

Don't misapply correct formulas! If you encounter a question about "stability", don't start writing formulas until you've understood the process and the system. For example, determine whether the issue is open-loop stability (the process is itself unstable) or closed-loop stability (making a good process go bad by using control). And don't get spooked by odd processes - think like a chemical engineer.

(1) It is probably fair to describe your checking account as a dynamic system. You are usually concerned with the balance $y(t)$, and how it is affected by the income $x_i(t)$ and bill payments $x_b(t)$, both with dimensions of money time⁻¹. Maybe you have an interest-bearing account, as well, so that the account earns money at a rate ky , where k has dimensions of time⁻¹.

- Express the Laplace Transform of the balance $y(s)$ as a function of inputs $x_i(s)$ and $x_b(s)$. Because checking accounts are not steady systems, there is no point in defining a reference condition; therefore, don't bother with deviation variables. Your initial balance in the account is y_o .
- Calculate the poles of the transfer functions. Is your system stable? Is there a problem with that?

(2) You undertake to control a process. Individual component transfer functions are given by

$$G_v = \frac{0.016}{3s+1} \quad G_m = \frac{50}{30s+1} \quad G_s = \frac{1}{10s+1}$$

where the subscripts are conventional, and the gains have been expressed in consistent units for your convenience. Use proportional control, and calculate the controller gain at the stability limit. Do this by two methods: "poles of the transfer function" and "Bode stability criterion".

(3) Speed up the sensor;

$$G_v = \frac{0.016}{3s+1} \quad G_m = \frac{50}{30s+1} \quad G_s = \frac{1}{5s+1}$$

What is the controller gain at the stability limit? (your choice of method)

(4) Speed up the process instead;

$$G_v = \frac{0.016}{3s+1} \quad G_m = \frac{50}{20s+1} \quad G_s = \frac{1}{10s+1}$$

What is the controller gain at the stability limit? (your choice of method)

(5) Please explain what's going on in (3) and (4). This is a mathematical question - I found it useful to approach it via Bode stability criterion applied to a set of first-order lags in series.

(Problems (2) through (5) are based on Smith, Carlos A. *Automated Continuous Process Control*. New York, NY: John Wiley, 2002. Sec. 6-2. ISBN: 0471215783.)