

# SOLID-LIQUID SEPARATION: FILTRATION

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**Downstream**  
**Processing Course**

# FILTRATION THEORY

Poiseuille's Law

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu(R_m + R_c)}$$

Filtration Rate

Broth  
Viscosity

Resistance to Flow

Cake Resistance

$$R_m = \frac{\alpha W}{A}$$

Specific Cake Resistance

$$\alpha = \alpha' \Delta P^S$$

V=filtrate volume

A=Filter area

t=Time

$\Delta P$ =Pressure Driving Force

$\mu$ =Broth viscosity

W=Mass of filter cake

R=Resistance

$\alpha$ =Specific cake resistance

S=Compressability factor

The filter resistance is much less than the cake resistance

$$R_c \ll R_m$$

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu \left( \frac{\alpha' \Delta P^S W}{A} \right)}$$

When the filter cake is incompressible,  $S=0$

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu \left( \frac{\alpha' W}{A} \right)}$$

When the filter cake is very compressible,  $S=1.0$

$$\frac{1}{A} \frac{dV}{dt} = \frac{1}{\mu \left( \frac{\alpha' W}{A} \right)}$$

# Flocculation of Cells

## Sedimentation Rate

$$V_s = \frac{d_p^2 (\rho_p - \rho_s)}{18\mu} g_c$$

## Filtration Rate

$$\frac{dV}{dt} = \frac{k\Delta P}{VS_0^2}$$

$$S_0 = K'(1/D_p)^2$$

$$S_0^2 = K''/D_p^4$$

$$\frac{dV}{dt} = \frac{k_0 \Delta P D_p^4}{V}$$

# Mechanism of Flocculation

- Particle bridging
- Charge neutralization
- Charge patch neutralization

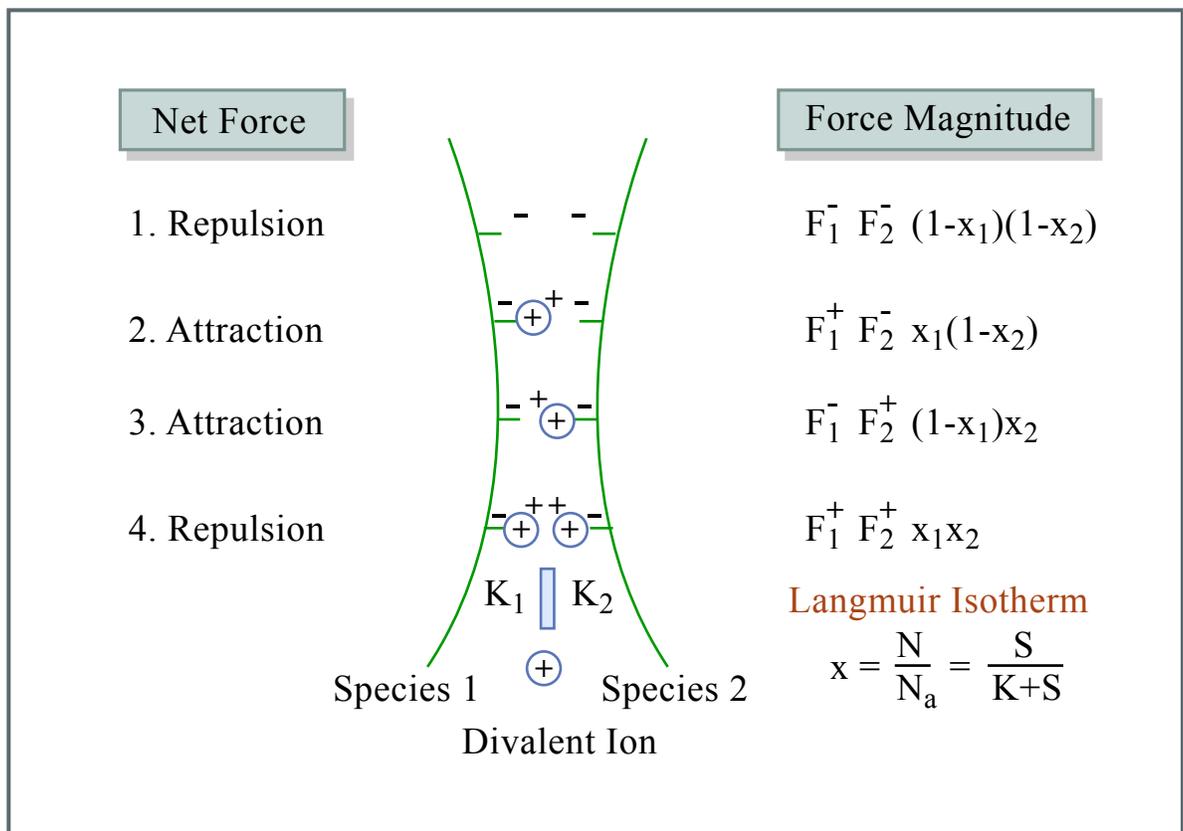


Figure by MIT OCW.

# Recovery of Penicillin G from 200 m<sup>3</sup> Fermentation

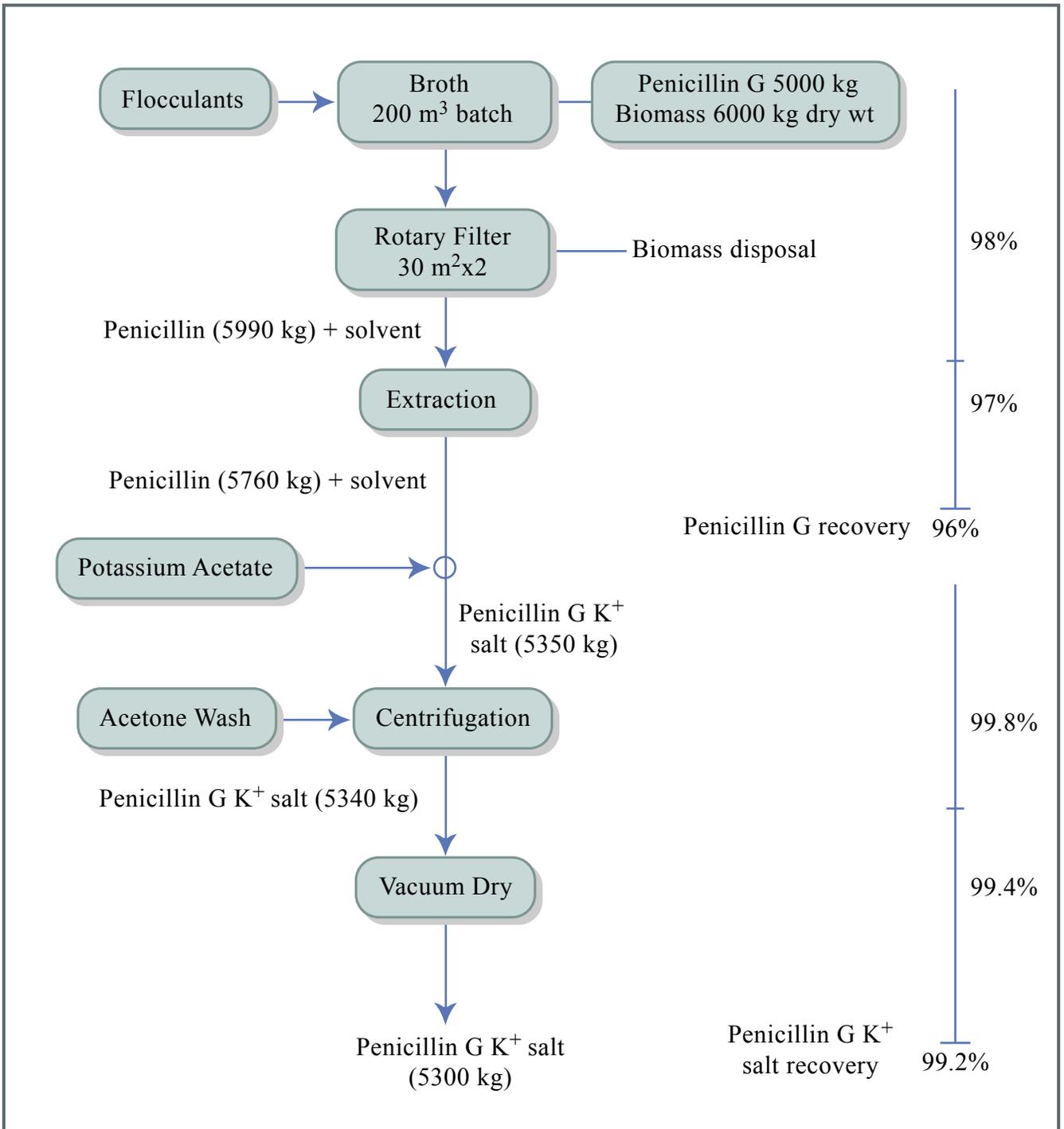


Figure by MIT OCW.