10.40 Thermodynamics **Fall 2003**

Problem Set 7

Problem 2

Using MatLab, Excel, or a similar software program, determine the value of *N* where the error in the Stirling approximation becomes less than 0.1%.

Solution:

We start by noting that :

$$N! = 1 \times 2 \times 3 \times ... \times (N-1) \times N \tag{1}$$

$$\ln(N!) = \ln(1) + \ln(2) + ... + \ln(N-1) + \ln(N) = \sum_{i=1}^{N} \ln(i) \tag{2}$$

Stirling's approximation is given by:

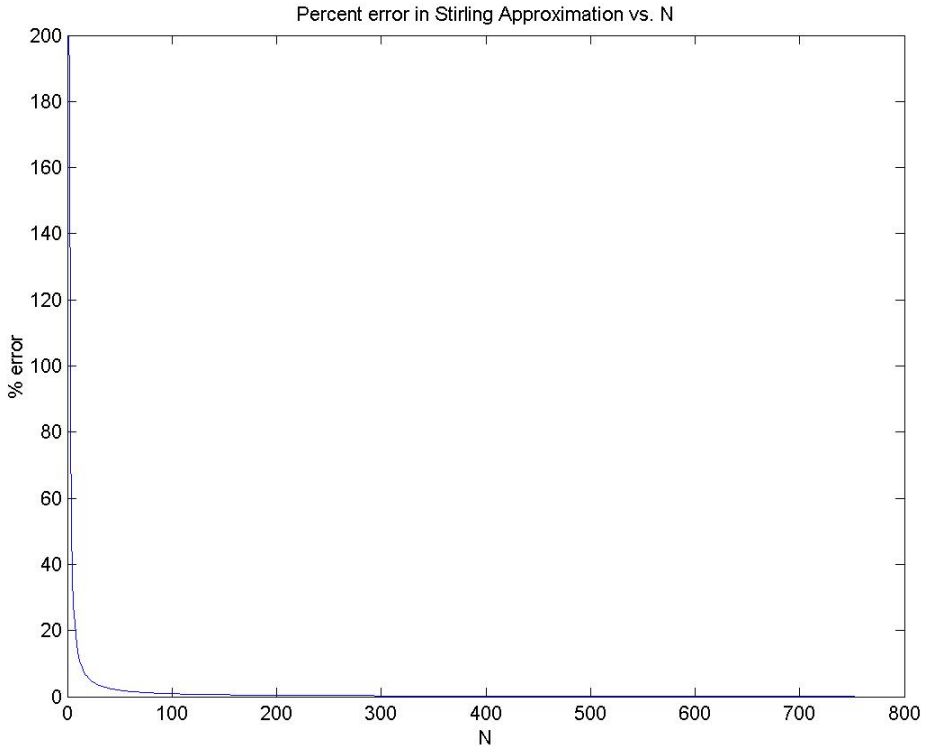
$$\ln(N!) = N \ln(N) - N \tag{3}$$

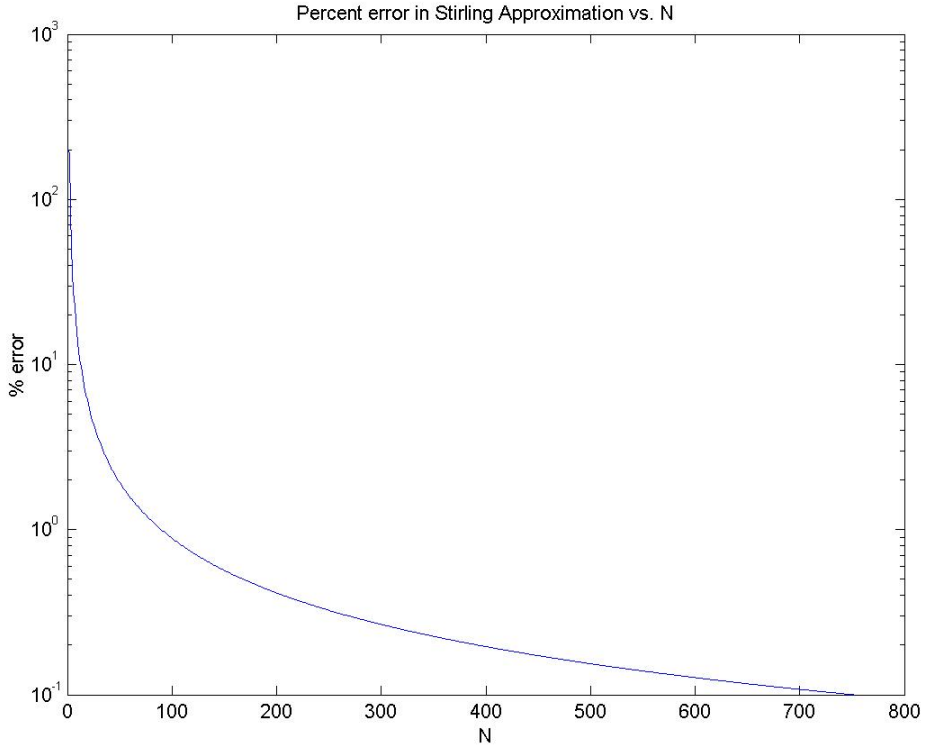
Therefore, we must write some sort of code or program that checks the percent difference between the actual value of *ln(N!)* and Stirling's approximation by iterating from N=1 upwards until the error is less than 0.1%. This is made very simple by that fact that:

$$\ln(N!) = \ln((N-1)!) + \ln(N) \tag{4}$$

Therefore, we can calculate *ln(N!)* by simply adding *ln(N)* to the answer from the previous iteration. Example code in Matlab is given below.

We find that the solution is $N = 752$. The two figures below illustrate how the %error varies as a function of *N*. The first shows how quickly the percent error falls as N increases. The second, which is a semi-log plot, shows more clearly where the %error = 0.1.


Percent error in Stirling Approximation vs. N
% error
N
0
20
40
60
80
100
120
140
160
180
200
0
100
200
300
400
500
600
700
800


Percent error in Stirling Approximation vs. N
% error
N
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
0
100
200
300
400
500
600
700
800

```
% Determine the value of N where the error in the Stirling approximation becomes less than 0.1%
% N! = 1 x 2 x 3 x ... x (N-1) x N
% ln(N!) = summation[ln(1) + ln(2) + ln(3) + ... + ln(N-1) + ln(N)]
% Stirling Approximation:  ln(N!) = N*ln(N) - N

% Declare variables.  Note that all variables set to values for N=1
N = 1;          % integer, any number
actual = 0;     % actual value of ln(N!)
stirling = -1;  % value of Stirling approximation of ln(N!)
error(1) = 200; % 1xN array of percent error between actual value of N! and Stirling approx
                % error(1) is technically infinite

% Caclculate error between N! and Stirling approx. until error < 0.1%
while (error(N) >= 0.1)
   N = N+1;
   actual = actual + log(N);
   stirling = N * log(N) - N;
   error(N) = ((actual - stirling) / actual) * 100;
end

% Plot the results
% Plot on regular axes
figure
plot(error)
title('Percent error in Stirling Approximation vs. N')
xlabel('N')
ylabel('% error')

% Plot on semilog axes
figure
semilogy(error)
title('Percent error in Stirling Approximation vs. N')
xlabel('N')
ylabel('% error')

disp(['The value of N where the error in the Stirling approximation becomes less than 0.1% is ' int2str(N)])
```