

Chapter 29:
**Economic Analysis of Investment in Real
Estate Development Projects,**
Part 2

Three considerations are important and unique about applying the NPV rule to evaluating investment in development projects as compared to investments in stabilized operating properties:

- 1. “*Time-to-Build*”:** Investment cash outflow occurs *over time*, not all at once up front, due to the *construction phase*.
- 2. *Construction loans*:** Debt financing for the construction phase is *almost universal* (even when the project will ultimately be financed entirely by equity).
- 3. *Phased risk regimes*:** Investment risk is very different (greater) between the construction phase (the *development investment per se*) and the stabilized operational phase. (Sometimes an intermediate phase, “*lease-up*”, is also distinguishable.)

We need to account for these differences in the methodology of how we *apply* the NPV Rule to development investments. . .

NPV = Benefits – Costs

The benefits and costs must be measured in an “*apples vs apples*” manner. That is, in dollars:

- As of the same point in time.
- That have been adjusted to *account for risk*.

As with all DCF analyses, time and risk can be accounted for by using *risk-adjusted discounting*.

Key is to identify: *opportunity cost of capital*

- Reflects amount of *risk* in the cash flows
- Can be applied to either *discount* CFs back in time, or
- To *grow* (compound) CFs forward in time
- e.g., to the projected time of completion of the construction phase.

Hereandnow Place:

- Twin buildings, \$75,000/mo net rent perpetuity
- OCC = 9%/yr (→ 0.75%/mo, → $1.0075^{12} - 1 = 9.38\%$ EAR)
- in total, V_0 is:

$$\frac{\$75,000}{1.0075} + \frac{\$75,000}{1.0075^2} + \dots = \frac{\$75,000}{.0075} = \$10,000,000$$

Typical investment deal for this stablized property:

$$NPV_0 = V_0 - P_0 = \$10,000,000 - \$10,000,000 = 0$$

Futurespace Centre:

- Across the street from Hereandnow.
- Will be same asset as Hereandnow, complete in 12 mos
- Constr cost \$1,500,000 X 4 payable @ mos 3, 6, 9, 12.
- First building complete in 6 mos.
- This is definitely HBU of site; irreversible commitment to develop now is appropriate

Development investment valuation question :

What is the price that can be paid today for the FutureSpace land site such that the development investment will be zero NPV? . . .

This is the value of the land, the price the FutureSpace land site would presumably sell for in a competitive market.
Hence, equivalently:

What is the NPV of the development project investment apart from the land cost? . . .

Answer:

$$NPV_0 = V_0 - P_0 = V_0 - (K_0 + Land)$$

So, what is V_0 ?, and what is K_0 ? For Futurespace Project . . .

First consider $V_0 \dots$

In 6 mos Futurespace One will be complete, expected to be worth:

$$\frac{\$37,500}{1.0075} + \frac{\$37,500}{1.0075^2} + \dots = \frac{\$37,500}{.0075} = \$5,000,000$$

And in 12 mos Futurespace Two is expected to be worth:

$$\frac{\$37,500}{1.0075} + \frac{\$37,500}{1.0075^2} + \dots = \frac{\$37,500}{.0075} = \$5,000,000$$

Thus, gross PV of project benefit is:

$$V_0 = \frac{\$5,000,000}{1.0075^6} + \frac{\$5,000,000}{1.0075^{12}} = \$9,352,000$$

Or, equivalently:

$$V_0 = \sum_{t=7}^{12} \frac{\$37,500}{1.0075^t} + \frac{\$10,000,000}{1.0075^{12}} = \$9,352,000$$

Why is Futurespace worth less than Hereandnow ? ...

Why do we cut off the analysis at month 12 ? ...

Now consider K_0 ...

Construction cost is 4 quarterly pmts of \$1,500,000 each.

These CFs have very little “risk” as capital mkt defines “risk”:

- *Low beta, low correlation w financial mkts.*

Hence: OCC for constr CFs near r_f , say **3%** per annum
(0.25%/mo, → 3.04% EAR).*

So, PV of construction costs is:

$$K_0 = \frac{\$1,500,000}{1.0025^3} + \frac{\$1,500,000}{1.0025^6} + \frac{\$1,500,000}{1.0025^9} + \frac{\$1,500,000}{1.0025^{12}} = \$5,889,000$$

* Note that by using a lower OCC for construction CFs, we discount them to a higher PV, thus causing construction costs to figure more prominently in the development investment decision.

In this sense we are treating construction cost as a greater “risk” factor to be considered in the decision.

This is not the capital market definition of “risk”, but it is consistent with common parlance.

Thus, excluding *Land* cost, we have Futurespace project valuation:

$$V_0 - K_0 = \$9,352,000 - \$5,889,000 = \$3,463,000.$$

If the price of the *Land* is x , then:

$$NPV_0 = \$3,463,000 - x .$$

For any *Land* price $\leq \$3,463,000$, the Futurespace project makes economic sense.

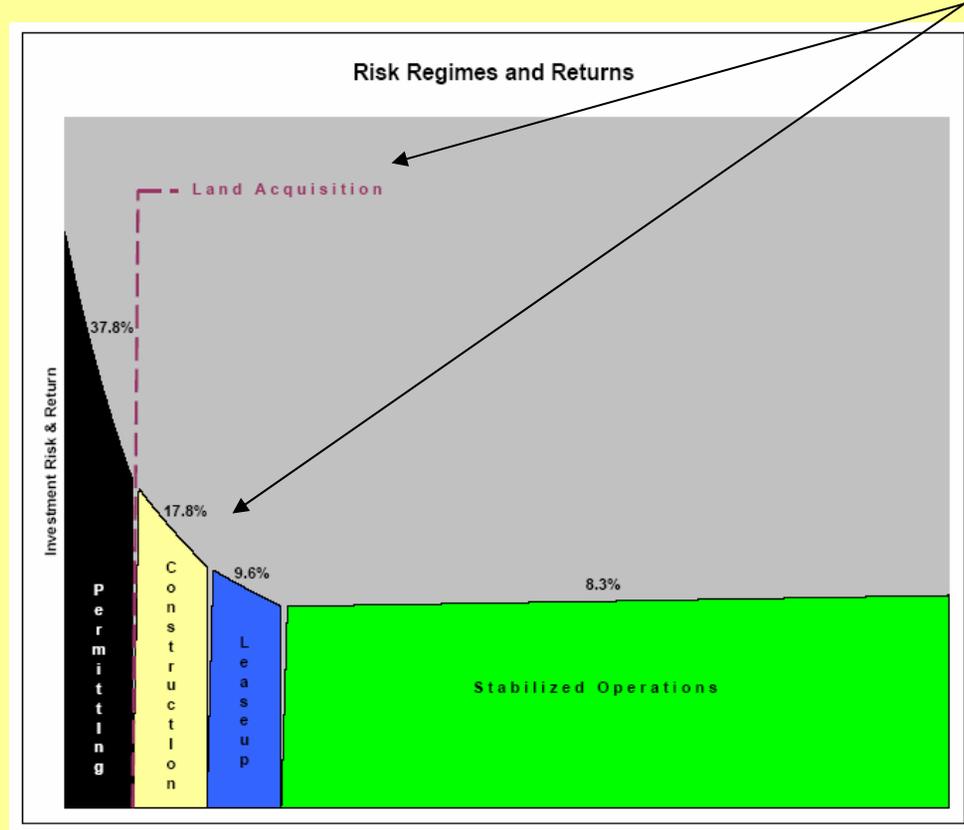
Because of the way we have defined economic value (based in market opportunity cost), if the project passes the above criterion, it should be possible to put together financial arrangements to make it happen (otherwise, \$\$\$ are being “left on the table” – recall: NPV rule based on wealth-maximization).

If the project does not pass the above criterion, it will either be difficult to put together financing, or at least one of the parties is likely to regret it later on if they did contribute...

Further investment analysis . . .

29.1.2. Operational Leverage and Estimation of the OCC for Development Investments

Recall that we are dealing with a high-risk/high-return phase (“style”) of investment (the yellow or yellow & blue phases):



How risky is this development project investment ? . . .

We can quantify the answer to this question.

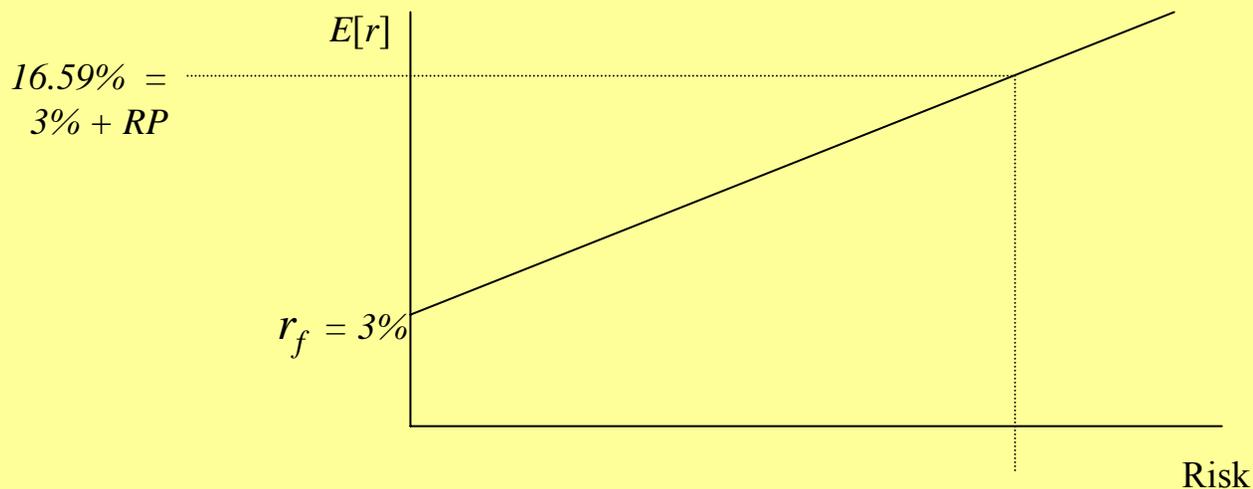
Futurespace:	Mo. 3	Mo. 6	Mo. 9	Mo.12
<i>K</i>	-\$1,500,000	-\$1,500,000	-\$1,500,000	-\$1,500,000
<i>V</i>	0	+\$5,000,000	0	+\$5,000,000
<i>Net CF</i>	-\$1,500,000	+\$3,500,000	-\$1,500,000	+\$3,500,000

Given that \$3,463,000 is the **market value** of the development project:

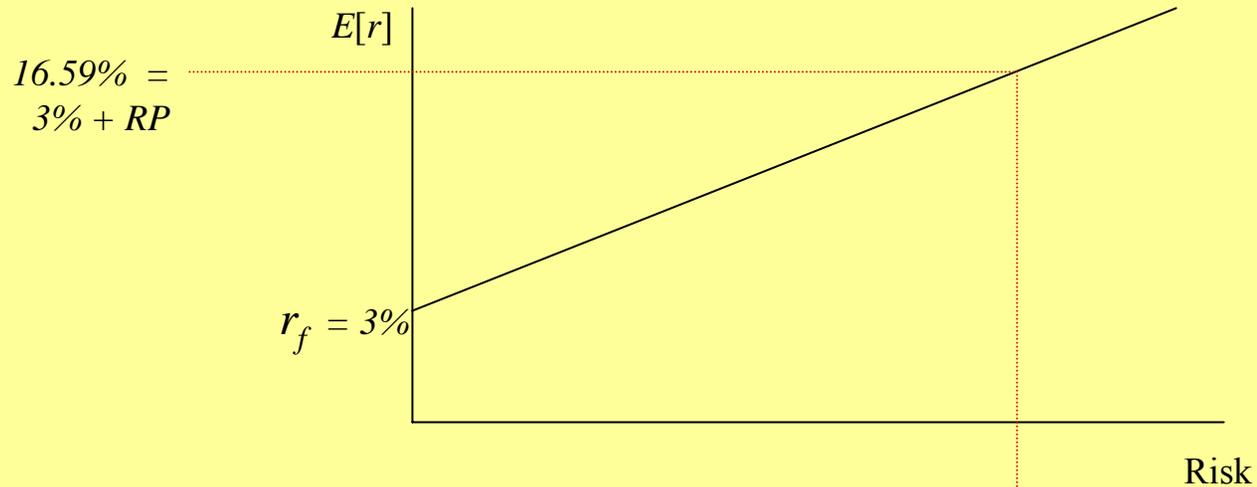
$$\$3,463,000 = \frac{-\$1,500,000}{(1 + IRR / mo)^3} + \frac{\$3,500,000}{(1 + IRR / mo)^6} + \frac{-\$1,500,000}{(1 + IRR / mo)^9} + \frac{\$3,500,000}{(1 + IRR / mo)^{12}}$$

$$\Rightarrow IRR = 1.29\% / mo, \Rightarrow (1.0129)^{12} - 1 = 16.59\% / yr.$$

The 16.59% going-in IRR for the development investment phase itself reflects the capital market's required *ex ante* risk premium:

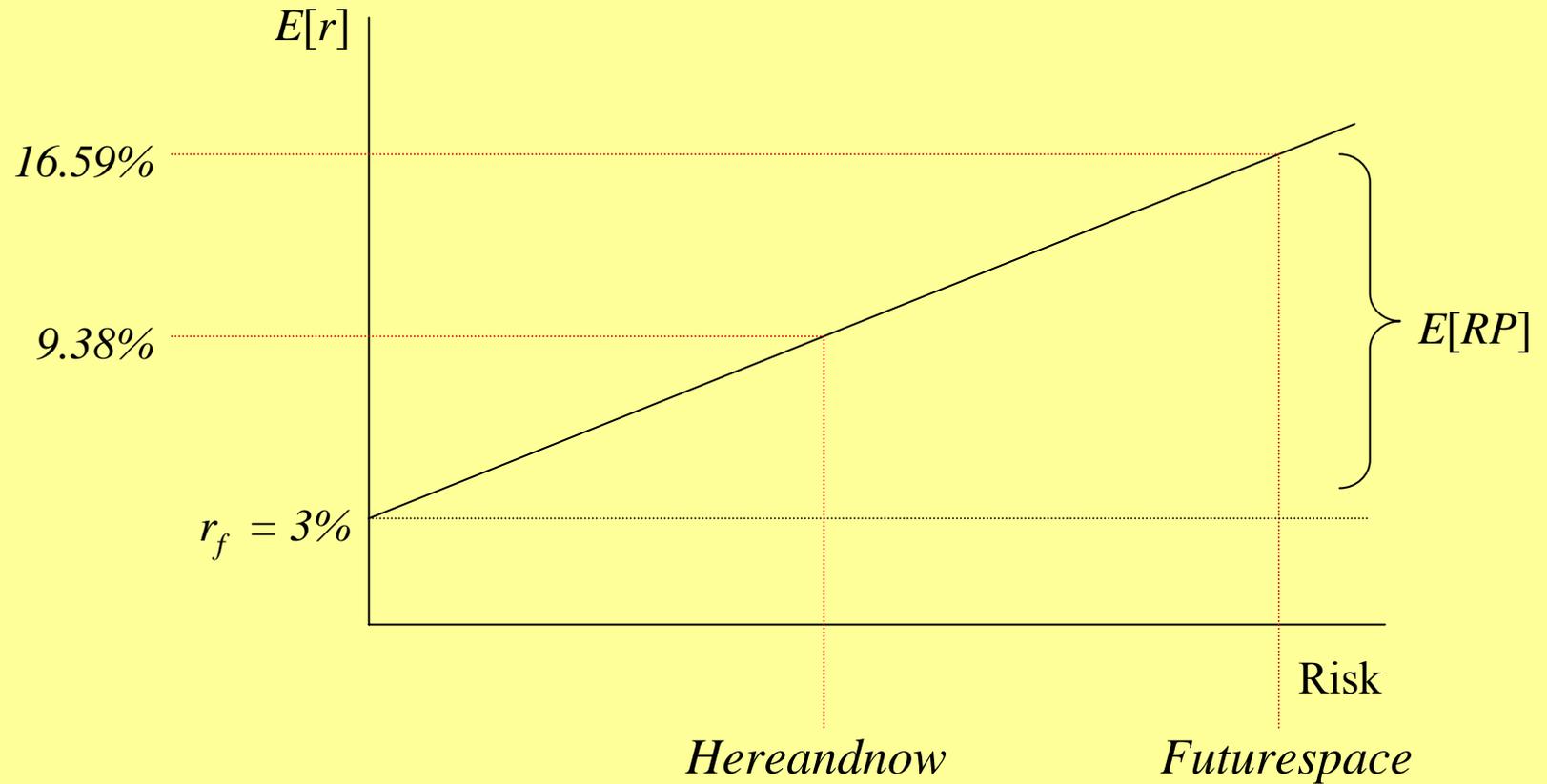


The Futurespace development project has...



$$\frac{16.59\% - 3.04\%}{9.38\% - 3.04\%} = \frac{13.55\%}{6.34\%} = 2.14 \text{ times}$$

the investment risk of an unlevered investment in stabilized property like what we are building in the Futurespace project.



If this relationship does not hold, then there are “super-normal” (disequilibrium) profits (expected returns) to be made somewhere, and correspondingly “sub-normal” profits elsewhere, across the markets for: *Land*, *Stabilized Property*, and *Bonds* (“riskless” CFs).

The added risk in Futurespace compared to Hereandnow:

$$\frac{16.59\% - 3.04\%}{9.38\% - 3.04\%} = \frac{13.55\%}{6.34\%} = 2.14 \text{ times}$$

reflects “*operational leverage*” in the development project.

$$\text{Recall: } NPV = V - P$$

Operational leverage arises whenever $P (= K + Land)$ does not occur entirely at time zero and is not perfectly positively correlated with the subsequent realization of V .

Bigger K relative to V , and/or later K in time relative to the realization of V (at time T), → Greater operational leverage.

Investments in stabilized properties have no operational leverage because the investment cost (P) occurs entirely at time zero.

Example of operational leverage:

Suppose asset values turn out to be *10% less* than previously anticipated . . .

Hereandnow:

$$\$10,000,000 = \sum_{t=1}^{12} \frac{\$75,000}{(1 + IRR / mo)^t} + \frac{\$9,000,000}{(1 + IRR / mo)^{12}}$$

$$\Rightarrow IRR = -0.087\% / mo, \Rightarrow (0.99913)^{12} - 1 = -1.04\% / yr.$$

Ex post return is $9.38\% - (-1.04\%) = 10.42$ points below *ex ante*.

Futurespace:

$$\$3,463,000 = \frac{-\$1,500,000}{(1 + IRR / mo)^3} + \frac{\$3,000,000}{(1 + IRR / mo)^6} + \frac{-\$1,500,000}{(1 + IRR / mo)^9} + \frac{\$3,000,000}{(1 + IRR / mo)^{12}}$$

$$\Rightarrow IRR = -1.19\% / mo, \Rightarrow (0.9881)^{12} - 1 = -13.42\% / yr.$$

Ex post return is $16.59\% - (-13.42\%) = 30.01$ points below *ex ante*:

➔ $30.01 / 10.42 = 2.9$ times the investment return risk.

Same kind of impact on risk and return as in Ch.13 (*“leverage”*).

In Ch.13 the effect was due to *financial leverage* (use of debt financing of the investment).

Here no debt financing is being employed (hence, no financial leverage).

How did we compute the OCC of the Futurespace development project (the 16.59%) ? . . .

We *backed it out*, that is:

We first computed the NPV of the project exclusive of *Land*:

$$V_0 - K_0$$

Then we assumed market value for *Land*:

$$NPV_0 = V_0 - P_0 = V_0 - (K_0 + Land) = 0$$

$$\rightarrow Land = V_0 - K_0$$

And then we derived the *IRR* implied by this (\$3,463,000) value for the *Land* (16.59%):

$$\$3,463,000 = \frac{-\$1,500,000}{(1 + IRR/mo)^3} + \frac{\$3,500,000}{(1 + IRR/mo)^6} + \frac{-\$1,500,000}{(1 + IRR/mo)^9} + \frac{\$3,500,000}{(1 + IRR/mo)^{12}}$$

$$\Rightarrow IRR = 1.29\%/mo, \Rightarrow (1.0129)^{12} - 1 = 16.59\%/yr.$$

We did not need to know the OCC of the development project investment in order to compute the NPV of that investment,

or (therefore) to determine whether the investment made economic sense or not.

We only needed to know the OCC of the project *benefit* (the $E[r_V] = 9.38\%$) and of the project *cost* (the $E[r_K] = 3.04\%$), together with the projected CFs of each of those:

$$NPV[*dvlpt*] = PV[V] - PV[K] - Land .$$

Nevertheless, we still found it useful to compute the OCC of the development project itself (the *16.59%*).

e.g., we used it to quantify the relative investment risk in the development investment vs a stabilized property investment (*2.14 times*).

But here we face a problem of consistency in concept or practice, regarding how we quantify the development project OCC . . .

The specific numerical value of the development phase OCC depends on the particular cash flow and value realization timing assumptions employed in the IRR computation.

Suppose,

instead of assuming that each of Futurespace's two buildings' values would be realized upon the completion of each building (in months 6 & 12), as we did in computing the 16.59% IRR,

we assume that we hold the entire project until its complete realization in month 12:

$$\$3,463,000 = \frac{-\$1,500,000}{(1 + IRR/mo)^3} + \frac{-\$1,500,000}{(1 + IRR/mo)^6} + \sum_{t=7}^8 \frac{\$37,500}{(1 + IRR/mo)^t} + \frac{-\$1,462,500}{(1 + IRR/mo)^9} + \sum_{t=10}^{11} \frac{\$37,500}{(1 + IRR/mo)^t} + \frac{\$8,537,500}{(1 + IRR/mo)^{12}}$$

$$\Rightarrow IRR = 1.07\%/mo, \Rightarrow (1.0107)^{12} - 1 = 13.58\%/yr.$$

Now it looks like the IRR (and hence the OCC) of this same development project is 13.58%, instead of 16.59%.

Which is the real (true) Futurespace OCC?...

Answer:

They can *both* be true!

(So long as they both represent realistic, feasible plans for disposition of the project.)

But this is not a very satisfying or practical answer for purposes where we need some *consistency* in quantifying development project investment OCC, for example,

For comparisons across different types of projects (e.g., for strategic planning purposes).

Also, this approach ignores the *ubiquity of use of construction loans* to finance the construction costs of development projects (even by deep-pocket institutions, such as pension funds).

The use of a construction loan pushes all of the construction cost cash outflows (from the developer/borrower's perspective) to the end of the construction phase ("time T "). . .

The “*Canonical Form*” of the Development Project OCC:

The idea is to use the ubiquity of the construction loan (in its classical form, with interest accrued until the end), to develop a simplified “stylized” representation of development project cash flows as occurring at two and only two points in time:

Time “zero”, and Time “ T ”

- *Time 0* = The moment when the *irreversible* decision to commit to development (construction) is made (opportunity cost of the land is incurred);
- *Time T* = The time when the construction phase is completed (and/or when lease-up is projected to be completed), resulting in a stabilized asset.

And then compute the standardized (“*canonical*”) IRR of the development project investment based on this 2-point cash flow assumption.

The canonical formula for the OCC of development investments can be expressed by the following condition of equilibrium across the markets for developable *land*, built *property*, and contractually fixed cash flows (*debt* assets):

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

Where:

V_T = Gross value of the completed building(s) as of time T .

K_T = Total construction costs compounded to time T .

$E[r_V]$ = OCC of the completed building(s).

$E[r_D] = E[r_K]$ = OCC of the construction costs (usually $\approx r_f$).

$E[r_C]$ = OCC of the development phase investment (“Canonical Form”).

T = The time required for construction.

The “*Canonical*” Formula:

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

LHS represents the investment in developable land.

RHS represents a way to *duplicate* this development investment:

- by investing in a combination of:
 - a long position in built property of the type being developed and
 - a short position (borrowing) in an asset that pays contractually fixed cash flows (debt) in the amount of the construction costs of the project.

The “*Canonical*” Formula:

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

As noted before, if this formula does not hold, then there will be “super-normal” (disequilibrium) profits (expected returns) to be made somewhere, and correspondingly “sub-normal” profits elsewhere, across the markets for: *Land*, *Stabilized Property*, and *Bonds* (“*riskless*” *CFs*).

In most cases, all of the variables in the canonical formula can be observed or estimated with relatively high confidence except for the OCC of the development phase investment, $E[r_C]$. Solving for $E[r_C]$ we obtain:

$$E[r_C] = \left[\frac{(V_T - K_T)(1 + E[r_V])^T (1 + E[r_D])^T}{(1 + E[r_D])^T V_T - (1 + E[r_V])^T K_T} \right]^{(1/T)} - 1$$

If you prefer, a simpler more intuitive (and equivalent) way to derive $E[r_C]$ is to first compute $V_0 - K_0$ using the previous method, and then derive the canonical development phase OCC as:

$$1 + E[r_C] = \left(\frac{V_T - K_T}{V_0 - K_0} \right)^{1/T}$$

$$\text{NPV} = \text{Benefits} - \text{Costs}$$

The benefits and costs must be measured in an “*apples vs apples*” manner. That is, in dollars:

- As of the same point in time.
- That have been adjusted to *account for risk*.

As with all DCF analyses, time and risk can be accounted for by using *risk-adjusted discounting*.

Key is to identify: *opportunity cost of capital*

- Reflects amount of *risk* in the cash flows
- Can be applied to either *discount* CFs back in time, or
- To *grow* (compound) CFs forward in time
- e.g., to the projected time of completion of the construction phase.

For the development project:

NPV exclusive of land cost =

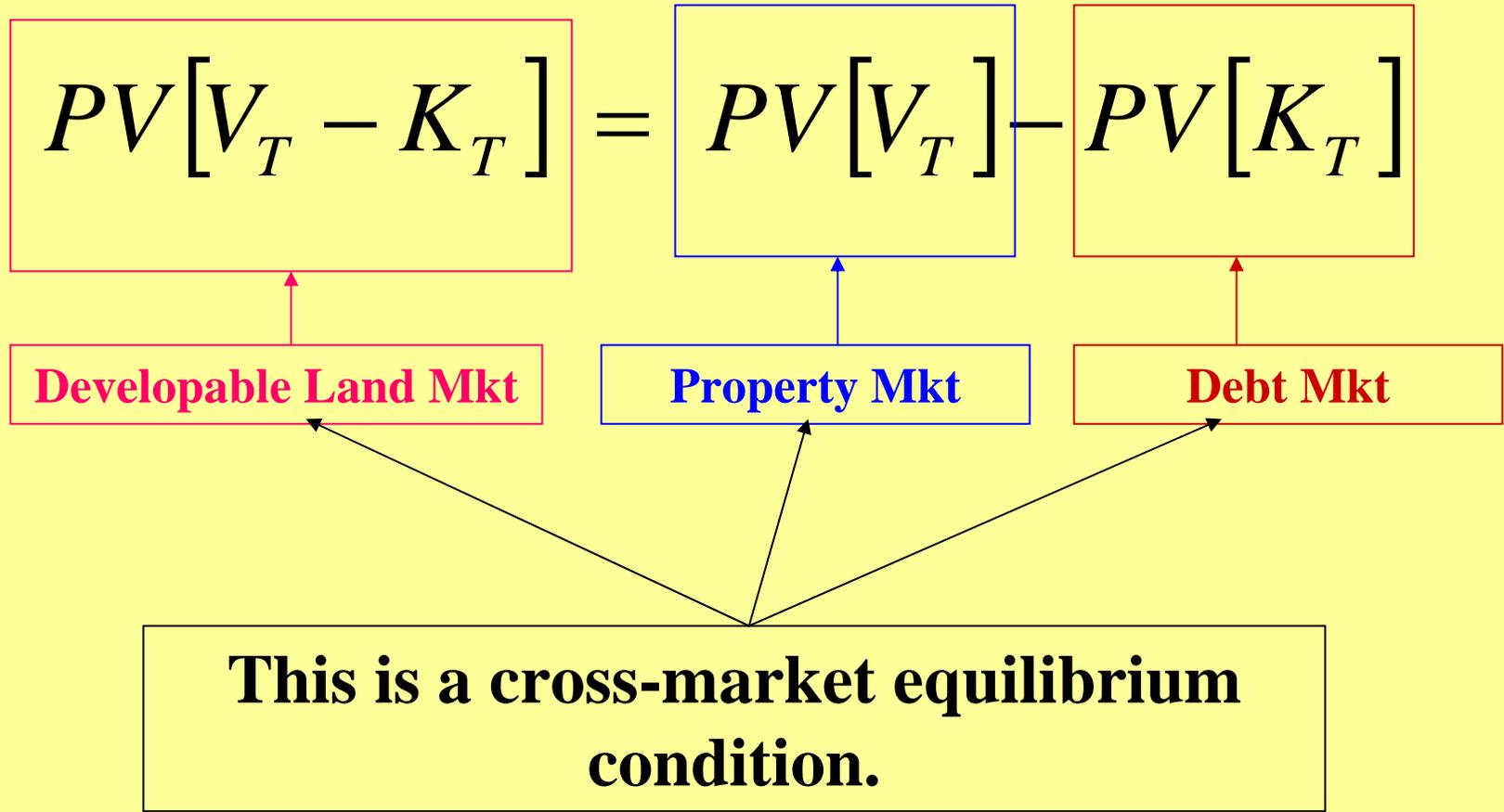
$$PV[V_T - K_T] = PV[V_T] - PV[K_T]$$

Where:

- V_T = Gross value of the completed building(s) as of time T .
- K_T = Total construction costs (excluding land) compounded to time T .
- $PV[V_T - K_T]$ = Present Value of the construction profit.
- $PV[V_T]$ = Present Value of the property to be built.
- $PV[K_T]$ = Present Value of the construction costs.
- T = The time required for construction.

For the development project:

NPV exclusive of land cost =



Here we simply expand the formula to show the relevant rates of return, the OCC rates in the three markets . . .

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

Where:

- V_T = Gross value of the completed building(s) as of time T .
- K_T = Total construction costs (excluding land) compounded to time T .
- $E[r_V]$ = OCC of the completed building(s), from the property mkt.
- $E[r_D] = E[r_K]$ = OCC of the construction costs (usually $\approx r_f$), from debt mkt.
- $E[r_C]$ = OCC of the development project (“Canonical Form”).
- T = The time required for construction.

Here we simply expand the formula to show the relevant rates of return, the OCC rates in the three markets . . .

$$\frac{V_T - K_T}{(1 + E[r_C])^T} = \frac{V_T}{(1 + E[r_V])^T} - \frac{K_T}{(1 + E[r_D])^T}$$

Developable Land Mkt **Property Mkt** **Debt Mkt**

This is still the same cross-market equilibrium condition.

Numerical Example of the Canonical Formula

Derivation of the canonical OCC for the Futurespace Project:

First compute the forward V_T value of the project *benefit* as of *Time T*:

$$\begin{aligned}V_T &= \$5,000,000(1.0075)^6 + \$5,000,000 \\&= \sum_{t=7}^{12} \$37,500(1.0075)^{12-t} + \$10,000,000 \\&= \$10,229,000\end{aligned}$$

Numerical Example of the Canonical Formula

Next compute the forward K_T value of the project *construction cost* as of *Time T*:

$$K_T = \$1,500,000(1.0025)^9 + \$1,500,000(1.0025)^6 + \$1,500,000(1.0025)^3 + \$1,500,000 = \$6,068,000$$

To obtain the projected net development profit as of month 12:

$$V_T - K_T = \$10,229,000 - \$6,068,000 = \$4,161,000.$$

Then substituting into the Canonical Formula we obtain the canonical OCC of the Futurespace Project:

$$\frac{\$10,229,000 - \$6,068,000}{(1 + E[r_C])^{12}} = \frac{\$10,229,000}{(1.0075)^{12}} - \frac{\$6,068,000}{1.0025^{12}}$$

\Rightarrow

$$E[r_C] = \left[\frac{(\$4,161,000)(1.0075)^{12}(1.0025)^{12}}{(1.0025)^{12}\$10,229,000 - (1.0075)^{12}\$6,068,000} \right]^{(1/12)} - 1 = 1.54\% / mo.$$

$$\Rightarrow (1.0154)^{12} - 1 = 20.16\% / yr.$$

Canonical OCC = 20.16% / year.

Numerical Example of the Canonical Formula

Alternatively (and equivalently):

$$\begin{aligned}1 + E[r_C] &= \left(\frac{V_T - K_T}{V_0 - K_0} \right)^{1/T} \\ &= \left(\frac{\$10,229,000 - \$6,068,000}{\$9,352,000 - \$5,889,000} \right)^{1/12} \\ &= \left(\frac{\$4,161,000}{\$3,463,000} \right)^{1/12} = 1.2016^{1/12} = 1.0154\end{aligned}$$

Canonical OCC = 20.16% / year.

Numerical Example of the Canonical Formula (Futurespace Project)

This canonical 20.16% exceeds the previously derived 16.59% and 13.58% OCC numbers because the canonical assumption involves more leverage, due to the assumption, in effect, of the use of a construction loan (all CFs at only Time 0 and Time T).

Returning to our risk comparison between the development project and the stabilized property, from the canonical perspective, Futurespace Centre has...

$$\frac{20.16\% - 3.04\%}{9.38\% - 3.04\%} = \frac{17.12\%}{6.34\%} = 2.70 \text{ times}$$

the risk of Hearandnow Place.

Relation of the Canonical Formula to the WACC:

As the Canonical Formula reduces the development project to a single-period investment (between Time 0 and Time T), the Canonical OCC can be equivalently derived using the “WACC” Formula that we introduced in Chapter 13:

$$E[r_C] = E[r_D] + LR(E[r_V] - E[r_D])$$

Defining LR as the leverage ratio: $V/(V-K)$, based on the *Time Zero* valuations of the asset to be built and the land value:

$$LR = \frac{V_0}{V_0 - K_0} = \frac{\$9,352,000}{\$9,352,000 - \$5,889,000} = 2.70$$

we have the development (land) OCC given as:

$$E[r_C] = 3.04\% + 2.70(9.38\% - 3.04\%) = 20.16\%.$$

Thus further illustrating how a development project investment may be thought of as like a levered investment in a stabilized property like the one being built.

In whatever manner the canonical OCC is determined, it will of course yield the same NPV of the development project (land value) as we derived originally:

$$\frac{\$4,161,000}{1.2016} = \frac{\$10,229,000}{1.0938} - \frac{\$6,068,000}{1.0304} = \$9,352,000 - \$5,889,000 = \$3,463,000$$

However, prior knowledge of this NPV is not necessary to ascertain the canonical OCC of the development project, as $E[r_C]$ is determined solely by the variables on the right-hand side of the Canonical Formula equation:

$$E[r_C] = \left[\frac{(V_T - K_T)(1 + E[r_V])^T (1 + E[r_D])^T}{(1 + E[r_D])^T V_T - (1 + E[r_V])^T K_T} \right]^{(1/T)} - 1$$

It should also be noted (for future reference) that the Canonical Formula is completely consistent with the real options valuation of the development project investment, once the option is at the point where immediate exercise (development) is optimal (our original assumption here).

Summarizing the advantages of the recommended procedure:

Consistent with underlying theory. (i.e., consistent with NPV Rule, based on Wealth Maximization Principle. Based on market opportunity costs, equilibrium across markets.)

Simplicity. Avoids need to make assumptions about permanent loan or form of permanent financing (equity vs debt).

Explicit identification of the relevant OCC. Identifies explicit expected return (OCC) to each phase (each risk regime) of the investment: Development, Lease-up (if appropriate), Stabilized operation.

Explicit identification of land value. Procedure results in explicit identification of current opportunity value of the land.

“Front-door” or “Back-door” flexibility possible. Procedure amenable to “backing into” any one unknown variable. E.g., if you know the land value and the likely rents, you can back into the required construction cost. If you know (or posit) all of the costs and values, then you can back into the expected return on the developer's equity contribution for the development phase. Or you can back into the implied land value (supportable price). 37

Do developers really use the “NPV Rule”? . . .

- Most don't use NPV *explicitly*.
- But remember: NPV ← *Wealth Maximization*.
- By definition, *successful* developers maximize their wealth.
- Thus, *implicitly* (if not explicitly), *successful* developers must (somehow) be employing the NPV Rule:
 - e.g., in deciding *which projects* to pursue,
 - An intuitive sense of correctly rank-ordering mutually-exclusive projects or designs by NPV, and picking those with the highest NPV (they may think of it as “best profit potential”), must be employed (by the most successful developers).
- Suggestion in Ch.29 is that by making this process more explicit, it may be executed better, or by more developers (i.e., making more developers “successful”),
- The NPV approach also should improve the ability of the development industry to “communicate” project evaluation in the “*language of Wall Street*” (e.g., “NPV”, “OCC”, phased risk regimes, risk/return “styles”).

Note that in this approach, there is no need to *pre-assume* what type of permanent financing will be used for the completed project.

There is no assumption at all about what will be done with the completed project at *time T*. It may be:

- Financed with a permanent mortgage,
- Financed or sold (wholly or partly) tapping external equity, or
- Held without recourse to external capital.

Project evaluation is independent of project financing, as it should be.*

** Unless subsidized (non-market-rate) financing is available contingent on project acceptance: Recall Chapter 14, the “APV” (Adjusted Present Value) approach to incorporating financing in the investment evaluation.*

Another important reason for this approach:

- Risk characteristics of **development phase** different from risk characteristics of **stabilized phase**.
- Thus, different OCCs, therefore:
- Two phases must be analyzed in two separate steps.

***29.2 Advanced Topic:**

The Relationship of Development Valuation to the Real Option Model of Land Value

The NPV & canonical OCC development project investment valuation and analysis procedure previously described is

Consistent with the *real option* model . . .

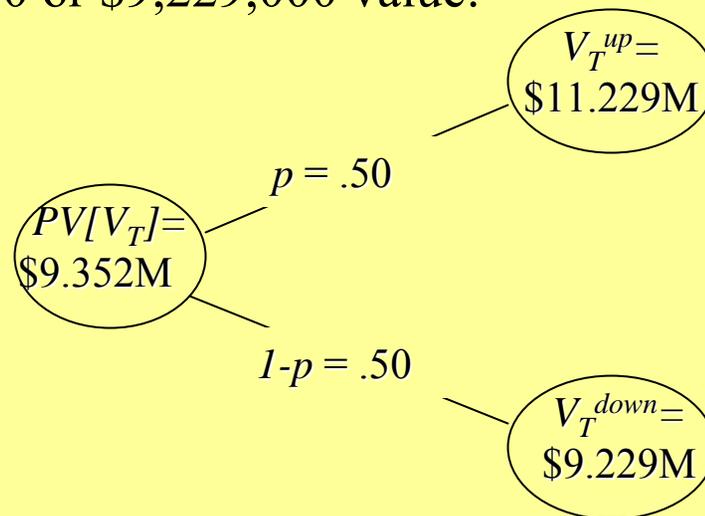
To see this, consider a binomial model of the preceding *Hereandnow / Futurespace* example . . .

To see this, consider a binomial model of the preceding *Hereandnow / Futurespace* example . . .

Suppose that risk in the property market is such that the *ex ante* expected value of the project at Time T , what we have labeled and quantified as:

$$V_T = \$10,229,000$$

is actually based on a binomial outcome possibility of a 50/50 chance of either \$11,229,000 or \$9,229,000 value:



With in any case the construction cost having a K_T value of \$6,068,000 (as before), and recalling that we have previously determined that the PV of this 1-yr forward claim is \$9,352,000.

Then applying the certainty-equivalence form of the DCF present value model that we first introduced in Chapter 10 Appendix C and which formed the basis of the binomial option value model we described in Chapter 28, we have the following present value computation for the FutureSpace project as of time zero:

$$PV[C_1] = \frac{E_0[C_1] - (C_1^{up} \$ - C_1^{down} \$) \frac{E_0[r_V] - r_f}{V_1^{up} \% - V_1^{down} \%}}{1 + r_f}$$

$$PV[C_1] = \frac{((0.5)(11.229 - 6.068) + (0.5)(9.229 - 6.068)) - ((11.229 - 6.068) - (9.229 - 6.068)) \frac{9.38\% - 3.04\%}{(11.229 - 9.229)/9.352}}{1.0304}$$

$$= \frac{\$4.161 - \$2.000 \left(\frac{6.34\%}{21.38\%} \right)}{1.0304} = \frac{\$4.161 - \$2.000(.2964)}{1.0304} = \frac{\$3.568}{1.0304} = \$3.463$$

Which is identical to the PV of the project (land value) that we found before.

Note that in percentage terms the development project investment outcome spread (which may be viewed as a measure of risk) is:

$$\frac{(\$11,229,000 - \$6,068,000) - (\$9,229,000 - \$6,068,000)}{\$3,463,000}$$

$$= \frac{\$5,161,000 - \$3,161,000}{\$3,463,000} = \frac{\$2,000,000}{\$3,463,000} = 57.75\%$$

Whereas that in (the *T*-period forward purchase of) the stabilized property is:

$$\frac{\$11,229,000 - \$9,229,000}{\$9,352,000} = \frac{\$2,000,000}{\$9,352,000} = 21.38\%$$

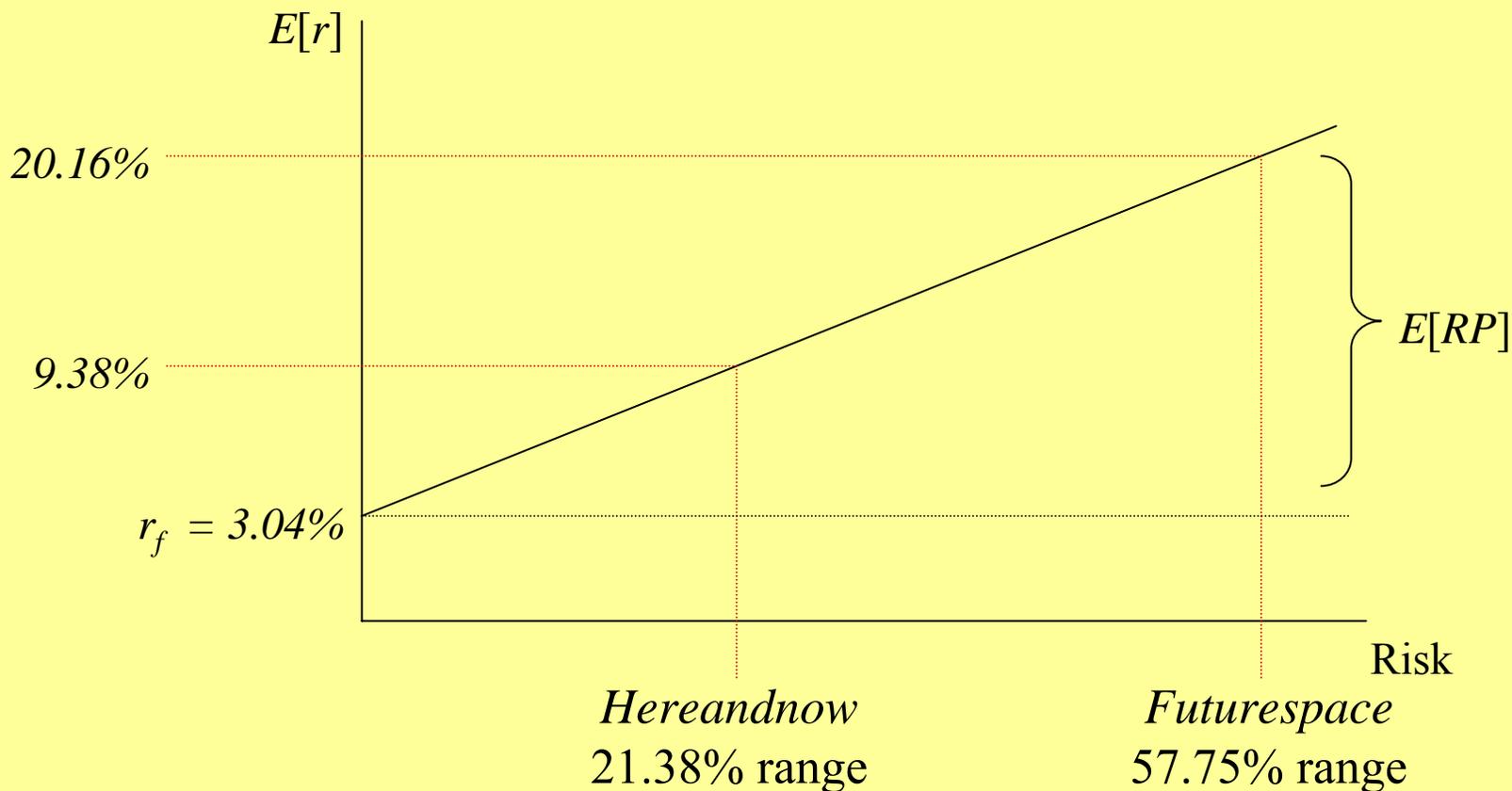
The ratio of these outcome spreads (risk):

$$\mathbf{57.75 / 21.38 = 2.70,}$$

is exactly the same as the ratio of the ex ante risk premia between the development project and its underlying asset (using the Canonical OCC):

$$\mathbf{(20.16\% - 3.04\%) / (9.38\% - 3.04\%) = 17.12\% / 6.34\% = 2.70.}$$

The “price of risk” (the ex ante investment return risk premium per unit of risk) must be the same across the relevant asset markets:



If this relationship does not hold, then there are “super-normal” (disequilibrium) profits (expected returns) to be made somewhere, and correspondingly “sub-normal” profits elsewhere, across the markets for: *Land*, *Stabilized Property*, and *Bonds* (“riskless” CFs).

29.3.1 How Developers Think About All This...

Developers don't usually *explicitly* apply the NPV Rule.

But remember . . .

Proof that Wealth Maximization implies the NPV Decision Rule:

- Suppose not.
- Then I could maximize my wealth and still contradict NPV Rule.
- I could choose a project with $NPV < 0$, or with NPV less than that of another mutually-exclusive feasible alternative.
- But if I did that I would be “leaving money (i.e. “wealth”) on the table”, taking less wealth when I could have more.
- This would not be wealth-maximization.
- Hence: Contradiction.

QED (“Proof by Contradiction”).

Thus, if our definition of a “successful” developer is one who *maximizes wealth*, then successful developers must be applying the NPV Rule (implicitly if not explicitly).

What sort of quantitative performance measures *do* developers look at ? . . .

- Sometimes, just the *SFFA* described earlier in Ch 28 (but that is only a *feasibility* analysis, not an *evaluation*). Other times:

- ***Profit Margin Ratio***

- ***Enhanced Cap Rate on Cost***

- ***Blended Long-run IRR***

But developers must apply these approaches in a manner that gives the same result as the NPV Rule, or else they are not maximizing wealth.

And their evaluations must be consistent with market equilibrium, or they “won’t be able to play the game”.

Profit Margin Ratio

Definition:

Expected Gross Value

Total Costs

(undiscounted)

Evaluation is then based on some conventional but *ad hoc* “Rule of Thumb”, such as **20%** (for a relatively quick project, when land is included in the cost).

e.g., for our *Futurespace* example project:

$$1 + \text{Margin} = \frac{V_T}{K + C} = \frac{\$10,000,000}{\$6,000,000 + C} = 1.20$$

⇒

$$\text{Land Value} = C = \frac{V_T}{1 + \text{Margin}} - K = \frac{\$10,000,000}{1.20} - \$6,000,000 = \$2,333,000$$

Profit Margin Ratio

e.g., for our *Futurespace* example project:

$$1 + \text{Margin} = \frac{V_T}{K + C} = \frac{\$10,000,000}{\$6,000,000 + C} = 1.20$$

⇒

$$\text{Land Value} = C = \frac{V_T}{1 + \text{Margin}} - K = \frac{\$10,000,000}{1.20} - \$6,000,000 = \$2,333,000$$

But we have seen that this answer is wrong, if the OCC of the built property is 9% and the riskfree rate is 3%.

The margin that would give the correct answer is 5.7%:

$$\frac{\$10,000,000}{\$6,000,000 + \$3,463,000} - 1 = 5.7\%$$

(This will provide the canonical expected return of 20.16% as we have seen.)

(A land price of \$2,333,000 will provide the developer with a (canonical) expected return of 78% !)

Profit Margin Ratio

e.g., for our *Futurespace* example project:

$$1 + \text{Margin} = \frac{V_T}{K + C} = \frac{\$10,000,000}{\$6,000,000 + C} = 1.20$$

⇒

$$\text{Land Value} = C = \frac{V_T}{1 + \text{Margin}} - K = \frac{\$10,000,000}{1.20} - \$6,000,000 = \$2,333,000$$

But perhaps the developer is applying the 20% margin with the *historical* cost of the land.

And perhaps the land was acquired some 1.77 years before, for the \$2,333,000 price, and has earned a speculative (*real option* based) return of 25%/year since then:

$$\$2,333,000 * (1.25)^{1.77} = \$3,463,000$$

Enhanced Cap Rate on Cost

Definition: Expected Initial Stabilized NOI/yr

Total Cost inclu Land (undiscounted)

Evaluation is then based on a required cap rate that is somewhat greater than what currently prevails in the market for stabilized properties of the type to be built.

e.g., for *Futurespace* example:

Market cap rate for stabilized is 9% (see *Hereandnow*),
so developer might require 10% cap rate for the *Futurespace* development
project:

$$\text{NOI} / 10\% = \$900,000 / 0.1 = \$9,000,000.$$

So, if total costs \leq \$9,000,000, project looks good:

$$\rightarrow \text{Land value} = \$9,000,000 - \$6,000,000 = \$3,000,000.$$

Blended Long-run IRR

Definition:

Append a projected multi-year (usually 10 years) operating phase cash flow projection to the development phase cash flow projection, including land cost (e.g., a $10+T$ year projection), and apply a hurdle IRR requirement that is greater than that for stabilized investments, by some *ad hoc* amount (e.g., maybe 100bps or so, for unlevered holding).

e.g., in our *Futurespace* example, if we extended the CF projection out another 9 years (108 months), and applied a 10% discount rate (instead of the 9% OCC for stabilized property) across the entire 10-year projection, we would get a PV of \$3,049,000, suggesting that this could be paid for the land.

However, from a capital markets perspective, this muddies the waters by mixing two very different investment styles together.

And from a development decision perspective it concatenates two different decisions that need not be fused.

29.3.2 Including Lease-up & Land Assembly:

Unblending the blended IRR

Survey: Ask developers what long-run (blended) IRR they will accept (or typically face) for projects at different stages.

Example, suppose developers say that for a project like Futurespace they would be looking for a long-run IRR of 9.00% for an investment in the completed stabilized property, but would require 9.73% to come in at the construction phase even if the building were pre-leased.

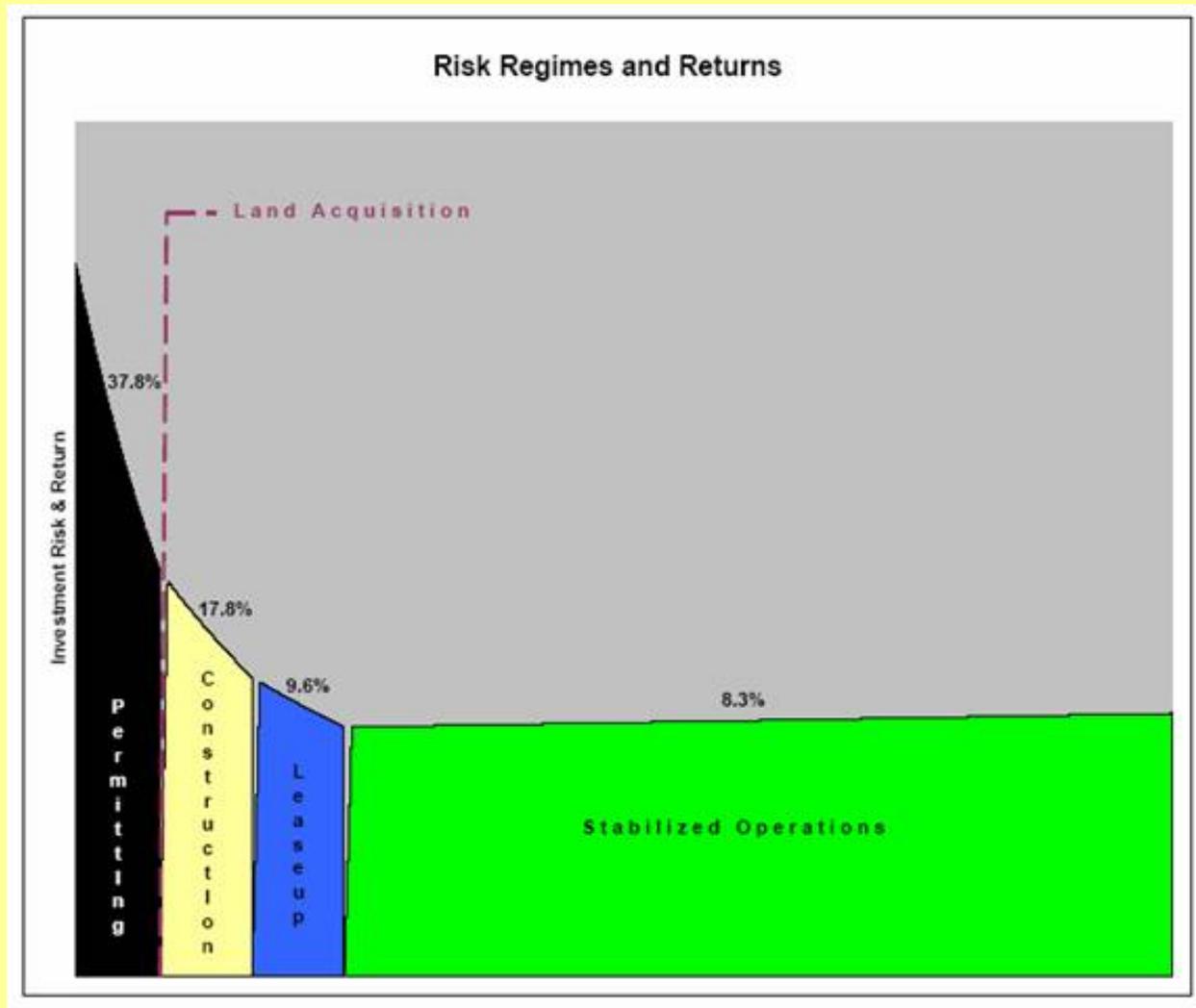
Implication is that the required going-in return expectation for the 1-year development phase alone is 20.16% (canonical).

Because 9% stabilized IRR gives \$10,000,000 building value, and 9.73% IRR at construction outset gives \$3,463,000 value of the construction project, which we saw implies 20.16% canonical devlpt phase IRR.

LR Blended Rate for Futurespace:										
Year:	1	2	3	4	5	6	7	8	9	10
(\$3,463,000)	(\$5,775,000)	\$900,000	\$900,000	\$900,000	\$900,000	\$900,000	\$900,000	\$900,000	\$900,000	\$10,900,000
9.73% = IRR										

Results of Tod McGrath survey of Boston area developers (2004):

Exhibit 29-2: An Example of Risk and Return Regimes for Phases of the Development Process. (Taken from a study of affordable housing development in Massachusetts.)



29.4 & Appendix 29: The Phasing Option . . .

The ability (and need) to break the development project into two or more sequential *phases* rather than committing to its complete construction all at once complicates the option model of development and land value (but is an important aspect of large projects).

The phased option can be modeled as a “compound option” :

An Option on an Option

Building the first phase gives you the option to build the second phase, and so on...

The Phasing Option . . .

The phasing option can be modeled using the binomial procedure as before, applying the same formulas as before (with *time-to-build* as appropriate), repeatedly, starting with the last phase (as a simple option), then the next-to-last phase is modeled as an option on the last phase, and so on...

That is, the next-to-last phase is an option whose “underlying asset” is another option, namely, the last phase.

Let’s walk through a simple numerical example . . .

Roth Harbor . . .

The following numerical example will demonstrate all of the option modeling techniques presented in this lecture . . .

***Roth Harbor* is a strategically and scenically located former brownfield site on the shore near the center of Wheatonville, ME.**

Wheatonville is a former shipbuilding city now booming with high-tech startups and an influx of young professionals and empty-nesters, creating a serious housing shortage.

The 50 acres of former industrial and warehouse property in Roth Harbor are currently zoned to allow 500 market-rate apartments to be developed.

The property is owned by *Ciochetti Enterprises LLC* (CEC), which has plans to build the 500 units in a single project, to be called *Rentleg Gardens*.

The Planning Commission of Wheatonville “*has a better idea...*”

Roth Harbor . . .

Rentleg Gardens:

Current apartment rents in Wheatonville suggest that the Rentleg Gardens apartments could charge gross rents of \$1100/mo, with operating expenses of \$6533/year per occupied unit, and average vacancy of 4%. Cap rates (y_V) on such properties are currently 8%.

If the 500 Rentleg Gardens units existed today, the property would be worth:

$$\mathbf{[(1100*12 - 6533)*0.96 / .08] * 500 = \$40,000,000}$$

based on a projected current NOI of \$3,200,000/yr and an average unit value of \$80,000.

Construction cost as of today would be \$32,000,000, with a projected deterministic (riskless) growth rate of 2%/yr. Construction would take 1 year (implying a bill due next year of: $(1.02)\$32$ million = \$32.64 M).

(Environmental cleanup of the site has already been done by CEC, and the site is now ready for development.)

Roth Harbor . . .

The Planning Commission's "Better Idea"...

"Roth Harbor Place":

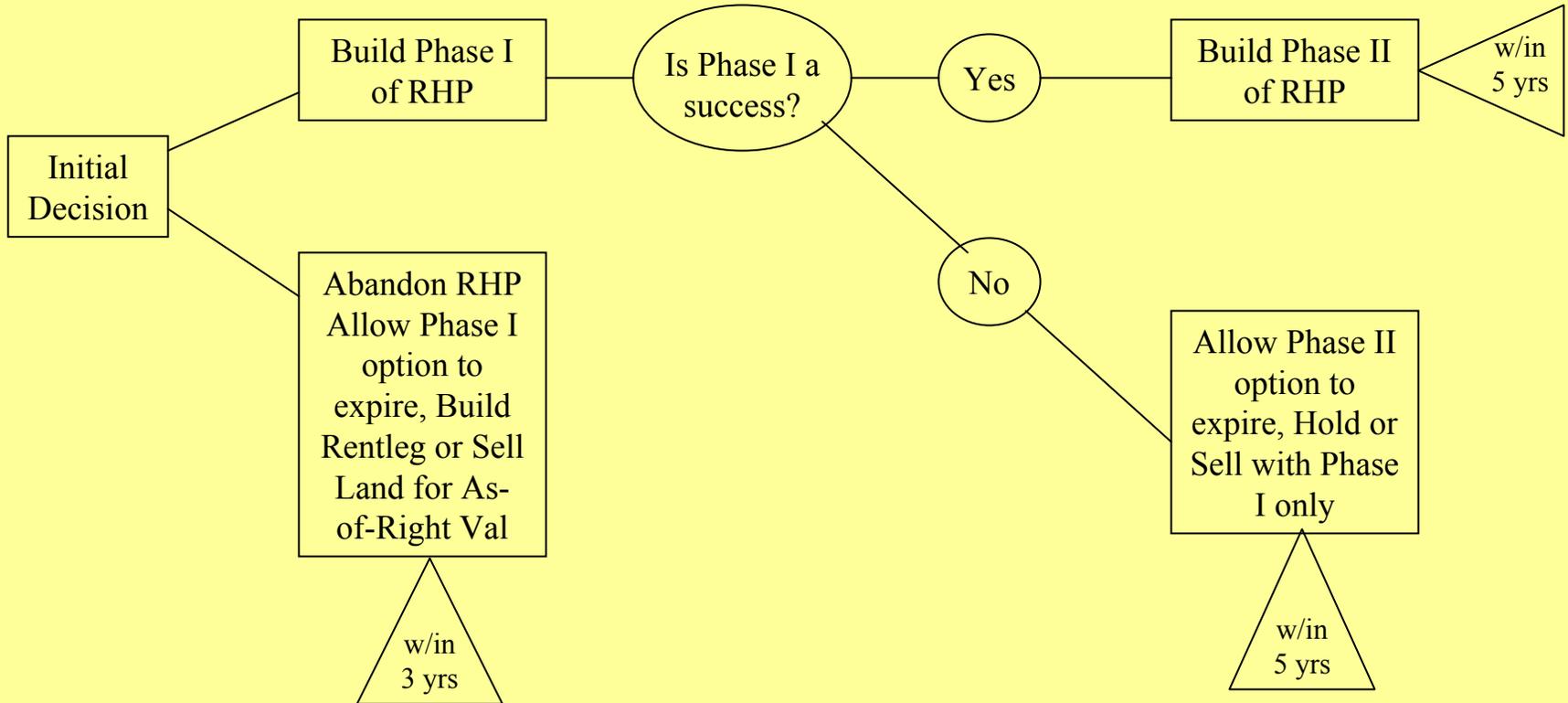
The PC would approve a *Special Zoning Exemption* that would allow *much greater density*, in a *two phase* development to be called "*Roth Harbor Place*" (RHP). In return, the landowner would commit to:

- 1. Provide mixed-income housing (approximately 25% of units below-mkt rent).**
- 2. Start construction on Phase I ("*Frenchman Cove*") no later than 3 years from now.**
- 3. Start construction on Phase II ("*Fisher Landing*") no later than 5 years from now.**

Phase II cannot be started until Phase I is complete, and if Phase I is not started within 3 years the special exemption expires and the land reverts to its previous as-of-right based value based on a project like Rentleg Gardens.

Roth Harbor . . .

Here is a *decision tree* representation of the RHP Project:



Roth Harbor . . .

The economics of the Roth Harbor Place proposal are as follows:

- **Phase I (Frenchman Cove):**
 - 900 units, At today's rents NOI = \$4,800,000/yr
 - At today's cap rate of $y_V = 8\%$, →
 - Current value $V_0 = \$60,000,000$
 - Construction cost as of today would be $K_0 = \$48,000,000$ and time-to-build is 2 years.
- **Phase II (Fisher Landing):**
 - 1600 units, At today's rents NOI = \$8,000,000/yr
 - At today's cap rate of $y_V = 8\%$, →
 - Current value $V_0 = \$100,000,000$
 - Construction cost as of today would be $K_0 = \$80,000,000$ and time-to-build is 2 years.
- **In both cases (as also with Rentleg Gardens):**
 - Market OCC for stabilized apartments (plus small lease-up risk premium) = 9%/yr, a 5% risk premium over $r_f = 4\%$.
 - Growth in construction costs $g_K = \text{infla}/\text{yr} = 2\%/\text{yr}$ (riskless).
 - Volatility of built property = 15%/yr.

Roth Harbor . . .

Important questions:

- **What is the current value of the Roth Harbor site based on its current as-of-right development option as typified by the Rentleg Gardens project? . . . This question is important because:**
 - **This value suggests how much the city might have to pay the current landowner to take over the site if they don't want to work together to create the RHP Project or if the city feels the project should be put out to an open bid.**
 - **This is the opportunity cost for the RHP Project, necessary to compute the NPV of the project.**
 - **Future values of this as-of-right asset represent the “abandonment value” of the RHP Project (if the developer decides not to pursue the project).**
 - **If the as-of-right project currently equals or exceeds its Samuelson-McKean “*hurdle value*” (V^*), this suggests urgency in getting the alternative RHP Project to supercede Rentleg Gardens, as the landowner should optimally immediately proceed with the as-of-right development in the absence of the special zoning exemption.**

Roth Harbor . . .

Important questions (*cont.*):

- **What is the value of the proposed special zoning exemption for the 2-phase RHP Project? . . . This question is important because:**
 - **It suggests how much someone (either the current landowner, or another entity via an open bid process if the city takes over the site) could profitably bid (to make their $NPV = 0$) for the RHP Project (e.g., how much the city could sell the site for with the special zoning exemption if the city takes the land).**
 - **It allows computation of the additional value created by the special zoning for the RHP Project, necessary to compute the NPV of the project.**
- **Do the economics of the RHP project make it realistic for immediate start of Phase I construction? That is, will an immediate construction start be sufficiently profitable (optimal) so that a private sector developer would not delay starting the project? . . . This question is important because:**
 - **The city for political reasons wants the site developed soon.**

Roth Harbor . . .

Step 1: Compute the as-of-right land value.

- **Clearly a job for the Samuelson-McKean Formula;**
- **Based on the Rentleg Gardens Project as the underlying asset;**
- **With 1-yr time-to-build . . .**

$$y_V = 8\%, y_K = (1+r_f)/(1+g_K)-1 = 1.96\%, \sigma = 15\%, K_0 = \$32, V_0 = \$40,$$

$$\begin{aligned}\rightarrow \eta &= \{y_V - y_K + \sigma^2/2 + [(y_K - y_V - \sigma^2/2)^2 + 2y_K\sigma^2]^{1/2}\} / \sigma^2 \\ &= \{.08 - .0196 + .15^2/2 + [(.0196 - .08 - .15^2/2)^2 + 2(.0196).15^2]^{1/2}\} / .15^2 \\ &= 6.63.\end{aligned}$$

$$\begin{aligned}V^* &= K_0(1+g_K)/(1+r_f)[\eta/(\eta-1)] = \$31.38[6.63/(6.63-1)] \\ &= \$31.38(1.178) = \$36.96.\end{aligned}$$

Roth Harbor . . .

Step 1 (cont.): Computing the as-of-right land value . . .

$$LAND = \begin{cases} (V^* - K/(1 + y_K)) \left(\frac{V/(1 + y_V)}{V^*} \right)^{\eta}, & \text{if } V/(1 + y_V) \leq V^* \\ V/(1 + y_V) - K/(1 + y_K), & \text{otherwise} \end{cases}$$

In this case:

$$V/(1 + y_V) = \$40/1.08 = \$37.04 > \$36.96 = V^*$$

Therefore:

$$\begin{aligned} LAND &= V/(1 + y_V) - K/(1 + y_K) = 40/1.08 - 32/1.0196 \\ &= \$37.04 - \$31.38 = \$5.65 \text{ million} \end{aligned}$$

The as-of-right land value (based on Rentleg Gardens) is \$5.65 million.

Furthermore, Rentleg Gardens exceeds its hurdle and so is ripe for immediate development. → Urgency in dealing with CEC.

Roth Harbor . . .

Step 2: Develop the as-of-right land value binomial tree, to provide the “abandonment value” contingencies in the RHP Project analysis

This is done by applying the preceding Samuelson-McKean Formula to $V_{i,j}$ and K_j in each i, j of a binomial tree.

The next slide shows binomial trees for seven years of (annual) projections of V , K , and $LAND$ based on Rentleg Gardens ($T/n = 1$ year).

Note: Use of such a small n (long period) will tend to bias the option valuation downward. In real world analysis a shorter period length (such as monthly rather than annual) would be preferable. Annual periods are used here for clarity of presentation.

Roth Harbor . . .

Year ("j"):	0	1	2	3	4	5	6	7
<i>Rentleg Gardens Expected Values:</i>		\$40.37	\$40.74	\$41.12	\$41.50	\$41.89	\$42.27	\$42.67
<i>'down' moves ("i"): Rentleg Gardens Value Tree (as if new, ex dividend):</i>								
0	40.00	42.59	45.35	48.29	51.42	54.76	58.30	62.08
1		32.21	34.29	36.52	38.88	41.40	44.09	46.94
2			25.93	27.61	29.40	31.31	33.34	35.50
3				20.88	22.23	23.67	25.21	26.84
4					16.81	17.90	19.06	20.30
5						13.53	14.41	15.35
6							10.90	11.60
7								8.77

Year ("j"):	0	1	2	3	4	5	6	7
<i>'down' moves ("i"): Rentleg Construction Cost Tree :</i>								
0	32.00	32.64	33.29	33.96	34.64	35.33	36.04	36.76
1		32.64	33.29	33.96	34.64	35.33	36.04	36.76
2			33.29	33.96	34.64	35.33	36.04	36.76
3				33.96	34.64	35.33	36.04	36.76
4					34.64	35.33	36.04	36.76
5						35.33	36.04	36.76
6							36.04	36.76
7								36.76

Year ("j"):	0	1	2	3	4	5	6
<i>'down' moves ("i"): Rentleg Land Value Tree (Samuelson-McKean, reflecting 1 yr time-to-build):</i>							
0	5.65	7.43	9.34	11.41	13.64	16.05	18.64
1		1.20	1.63	2.21	3.00	4.07	5.52
2			0.26	0.35	0.47	0.64	0.86
3				0.05	0.07	0.10	0.14
4					0.01	0.02	0.02
5						0.00	0.00
6							0.00

Roth Harbor . . .

Step 3: Compute the value of the option to build Phase II (*Fisher Landing*), by:

- **First build the Fisher Landing underlying asset value tree forward in time, through Year 7 (the latest it can be obtained, if the option is exercised at its expiration time in Year 5, given the 2-year time-to-build requirement), starting from time 0 where $V_0 = \$100\text{M}$.**
- **Build the corresponding Fisher Landing construction cost tree forward in time, through Year 7, starting from $K_0 = \$80$ million.**
- **Then build the call option value tree through Year 5 (option expiration), working backwards from Year 5 to time 0. Option exercise gets completed Fisher Landing 2 yrs after exercise.**

Note: The Fisher Landing option cannot be obtained prior to Year 2, the earliest possible completion date of Phase I. Thus, we won't end up using the Year 0 and Year 1 values of this option, but we might as well calculate them anyway.

The next slide shows these three trees.

Roth Harbor . . .

Year ("j"):	0	1	2	3	4	5	6	7
<i>Fisher Landing Expected Values:</i>	\$100.93	\$101.86	\$102.80	\$103.76	\$104.72	\$105.69	\$106.66	
<i>'down' moves ("i"): Fisher Landing Underlying Asset Value Tree (as if new, ex dividend):</i>								
0	100.00	106.48	113.38	120.73	128.56	136.89	145.76	155.21
1		80.52	85.73	91.29	97.21	103.51	110.22	117.36
2			64.83	69.03	73.50	78.27	83.34	88.74
3				52.20	55.58	59.18	63.02	67.10
4					42.03	44.75	47.65	50.74
5						33.84	36.03	38.37
6							27.24	29.01
7								21.94

Year ("j"):	0	1	2	3	4	5	6	7
<i>'down' moves ("i"): Fisher Landing Construction Cost Tree:</i>								
0	80.00	81.60	83.23	84.90	86.59	88.33	90.09	91.89
1		81.60	83.23	84.90	86.59	88.33	90.09	91.89
2			83.23	84.90	86.59	88.33	90.09	91.89
3				84.90	86.59	88.33	90.09	91.89
4					86.59	88.33	90.09	91.89
5						88.33	90.09	91.89
6							90.09	91.89
7								91.89

<i>Value of Option on Phase II (Fisher Landing), reflecting 2-yr time-to-build:</i>						
Year ("j"):	0	1	2	3	4	5
<i>'down' moves ("i"):</i>						Opt Expires
0	8.78	12.80	17.15	21.85	26.92	32.40
1		0.44	0.75	1.29	2.21	3.78
2			0.00	0.00	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

Roth Harbor . . .

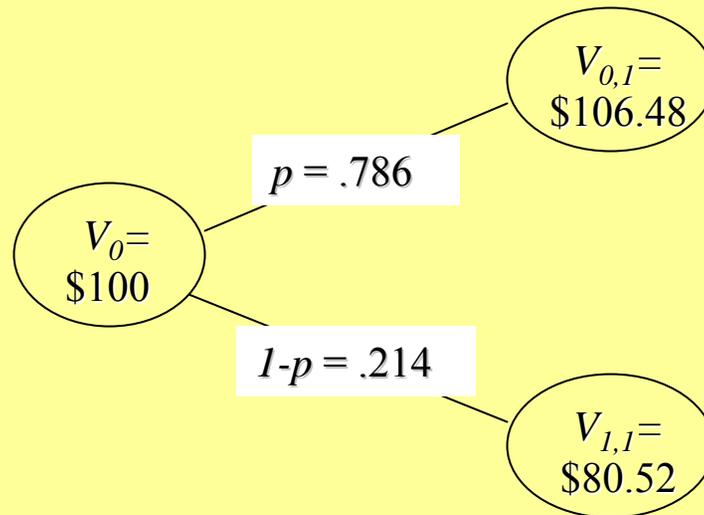
For example . . .

$$V_{0,1} = uV_{0,0}/(1 + y_V) = (1 + \sigma\sqrt{T/n})V_t/(1 + y_V) = (1.15)\$100/(1.08) = \$106.48$$

$$V_{1,1} = dV_{0,0}/(1 + y_V) = (1/u)V_{0,0}/(1 + y_V) = (1/1.15)\$100/(1.08) = \$80.52$$

$$K_1 = (1 + r_f)K_0/(1 + y_K) = (1 + g_K)K_0 = (1.02)\$80 = \$81.60$$

$$p = \frac{(1 + r_V) - 1/(1 + \sigma\sqrt{T/n})}{(1 + \sigma\sqrt{T/n}) - 1/(1 + \sigma\sqrt{T/n})} = \frac{1.09 - 1/1.15}{1.15 - 1/1.15} = 0.786$$



Roth Harbor . . .

And the option value is given by:

$$C_{i,j} = \text{Max} \left\{ \frac{V_{i,j}}{(1+y_V)^2} - \frac{K_{i,j}}{(1+y_K)^2}, \frac{(pC_{i,j+1} + (1-p)C_{i+1,j+1}) - (C_{i,j+1} - C_{i+1,j+1}) \left[\frac{r_V - r_f}{(1+\sigma\sqrt{T/n}) - 1 / (1+\sigma\sqrt{T/n})} \right]}{1+r_f} \right\}$$

For example:

$$C_{0,0} = \text{Max} \left\{ \frac{100}{(1.08)^2} - \frac{80}{(1.0196)^2}, \frac{((.786)12.80 + (.214)0.44) - (12.80 - 0.44) \left[\frac{9\% - 4\%}{(1.15) - 1 / (1.15)} \right]}{1.04} \right\}$$

$$= \text{Max} \{ \$8.78, \$7.65 \} = \$8.78$$

Roth Harbor . . .

Step 4: Compute the value of the compound option to build Phase I, which obtains both *Frenchman Cove* and also the option to build *Fisher Landing*, by:

- **First step the Phase II option value we just calculated back two periods in time, to obtain its present value in each period as a part of the underlying asset for the Phase I compound option, reflecting the Phase I time-to-build. (i.e., if you exercise the Phase I option, you will get the Phase II option only after a 2-yr lag. We want to know the PV of the Phase II option as of the time when the Phase I option may be exercised.)**
- **Then develop the other part of the Phase I option's underlying asset value by building the Frenchman Cove value tree forward in time, through Year 5 (the last year it could be obtained, as the Phase I option expires in Year 3 and the project takes 2 years to build), starting from $V_0 = \$60$ million.**
- **Build the corresponding Frenchman Cove construction cost tree forward in time, through Year 5, starting from $K_0 = \$48$ million.**
- **Finally build the Phase I compound call option value tree working backwards in time from Year 3 (option expiration) to time 0. Option exercise gets completed Frenchman Cove + Phase II option, both 2 yrs after exercise. Abandonment for as-of-right land value is always an alternative to either holding or exercising the Phase I option.**

Roth Harbor . . .

PV of 1 period delayed receipt of Phase 2 option value:					
Year ("j"):	0	1	2	3	4
'down" moves ("i"):					
0	7.65	10.30	13.25	16.57	20.36
1		0.44	0.75	1.29	2.21
2			0.00	0.00	0.00
3				0.00	0.00
4					0.00

PV of 2 period delayed receipt of Phase 2 option value:				
Year ("j"):	0	1	2	3
'down" moves ("i"):				
0	6.19	8.03	10.17	12.73
1		1.20	1.63	2.21
2			0.26	0.35
3				0.05

Here we are simply applying the certainty-equivalence valuation formula 1 period at a time to the Phase II option value (previously calculated).

For example . . .

Roth Harbor . . .

PV in the 0,1 node of Year 1 of the Phase II option in Year 2 is:

$$PV_{0,1}[C_2] = \frac{((.786)17.15 + (.214)0.75) - (17.15 - 0.75) \left[\frac{9\% - 4\%}{(1.15) - 1/(1.15)} \right]}{1.04} = \$10.30$$

PV in the 0,0 node of Year 0 (the present) of the Phase II option in Year 2 is:

$$PV_0[C_2] = PV_{0,0}[PV_1[C_2]] = \frac{((.786)10.30 + (.214)0.44) - (10.30 - 0.44) \left[\frac{9\% - 4\%}{(1.15) - 1/(1.15)} \right]}{1.04} = \$6.19$$

This PV_t of the Phase II option as of time t is part of what one obtains in time t by exercising the Phase I option in time t .

The other part of what one obtains is the PV_t as of time t of the Frenchman Cove development project (which would be completed 2 years later):

$$PV_t[V_{t+2} - K_{t+2}] = \frac{E_t[V_{t+2}]}{(1 + r_V)^2} - \frac{K_{t+2}}{(1 + r_f)^2} = \frac{V_t}{(1 + y_V)^2} - \frac{K_t}{(1 + y_K)^2}$$

Roth Harbor . . .

Here are the Frenchman Cove underlying asset value and construction cost trees, obtained in the usual manner:

Year ("j"):	0	1	2	3	4	5
<i>Frenchman Cove Expected Values:</i>		\$60.56	\$61.12	\$61.68	\$62.25	\$62.83
<i>'down' moves ("i"): Frenchman Cove Underlying Asset Value Tree (as if new, ex-dividend):</i>						
0	60.00	63.89	68.03	72.44	77.13	82.13
1		48.31	51.44	54.77	58.32	62.10
2			38.90	41.42	44.10	46.96
3				31.32	33.35	35.51
4					25.22	26.85
5						20.30

Year ("j"):	0	1	2	3	4	5
<i>'down' moves ("i"): Frenchman Cove Construction Cost Tree:</i>						
0	48.00	48.96	49.94	50.94	51.96	53.00
1		48.96	49.94	50.94	51.96	53.00
2			49.94	50.94	51.96	53.00
3				50.94	51.96	53.00
4					51.96	53.00
5						53.00

Roth Harbor . . .

The value of the Phase I option is then calculated working backwards in time from its expiration in Year 3.

The value in Year 3 is the maximum of either: (i) the as-of-right land value (land value based on the Rentleg Gardens Project, which is the “abandonment value” of the RHP Project); or (ii) the value of immediate exercise of the Phase I option (which obtains the completed Frenchman Cove Project plus the Phase II option, both 2 years later):

$$C_3 = \text{Max} \{ \text{As-of-right Land Value}_3, PV_3[V_5 - K_5] + PV_3[\text{Ph.II Opt}_5] \}$$

The value in any earlier year is the current value of the maximum of either of the above two alternatives (i) and (ii) plus the third alternative of holding the “live” option unexercised for at least one more year:

$$C_t = \text{Max} \{ \text{As-of-right Land Value}_t, PV_t[V_{t+2} - K_{t+2}] + PV_t[\text{Ph.II Opt}_{t+2}], \\ PV_t[C_{t+1}] \}$$

Roth Harbor . . .

Here is the Phase I option value tree:

<i>Value of Option on Phase I (Frenchman Cove), reflecting 2-yr time-to-build:</i>				
Year ("j"):	0	1	2	3
'down" moves ("i"):	Opt Expires			
0	11.46	15.71	20.45	25.84
1		1.20	1.63	2.21
2			0.26	0.35
3				0.05

The Phase I option is worth \$11.46 million:

$$C_{0,0} = \text{Max} \left\{ 5.65, \frac{60}{(1.08)^2} - \frac{48}{(1.0196)^2} + 6.19, \frac{((.786)15.71 + (.214)1.20) - (15.71 - 1.20) \left[\frac{9\% - 4\%}{(1.15) - 1/(1.15)} \right]}{1.04} \right\}$$

$$= \text{Max} \{ \$5.65, \$11.46, \$9.63 \} = \$11.46$$

This \$11.46 million is in fact the value of the Roth Harbor land with the special zoning exemption, the value of the RHP Project option.

All of these calculations are available in a downloadable Excel file.

Roth Harbor . . .

Note that since the \$11.46 million valuation of the Phase I option is in fact the value of immediate exercise of the option, our model of option value is telling us that it is in fact optimal for the landowner to immediately begin construction on Phase I of the RHP Project.

Our real options analysis has therefore now allowed us to answer rigorously all three of the important questions that we set out to answer:

- 1. The current value of the site based on its pre-existing as-of-right development option (Rentleg Gardens) is **\$5.65 million**.**
- 2. The value of the site with the proposed special zoning exemption for the 2-phase RHP Project is **\$11.46 million**. The incremental value added over the pre-existing land value by the special zoning for the RHP Project is: $\$11.46 - \$5.65 = \mathbf{\$5.81 \text{ million}}$.**
- 3. The RHP Project (like the Rentleg Gardens Project) is “ripe” for **IMMEDIATE DEVELOPMENT**.**

Roth Harbor . . .

The preceding analysis included in a rigorous manner both the opportunity cost of capital (OCC) and the value of flexibility and phasing in the possible development projects.

The analysis allows us to say:

- 1. A fair price for a “taking” of the Roth Harbor site based on its pre-existing rights would be \$5.65 million.**
- 2. A fair bid for the site with the proposed special zoning exemption for the two-phase RHP Project would be \$11.46 million.**
- 3. Our best guess of the NPV of the special zoning exemption is \$5.81 million.**
- 4. A recipient of the site with the special zoning exemption would likely seek to immediately begin construction on Phase I.**
- 5. There is some urgency in closing an agreement with the current landowner, as the as-of-right project is also ripe for immediate development (the current owner is suffering an opportunity cost by not proceeding with that development).**

Roth Harbor . . .

Here is the Phase I option optimal exercise tree*:

Year ("j"):	0	1	2	3
'down" moves ("i"): <i>1st Phase Optimal Exercise</i> :				
0	exer	exer	exer	exer
1		hold	hold	sell
2			hold	sell
3				sell

Corresponding to the following Frenchman Cove value contingencies:

Year ("j"):	0	1	2	3
<i>Frenchman Cove Expected Values:</i>		\$60.56	\$61.12	\$61.68
'down" moves ("i"): <i>Frenchman Cove Underlying Asset Value Tree</i> (as if				
0	60.00	63.89	68.03	72.44
1		48.31	51.44	54.77
2			38.90	41.42
3				31.32

Which have the following probabilities of occurrence as of time 0:

Year ("j"):	0	1	2	3
'down" moves ("i"): <i>Contingency Probabilities:</i>				
0	100.0%	78.6%	61.8%	48.6%
1		21.4%	33.6%	39.7%
2			4.6%	10.8%
3				1.0%

Roth Harbor . . .

Here is the Phase I option opportunity cost of capital tree:

Year ("j"):	0	1	2	3
'down" moves ("i"): Roth Harbor OCC:				
0	30.85%	30.71%	30.44%	24.84%
1		27.37%	27.37%	27.37%
2			27.37%	27.37%
3				27.37%

Based on the following formula introduced earlier:

$$C_{i,j} = \frac{CEQ[C_{j+1}]}{1+r_f} = \frac{E_j[C_{j+1}]}{1+r_f + E[RP_{C_{i,j}}]} = \frac{E_j[C_{j+1}]}{1+OCC_{i,j}}, \Rightarrow 1+OCC_{i,j} = (1+r_f) \frac{E_j[C_{j+1}]}{CEQ[C_{j+1}]}$$

For example, for the initial OCC:

$$\begin{aligned}
 1+OCC_{0,0} &= (1+r_f) \frac{E_0[C_1]}{CEQ_0[C_1]} = \\
 &= 1.04 \frac{(.786)15.71 + (.214)1.20}{((.786)15.71 + (.214)1.20) - (15.71 - 1.20) \left[\frac{9\% - 4\%}{(1.15) - 1/(1.15)} \right]} = 1.04 \frac{\$12.60}{\$10.02} = 1.3085
 \end{aligned}$$

Roth Harbor . . .

Thus, the RHP Project currently has over *5 times* the amount of investment risk (as evaluated by the capital market) as does an unlevered investment in a completed apartment property, as indicated by:

$$\frac{E[RP_C]}{E[RP_V]} = \frac{30.85\% - 4\%}{9\% - 4\%} = \frac{26.85\%}{5.00\%} = 5.37$$

Roth Harbor . . .

How would the RHP Project be evaluated according to typical current practice in the real estate investment business? . . .

Typically, a conventional DCF analysis would be applied:

- **Ignoring the flexibility (but not ignoring the expected phasing) in the project;**
- **Using a non-rigorous ad hoc OCC as the discount rate.**

Thus:

- **A particular phasing scenario is assumed (e.g., *each phase will begin as soon as possible*);**
- **A particular discount rate is assumed (e.g., “20%”, *because it’s a nice round number and consistent with the “conventional wisdom” for required returns on development projects.*)**

Let’s see how this might be done for our RHP Project example . . .

Roth Harbor . . .

If both Phase I and Phase II are built as soon as possible, the net cash flow from the liquidation of each phase would be obtained in Years 2 and 4 respectively.

Using the projected values of $E[V_2]$ and K_2 , and of $E[V_4]$ and K_4 , in years 2 and 4 for Frenchman Cove and Fisher Landing respectively, we get the following net cash flow projection for the RHP Project as a whole:

Year:	1	2	3	4
Net Cash:	0	\$61.12 – \$49.94 = \$11.18	0	\$103.76 – \$86.59 = \$17.16

Discounting these cash flows @ 20% gives a gross PV for the RHP Project of:

$$PV = \frac{\$11.18}{1.20^2} + \frac{\$17.16}{1.20^4} = \$16.04$$

Thus, the conventional procedure would suggest a bid price of **\$16.04** million for the RHP site with the special zoning exemption. As compared to **\$11.46** million using the real options approach.

Roth Harbor . . .

In this example the conventional approach has substantially over-estimated the more rigorously estimated value, due primarily to*:

- **Using a discount rate that is too small (20% vs OCC = 30.85%).**
- **Assuming the most optimistic project schedule (both phases implemented, and each as soon as possible).**

While in the above example the conventional approach over-values the development project, in general, the conventional approach may either over- or under-estimate the project value, relative to the real options valuation . . .

Our point is not that the conventional approach is systematically biased.

Summarizing Real Options vs Conventional DCF . . .

While a systematic bias (in one direction) in the conventional approach may exist, . . .

Well functioning investment markets for land, built properties, and real estate debt instruments, should cause the conventional practice to tend to get valuation about right on average.

Otherwise opportunities for “*super-normal*” profit (excess investment returns) would be widespread.

The fact that our real options model is based fundamentally on the elimination of super-normal (“arbitrage”) profit, suggests that the options approach and the conventional approach should tend to agree on average (across projects and over time).

Summarizing Real Options vs Conventional DCF . . .

The key difference between real options vs conventional valuation, however, is that:

- **The options approach is more *rigorous*, providing a valuation that is less error-prone, more likely to be “more correct” in more individual instances (even if not on average across all projects). And through this rigor, . . .**
- **The options approach provides a deeper understanding of: (i) the sources of the project value; and (ii) the true nature of the project investment risk and return;**
- **The conventional approach is based on *ad hoc* assumptions regarding project execution and OCC, ignoring important realities of the project such as its flexibility.**
- **Even if the conventional approach gives a correct answer in a given case (i.e., the same valuation as the options approach), there is no way in itself to know *whether* the valuation is correct, or *why* it is correct if it is correct (except by basing it on the more rigorous options approach).**

Summarizing Real Options vs Conventional DCF . . .

The previous conventional DCF valuation approach used net cash flows and an ad hoc discount rate.

An alternative approach to applying conventional DCF valuation uses gross cash flows and rigorous OCC discount rates:

$$PV = PV[V_2^I - K_2^I] + PV[V_4^II - K_4^II]$$

$$= \left(\frac{E[V_2^I]}{(1+r_v)^2} - \frac{K_2^I}{(1+r_f)^2} \right) + \left(\frac{E[V_4^II]}{(1+r_v)^4} - \frac{K_4^II}{(1+r_f)^4} \right) = \left(\frac{V_0^I}{(1+y_v)^2} - \frac{K_0^II}{(1+y_k)^2} \right) + \left(\frac{V_0^I}{(1+y_v)^4} - \frac{K_0^II}{(1+y_k)^4} \right)$$

Year:	1	2	3	4
Gross Cash:	0	$E_0[V_2] = \$61.12$ $K_2 = \$49.94$	0	$E_0[V_4] = \$103.76$ $K_4 = \$86.59$

$$PV = \left(\frac{\$61.12}{1.09^2} - \frac{\$49.94}{1.04^2} \right) + \left(\frac{\$103.76}{1.09^4} - \frac{\$86.59}{1.04^4} \right) = \$5.27 - \$0.52 = \$4.75$$

Although the OCCs and cash flow projections used in this procedure are well justified, this procedure implicitly assumes an irreversible commitment at time 0 to complete the entire project as scheduled. It thus ignores the flexibility that actually exists, and thereby systematically under-estimates the project present value (in this case: \$4.75 million versus \$11.46 million).

Back to the “Big Picture”: Types of Development Options

We have now presented an in-depth and practical methodology for addressing two of the common types of options found in development projects:

- “Wait Option”: The option to delay start of the project construction;
- “Phasing Option”: The breaking of the project into sequential phases rather than building it all at once;

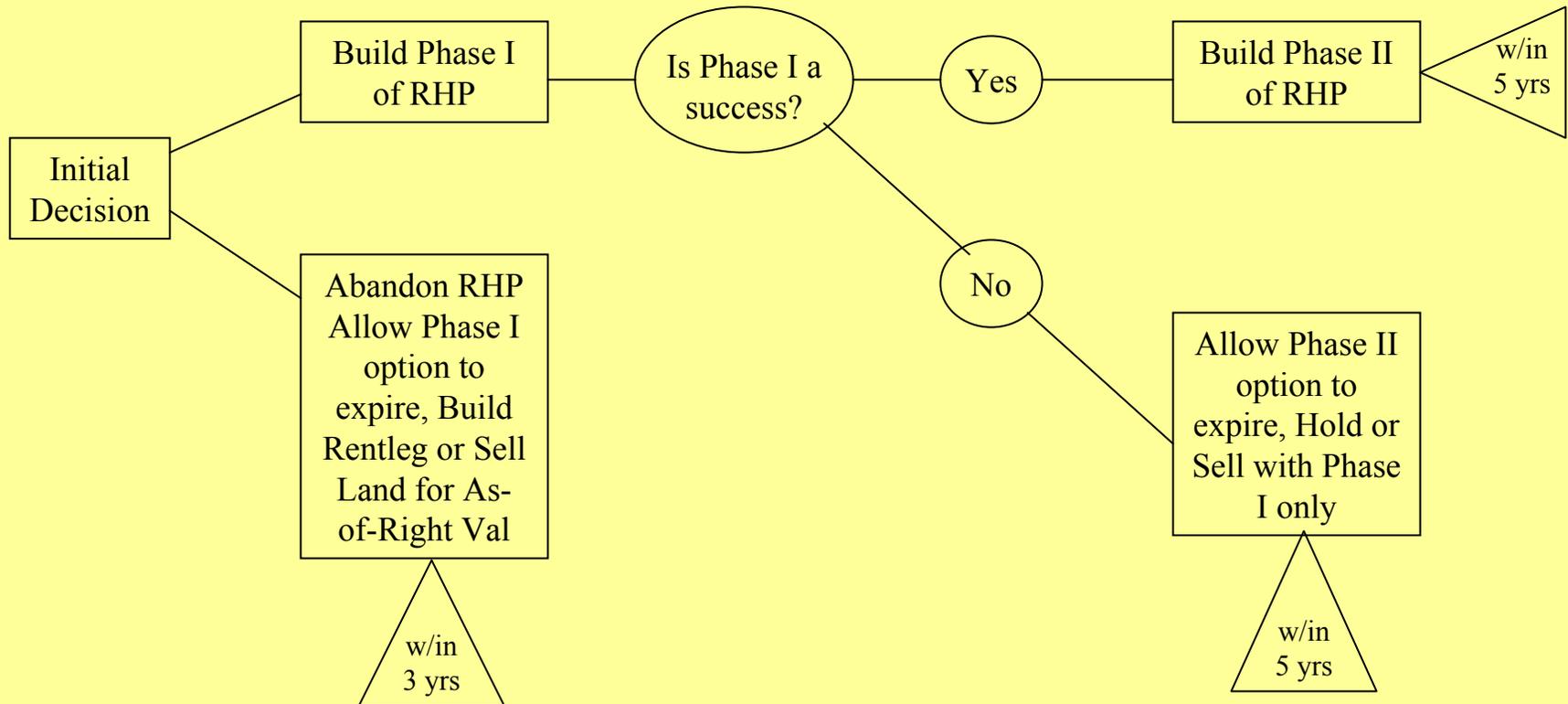
The third type:

- “Switch Option”: The option to choose among alternative types of buildings to construct on the given land parcel.

Requires much more advanced technical capability. Suffice it to say at this point that: *optionality* (rights without corresponding obligations) never reduces value.

The “Big Picture”: Project Design “Architecture”

Step back and look at the bigger picture of the Roth Harbor Place Project. Recall our *decision tree* representation of the RHP Project...



Project design “architecture” refers to how this overall structure of the project is designed . . .

The “Big Picture”: Project Design “Architecture”

Project design “architecture” refers to how the overall structure of the project is designed. For the RHP Project:

- **Why were there 2 phases, not 3, or 1, or 4? . . .**
- **Why did the options have to expire after 3 and 5 years? . . .**
- **Why 900 units in Phase I and 1600 in Phase II? . . .**
- **Why a total of 2500 units, not 3000 or 2000? . . .**

These overarching design questions can have a huge impact on project value.

As yet we have no comprehensive, systematic and rigorous method of optimizing project design architecture.*

The real options valuation theory presented in this lecture does not solve this problem.

But it does provide a framework and metric by which mutually exclusive alternative project architectures can be evaluated and rank-ordered, by which the best architecture can be selected from among alternatives... →

The architecture with the highest net value based on its real options valuation is the best architecture.