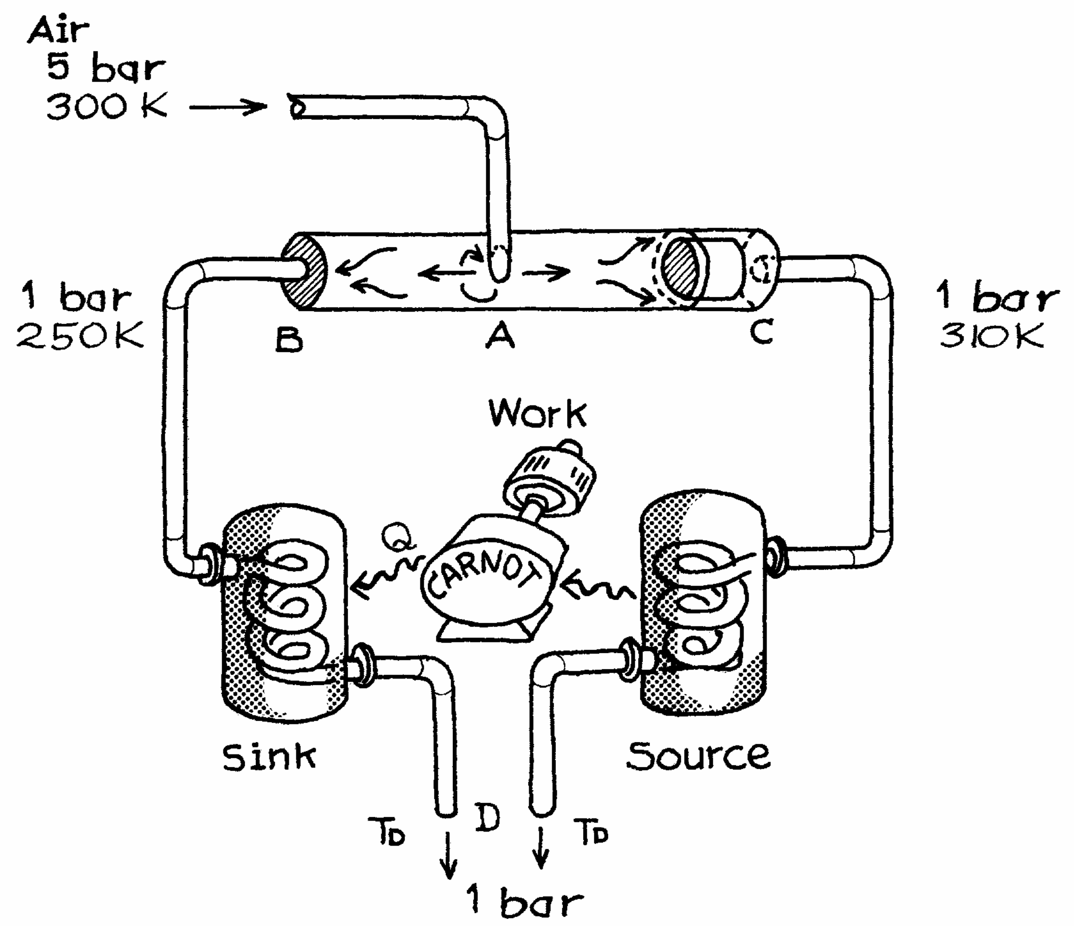
10.40 Thermodynamics **Fall 2003**
Problem Set 2

Problem 4.3 Text

A Hilsh vortex tube for sale commercially is fed with air at 300 K and 5 bar into a tangential slot near the center (Point A in figure). Stream B leaves from the left end at 1 bar and 250 K; stream C leaves at the right end at 1 bar and 310 K. These two streams then act as a sink and source for a Carnot engine and both streams leave the engine at 1 bar and T_D. Assume ideal gases that have a constant heat capacity C_p = 29.3 J/mol K.

(a) If stream A flows at 1 mol/s, what are the flow rates of streams B and C?
(b) What is T_D?
(c) What is the Carnot power output per mole of stream A?
(d) What is the entropy change of the overall process per mole of A?
(e) What is the entropy change in the Hilsh tube per mole of A?
(f) What is the maximum power that one could obtain by any process per mole of A if all heat were rejected or absorbed from an isothermal reservoir at T_D?



Solution:
(a) The flow rates for streams B and C can be calculated by performing a 1st Law Energy Balance, using the Hilsh vortex tube as the system.

System: Hilsh vortex tube
Open
Adiabatic
Rigid

$$d\underline{U} = 0 = \delta Q + \delta W + H_{in}\,\delta n_{in} - H_{out}\,\delta n_{out}$$
$$\delta Q = \delta W = 0$$
$$H_{in} = h_A$$
$$\delta n_{in} = dn_A$$
$$\delta n_{out} = dn_B + dn_C$$
$$H_{out}\,\delta n_{out} = h_B\,dn_B + h_C\,dn_C$$

Therefore, $h_A\,d\dot{n}_A = h_B\,d\dot{n}_B + h_C\,d\dot{n}_C$

and, $\dot{n}_B + \dot{n}_C = 1$ mol/s

with $h_A = C_pT_A + h^o,\ h_B = C_pT_B + h^o,\ h_c = C_pT_C + h^o,$ the 1st Law expression reduces to, $(1)(300) = \dot{n}_B(250) + (1-\dot{n}_B)310$

$$\dot{n}_B = 0.166 \text{ mol/s}$$
$$\dot{n}_C = 0.833 \text{ mol/s}$$

(b) Part (b) is very similar to Example 4.5. The major difference is that whereas in Example 4.5 we are given that the cold reservoir temperature is constant and asked to find the max. work, in this problem both the hot and cold reservoir temperatures change in the process and we are asked to find the final temperature of the two streams, T_D.

There are several ways to start this problem. Following Example 4.5, imagine that streams B and C are connected by an infinite number of Carnot engines. Picking a single, i^{th} engine as our system, we can start with a set of equations relating efficiency, work, heat interactions, and temperature:

$$\eta_i = \frac{-\delta W_i}{\delta Q_{Ci}} = \frac{\delta Q_{Ci} + \delta Q_{Bi}}{\delta Q_{Ci}} = \frac{T_{Ci} - T_{Bi}}{T_{Ci}} \qquad (1)$$

It should be noted that δQ_{Bi} is assumed to be negative in the above derivation. We also could have realized that since we are working with a reversible Carnot engine, the entropy changes in the hot and cold stream sum to zero and we can use the Clausius expression (Eq. 4-26 in the book). Either way, we eventually end up with:

$$\frac{\delta Q_{Ci}}{T_{Ci}} + \frac{\delta Q_{Bi}}{T_{Bi}} = 0 \tag{2}$$

Performing a 1st Law Balance on the differential part of the hot stream, stream C, that interacts with this ith Carnot engine shows that:

$$\underbrace{\cancel{d\underline{U}_i}}_{=0,\ SS} = \delta Q_{Ci} + \underbrace{\cancel{\delta W_{Ci}}}_{=0} + (\underbrace{H_{in} - H_{out}}_{=dH,\ \text{only differential change}})\delta n_C \tag{3}$$

Dividing by dt, and substituting for dH:

$$\dot{Q}_{Ci} = -\dot{n}_C C_p dT_C \tag{4}$$

Likewise, a material balance on cold stream, stream B, gives:

$$\dot{Q}_{Bi} = -\dot{n}_B C_p dT_B$$

Notice that our previous assumption that δQ_{Bi} is negative with respect to our Carnot Engine system was correct. Substituting these into equation (2) gives:

$$\frac{\dot{n}_C C_p dT_C}{T_{Ci}} + \frac{\dot{n}_B C_p dT_B}{T_{Bi}} = 0 \tag{5}$$

Integrating over the infinite number of Carnot engines from the inlet conditions to the exit, where both streams are at the same temperature T_D, and plugging in values gives:

$$\dot{n}_C \int_{T_C}^{T_D} \frac{dT}{T} + \dot{n}_B \int_{T_B}^{T_D} \frac{dT}{T} = 0 \tag{6}$$

$$T_D = 299.1 \text{ K}$$

(c) Performing a 1st Law Energy balance over the entire Hilsh tube and Carnot Engine Process gives:

$$d\underline{U} = 0 = \delta Q + \delta W + h_A dn_A - h_D dn_D$$

$$\delta Q = 0$$

$$dn_D = dn_A$$

$$\delta \dot{W} = -d\dot{n}_A(h_A - h_D) = -d\dot{n}_A C_p(T_A - T_D)$$

$$\dot{W} = -(1)(29.3)(300 - 299.1) = -26.4 \text{ J/s}$$

(d) To determine the entropy change for the overall process, we considered a closed system consisting of one mole of gas (considering an open system for our process leads to a dead end). Since entropy is a state function, we can consider only the end states of the gas, and imagine that there is some reversible process that connects these two end states. Applying the combined 1st and 2nd Law (Eq. 4-34 in the book):

$$d\underline{U} = Td\underline{S} - Pd\underline{V} \tag{7}$$

$$d\underline{S} = \frac{d\underline{U}}{T} + \frac{P}{T} d\underline{V} \quad (8)$$

Noting that the number of moles is constant (N=1 mole) and that:

$$d\underline{V} = d\left(\frac{NRT}{P}\right) = NR\left(\frac{1}{P}dT - \frac{T}{P^2}dP\right) \quad (9)$$

We now have:

$$d\underline{S} = N\left(\underbrace{\frac{C_V}{T}dT + \frac{R}{T}dT}_{C_V+R=C_P} - \frac{R}{P}dP\right) \quad (10)$$

$$d\underline{S} = N\left(\frac{C_P}{T}dT - \frac{R}{P}dP\right) \quad (11)$$

Integrating between our initial Stream A conditions and our final Stream D conditions gives:

$$\Delta\underline{S} = N\left(C_P \, ln(\frac{T_D}{T_A}) - R \, ln(\frac{P_D}{P_A})\right) \quad (12)$$

Substituting values, we find that $\Delta\underline{S}$ = 13.3 J/K for one mole of stream A.

(e) The entropy change in the Hilsh tube is the same as (d) since the Carnot step is reversible.

(f) The steady state maximum work for a flow entering at A and leaving at D with an environment at D, steady state is obtained using an energy and entropy balance:

$$d\underline{U} = 0 = \delta Q_{rev} + \delta W_{\max} + H_{in}\,\delta n_{in} - H_{out}\,\delta n_{out}$$
$$d\underline{S} = 0 = \delta Q_{rev} / T_D + S_{in}\,\delta n_{in} - S_{out}\,\delta n_{out}$$

Combining,

$$-\delta W_{\max} = (H_{in} - T_D S_{in})\delta n_{in} - (H_{out} - T_D S_{out})\delta n_{out}$$
$$\dot{W}_{\max} = -\dot{n}[(H_{in} - T_D S_{in}) - (H_{out} - T_D S_{out})]$$
$$= -\dot{n}[(H_{in} - H_{out}) - (T_D(S_{in} - S_{out}))]$$
$$= -\dot{n}[C_p(T_A - T_D) - T_D(S_A - S_D)]$$
$$= -(1)[(29.3)(300 - 299.1) - (299.1)(-13.3)]$$
$$\text{or } \dot{W}_{\max} = -4000 \text{ J/s}$$