o C_p \ln \frac{T_o}{T_f}$$

$$W_{\max} = C_p[T_o - T_i] - T_o C_p \ln \frac{T_o}{T_i} + \Delta H_f[1 - T_o/T_f] \tag{8}$$

which will give the same result as Eq. (6).

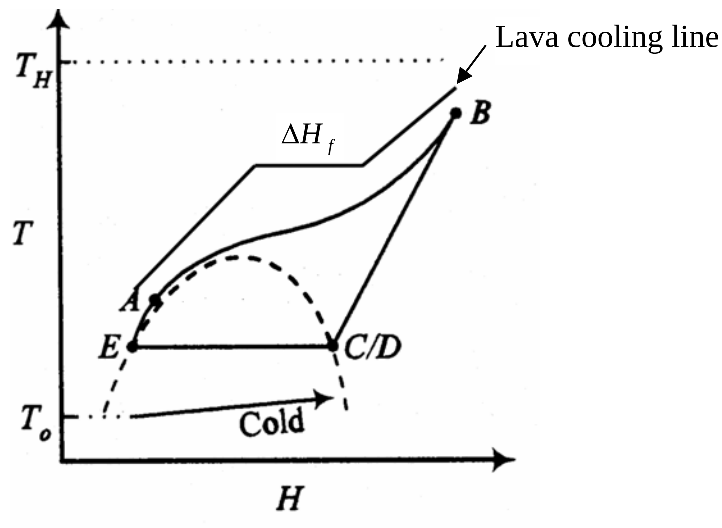
(b)



Liquid lava is sprayed into the counter-current heat exchanger and forms small liquid droplets. The droplets cool and solidify before hitting the bottom and being swept away by a rotating drum. An alternate heat exchange process could involve pumping the liquid lava until it solidifies. The hot, solid lava would then be crushed into a powder in order to continue the heat exchange process.

(c)

In order to examine the irreversibilities in the process, we can examine the enthalpy-temperature diagram for the process. It is similar to Figure 14.8 (b), a supercritical Rankine cycle.



The larger ΔT between the working fluids/solid is in the heat exchange process, the greater the exergy loss. One of the major irreversibilities in the design are in the lava/water heat exchanger, particularly when the lava is fusing. Other irreversibilities in the system are due to the frictional pressure drop in the spray nozzle and turbine. The pump irreversibilities are due to frictional losses and non-reversible cooling.

(d)

Examining figure 14.16, the cycle efficiency of the proposed Rankine cycle can be approximated by the "steam Rankine and gas turbine cycles" category since this category covers the appropriate temperature range and is a similar cycle. At 1730°C, the cycle efficiency is approximately 40%. To determine the net work produced by the proposed cycle, calculate the utilization efficiency from the cycle and Carnot efficiencies as follows.

$$\eta_u = \frac{\eta_{cycle}}{\eta_{Carnot}} = \frac{0.4}{\frac{T_H - T_C}{T_H}} = 0.46$$

$$\eta_u = \frac{\dot{W}_{net}}{\dot{n}\Delta B} = \frac{W_{net}}{W_{\max}}$$

$$W_{net} \approx -0.58 \text{ kJ}$$

The actual work produced (0.58 kJ) is well below CTI's claim of 1.2 MJ.