

Problem Set #3 + Answers (Answer to 4 B Corrected)

1) *This problem builds on Problem 2 of the last problem set.* Consider a different plan to raise farm incomes. As before, the government announces a price for soft white wheat of \$7.50 a bushel **but it does not buy any wheat directly**. Rather, it tells farmers:

Sell as much wheat as you want on the open market to consumers and collect official receipts that document how much wheat you sold and the market price at which you sold it. Turn in those receipts to the government who will then pay you an amount equal to:

$$\text{Check} = Q^{\text{sold}}(\$7.50 - \text{Market Price})$$

In other words, for every bushel of wheat a farmer sells, the government will pay the farmer the difference between \$7.50 a bushel and the market price at which the farmer sold the wheat.

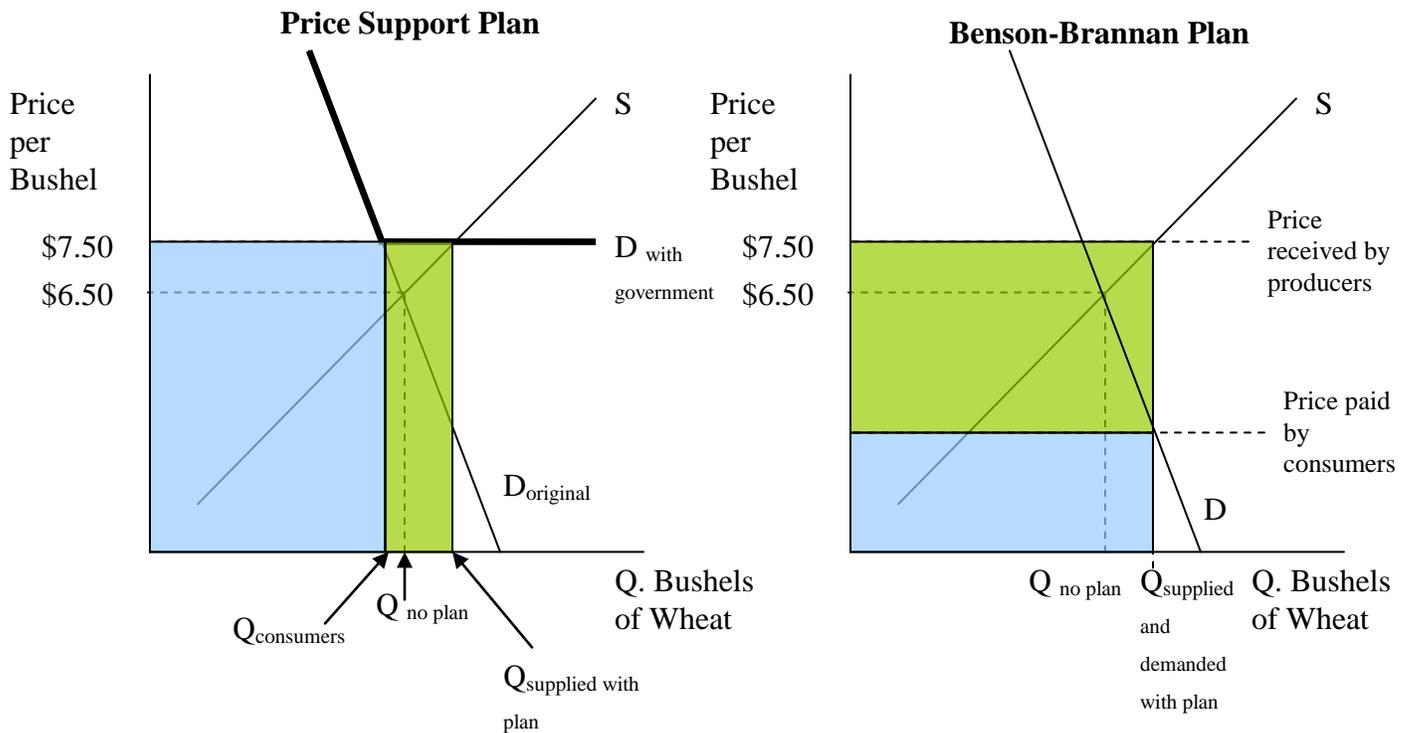
Last week's plan was called a price support plan. The current plan is called the Benson-Brannan after Charles Brannan, Harry Truman's Secretary of Agriculture, and Ezra Taft Benson, Dwight Eisenhower's Secretary of Agriculture.

Draw the Besnon-Brannan Plan. Using your diagram and the diagram you developed last week, compare the two plans along the following four dimensions:

- Total Wheat Produced
- Total Wheat Going to Consumers
- Total Income to Farmers (from Consumers + Government)
- Cost to Government of each Plan (the clue to answering this last question is buried in the beginning of Problem 2 from last week.)

Answer:

Lauren was good enough to graph the plans:



In thinking about the Benson-Brannan Plan, the starting point is recognizing that farmers will produce as if the price of a bushel of wheat is \$7.50. They don't care where this money comes from – i.e. consumers pay a much lower price and government makes up the difference between what consumers pay and \$7.50. Once you draw the quantity supplied where \$7.50 intersects the supply curve, you can answer most of the other questions by filling in the drawing as Lauren has done.

The only tricky part is the last question - in which plan does the government pay more - (which needed the email "hint" because I forgot to include the hint in the statement of Problem 2.

The "hint" said that farmers need income support because continued increases in supply over time drive down farm income. Since more supply means lower income, you should recognize that that the demand for agricultural goods must be inelastic. Given this fact, you can reason as follows:

- In the BB plan, consumers pay a lower price than in the PS plan.
- Since demand is inelastic, this must mean that consumers pay less total

dollars in the BB plan than in the PS plan.

- You know from earlier calculations that income going to farmers is equal in the two plans.
- If total farm income is equal in the two plans and consumers pay less in total in the BB plan, it follows that government must pay more total dollars in the BB plan.

2) You have a utility function defined over two goods, hot dogs and glasses of beer.

$$U(B,H) = 2\ln(B) + 2\ln(H)$$

with: $P_{\text{beer}} = \$0.80/\text{glass}$, $P_{\text{hot dog}} = \$1.60/\text{hot dog}$. Your income = \$9.60.

a) Solve your consumer's choice problem to find the combination of hot dogs and beer that maximizes utility.

Answer: Set up two equations beginning with the equality of Marginal Utility/Price across commodities:

$$MU_B / P_B = MU_H / P_H \quad \text{or} \quad (2/B)/\$0.8 = (2/H)/\$1.60$$

rearranging terms: $\$.8B = \$1.60H$ or $B = 2H$ – that is, in the optimal solution, you will buy two beers for hotdog.

Then substitute this result into the second equation, the budget constraint:

$$\$0.80B + \$1.60H = \$9.60$$

$$\$0.80 * 2H + \$1.60H = \$9.60, \text{ or}$$

$$\text{or } H = 3 \text{ hot dogs and } B = 6 \text{ Beers}$$

b) Suppose that \$.80 is the normal price for a glass of beer but during happy hour, the price drops to \$.40 per glass while hot dogs stay at \$1.60. What is the utility maximizing combination of beer and hot dogs at these prices?

$$MU_B / P_B = MU_H / P_H \quad \text{or} \quad (2/B)/\$0.40 = (2/H)/\$1.60$$

Rearranging terms: $\$.40B = \$1.60H$ or $B = 4H$ - you will buy 4 beers for every hot dog.

Putting this back into the budget constraint: $\$.40 * 4H + \$1.60H = \$9.60$

Or $H = 3$ hot dogs and $B = 12$ beers.

c) How much, in total, did you spend on beer when the price was \$.80/glass? How much in total did you spend on beer when the price was \$.40 per glass? What does this comparison tell you about the elasticity of demand for beer that comes from this utility function?

Answer: (You need to read this carefully). When beer was \$.80 cents, you bought 6 beers spending a total of \$2.40. When beer was \$.40, you bought 12 beers spending, again, \$2.40. Since the price has fallen and total expenditure on beer has not changed, the elasticity of demand for beer must be $e = -1.0$ (i.e. unit elastic = expenditure is the same regardless of price).

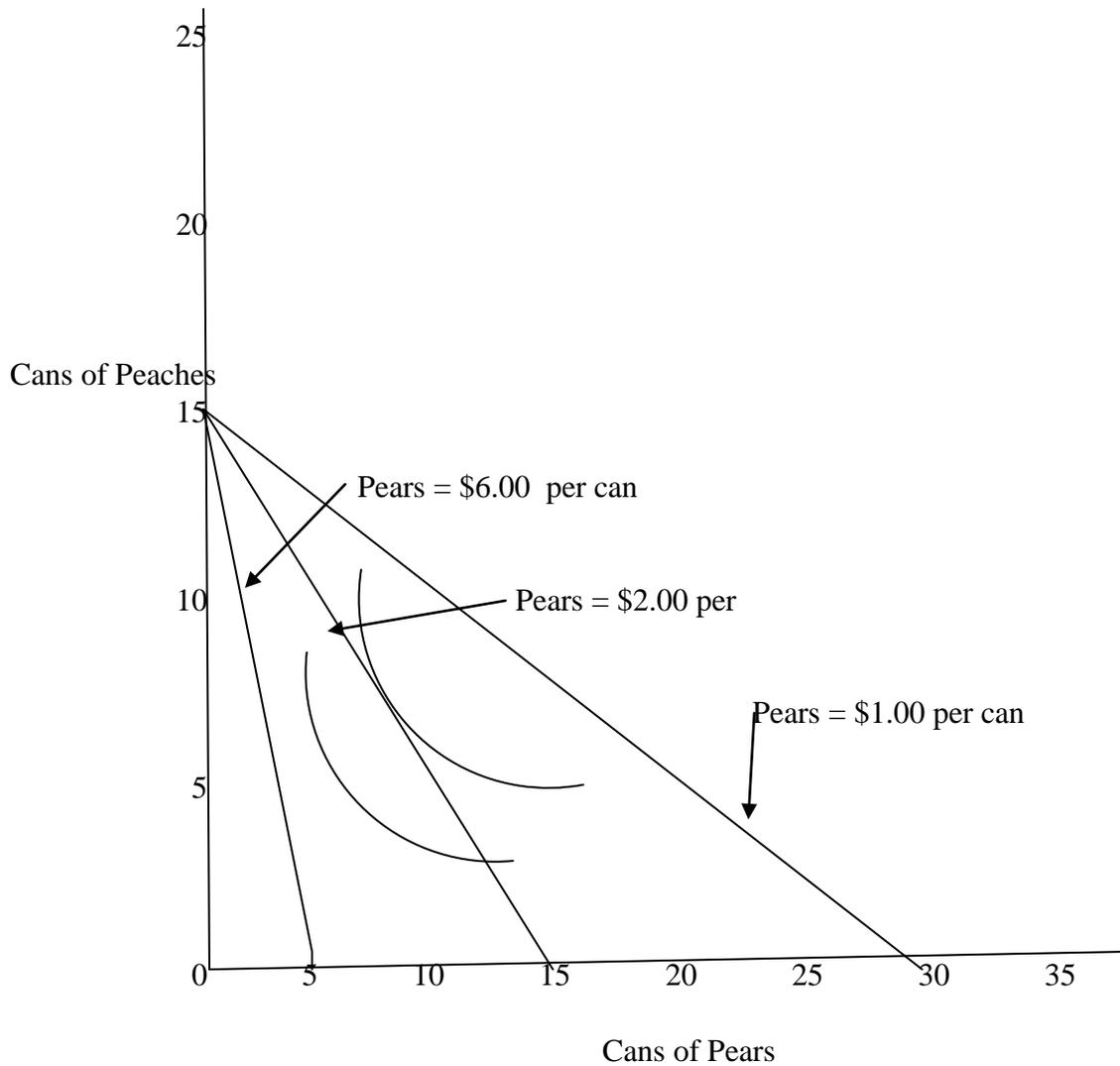
Alternatively, you might try to calculate the demand using the standard formula with using the percentage change in quantity and the percentage change in price. The problem with this formula for elasticity is that it is designed to estimate elasticity over small changes in quantity and price. When applying the formula to big changes– e.g. this 50% cut in price – the formula breaks down because it is a straight line approximation to a curve. A somewhat different formula applies here – what is called arc elasticity – and we can discuss take this issue in the review session.

d) Suppose the Massachusetts State Legislature, appalled at the extent of drunk driving, requires all restaurants and bars to institute a safe-drinking program that bans happy hour prices and limits each customer to three beers. What would your utility maximizing solution be in this case? (Don't jump to the mathematics but rather reason this one through).

Answer: Because of the state ban, we are back to a price of beer of \$.80 per glass where your utility maximizing solution is 6 beers. The state sets a maximum of 3 beers so the closest you can get is to buy 3 beers and use the rest of your money on hot dots (non stiation).

$$B = 3, H = (\$9.60 - \$2.40)/\$1.60 = 4.5 \text{ hot dogs (don't ask).}$$

3) Consider the following diagram:



Price of a Can of Peaches = \$2.00, Income = \$30.00

- Fill in several more indifference curves. Then draw a budget constraint assuming the Price of a Can of Pears = \$2.00.
- Using the indifference curves you have drawn, plot one point on a demand curve for cans of pears.
- Draw several other budget lines corresponding to the price of a can of pears varying between \$6.00 per can and \$1.00 per can. For each budget line, read off the number of cans of pears in

your utility maximizing solution and use that information to start filling in other points on your demand curve.

d) As part of your homework, include either the original or a Xerox of your indifference curve map (above) and the demand curve you created.

Answer: I have drawn the budget constraints. The rest of the problem depends on the indifference curves you have drawn.

4) a) In Problem (3) above, as the price of a can of pears fell from \$6.00 to \$1.00, describe what happened to the number of cans of **Peaches** you bought. How do you explain this?

Answer: They likely changes as well, either rising or declining. This is due to the INCOME EFFECT that we defined in class – the fact that as the price of a can of pears fell, you had more purchasing power (one way to see this is to recognize that as the price of a can of pears falls and the budget line swings out, you can buy combinations of peaches and pears that you previously could not afford.)

b) The demand elasticity we have studied in class is formally called the “own-price elasticity” – i.e. the sensitivity of a product’s demand to its own price. We can also define a “cross-price elasticity” which is the percentage change in demand of one good caused by a 1 percent change in the price of a different good.

Suppose we consider the budget of a typical suburban family and focus on two cross-elasticities of demand (for simplicity, I write peaches instead of cans of peaches, etc):

$$\epsilon_{\text{peaches, pears}} = (\Delta Q_{\text{peaches}}/Q_{\text{peaches}})/(\Delta P_{\text{pears}}/P_{\text{pears}})$$

$$\epsilon_{\text{peaches, gasoline}} = (\Delta Q_{\text{peaches}}/Q_{\text{peaches}})/(\Delta P_{\text{gallon of gasoline}}/P_{\text{gallon of gasoline}})$$

Discuss what sign each elasticity will likely have. Disregarding signs for a moment, explain which elasticity you expect to be larger in magnitude and why?

Answer: For $\epsilon_{\text{peaches, pears}}$, you can’t predict the sign. On the one hand, peaches and pears are substitutes so IF WE HELD INCOME CONSTANT—a fall in the price of pears would lead to more pears and less peaches = a negative sign. But there is also an income effect which may or may not be large enough to offset the substitution effect. You should be able to illustrate both of these possibilities on the kind of diagram you used in problem 3.

Corrected Answer: In giving an estimate of a cross-price elasticity, we need to be clear about whether or not we are holding income constant. (When we speak of “holding income constant” in this situation, we are referring to holding utility constant – not holding dollar income constant.) If we hold income constant, we can make a simple argument that ϵ peaches, pears should be positive:

- Peaches and pears are substitutes
- A fall in the price of pears will cause you to buy more pears
- If you buy more pears, holding income (utility) constant and you buy more pears, you will buy less peaches.
- Therefore the change in the price of pears and the change in the quantity of peaches are both negative and so the cross elasticity is positive (I said it was negative in the answer above which was wrong).

What we have just described – i.e. the effect of a price change with income held constant – is the **substitution effect – the effect of a price change with utility held constant**. But if we let the price of pears fall without holding utility constant, there will also be an **income effect which captures the effect of increased purchasing power**. This income effect could offset the substitution effect so that you end up buying an increased quantity of peaches. You should be able to illustrate both of these possibilities on the kind of diagram you used in problem 3.

For ϵ peaches, gasoline, we can start by assuming that a typical suburban family does a lot of driving. This means that a rise in the price of gasoline will have a significant impact on its budget which will mean reduced consumption in other areas. This means the quantity of peaches purchased will fall and so the sign of the elasticity is negative. Because gasoline takes large fraction of income compared to peaches or pears, we can also assume that the effect of a 1% price rise in gasoline on the quantity of peaches should be larger in absolute magnitude than the than the effect of a 1% rise in the price of pears.

5) (The Shoup reading may be helpful here). Analysts in the Burlington VT. Police Department allocate police patrols among three districts, each containing about 41,000 residents. They have applied statistical analysis to historical crime data to estimate the relationship between the number of patrols in a district and its crime rate. Since the demographics of the three districts are quite different (the student district is the worst), the patrol-crime rate function varies by district as well.

Specifically:

$$\text{District 1: Crime Rate/1000 residents} = 42 - 3P^{.73}$$

$$\text{District 2: Crime Rate/1000 residents} = 67 - 5P^{.58}$$

$$\text{District 3: Crime Rate/1000 residents} = 30 - 4P^{.87}$$

Where: P is the number of patrols per day.

a) Choose ONE of these three functions and take its first derivative to demonstrate that:

- Adding more patrols in the district always reduces the crime rate
- Each additional patrol reduces crime by less than the previous patrol

Answer: We can use the function for district 3. We know from our discussion of first derivatives that the derivative of the Crime Rate function with respect to police patrols - $d\text{Crime Rate}/dP$ - equals the sum of the derivatives of the two terms in the expression.

$d30/dP = 0$ (i.e. the derivative of a constant = zero.)

$d(-4P^{.87})/dP$ is an example of the general formula we have seen:

If $Y = aX^b$, then $dY/dX = abX^{b-1}$

In this case, the derivative of the second term is $= -4 \cdot .87P^{-.13}$

So $d\text{Crime Rate}/dP = 0 - 4 \cdot .87P^{-.13} = -4 \cdot .87P^{-.13}$

“Adding more patrols always in the district always reduces crime” - to demonstrate this, you can see that for any value of P, the first derivative will be negative and so the crime rate will be reduced.

“ Each additional patrol reduces crime by less than the previous patrol” – to demonstrate this, note that in the first derivative P has a negative exponent. In other words:

$$d\text{CrimeRate}/dP = -4 \cdot .87P^{-.13} = \frac{-4 \cdot .87}{P^{.13}}$$

As P grows larger, the first derivative gets smaller in absolute size. This means that each additional patrol reduces the crime rate by less than the previous patrol.

b) The department staffs 60 patrols per day. The analysts have chosen to allocate 20 patrols to each district in the name of equality. If the analysts wanted to reduce the city’s crime rate as much as possible, is this the allocation they would choose? Explain why or why not (You don’t have to calculate a new allocation even if one is needed.).

Answer: If you take a step back, you can see that the shape of these crime reduction functions are similar to utility functions: more patrols reduce crime but at a diminishing rate (positive marginal utility but diminishing marginal utility). Continuing the analogy, we can evaluate the first derivative for each district at 20 patrols:

$$d\text{CrimeRate}/dP \text{ in District 1} = -1.32$$

$$d\text{CrimeRate}/dP \text{ in District 2} = -.82$$

$$d\text{CrimeRate}/dP \text{ in District 3} = -2.35$$

The reasoning here is like the reasoning in a utility function – can you see a reallocation of patrols that would produce an overall reduction in the crime rate. If you can, then (20,20,20) does not reduce the city’s crime rate as much as possible. Using the derivatives above, if you were to remove a patrol from District 2, you would increase the crime rate by roughly .82. If you then added that patrol to District 3, you would reduce the crime rate by -.2.35. The result would be a net reduction in the crime rate. So (20,20,20) does not minimize the city’s crime rate.

c) Dividing the 60 patrols equally among the districts is one definition of equality but there are others. Describe one or two other definitions of equality that the analysts might have pursued.

Ans: One alternative definition might be to allocate patrols so that the crime rate is equal across districts. A second alternative is the equality of reduction at the margin - what we just discussed.

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