Nim Games

Static Nim is a one-pile game between two players. The rules of the game are as follows.

- There are *n* tokens arranged in a pile.
- On each turn, a player can take up from 1 to *k* tokens from the pile.
- The player who removes the last token wins.

1. Play one-pile static nim with 20 tokens, taking 1, 2, or 3 tokens per turn.
   a) Is it better to go first or second? Or does it not matter?

   b) Is there a winning strategy (a way for one player to always win)?

   c) How would your strategy change if *n* (the start number of tokens in the pile) is a different number?

   d) How would your strategy change if *k* (the highest number of tokens you can take on your turn) is a different number?

   e) How can you generalize the winning strategy for a pile of any *n* tokens, taking from 1 to *k* tokens per turn?

2. One-pile Nim games can be denoted as N(X; a, b, c), where X is the number of tokens initially in the pile, and a, b, c, …, are the possible number of objects that can be removed. Analyze the following games.

N(20; 1, 3, 5)

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

N(20; 1, 2, 4, 8, 16) – powers of 2

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

N(20; 2, 3, 5, 7, 11, 13, 17) – Primes

20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

If given the choice, should you make the first move in this game? Explain why or why not.

Identity Nim is a variation of one-pile nim. The rules of the game are as follows.

- There are *n* tokens arranged in a pile.
- On the first turn, the player can take any number from the pile (up to *n-1*).
- On each subsequent turn, a player can take up to the number taken in the previous move.
- The player who removes the last token wins.

3. Play identity nim with 20 tokens. Is it better to go first or second? What's the winning strategy? (hint: the winning strategy can be devised using an understanding of odd/even and binary place value)

   Challenge: Generalize a winning strategy to play identity nim for any starting *n* tokens.

4. More Nim variations (ideas for the games assignment)

(see Wendy for short story that provides the context for this game)

The Thirty-One game

From a deck of cards remove the A, 2, 3, 4, 5, 6 of each suit and lay the 24 cards face up on the table. Two players take turns turning over a card, and the number is added to a running total. Whoever turns the last card to make exactly 31 wins. The ace counts as the number one.

- How is this game similar to static nim? How does it differ?
- Is there a way to force a win? Is the winning advantage for the first or second player?

Thirty-One with Dice

Use a single die. The starting number is fixed by a chance roll of the die. Thereafter each player gives the die a quarter-turn, in any direction he pleases, to bring a new number face up. A running total is kept of the numbers, and he wins who reaches the total 31 or forces his opponent to go over 31.

- What number or numbers, turned up by the random roll, spell victory for the first player?
- What is the system whereby he can preserve his advantage and force the win?

11.124 Introduction to Education: Looking Forward and Looking Back on Education
Fall 2011