

Chapter 14:

After-Tax Investment Analysis

***Going from the “property before-tax cash flows” (PBTCF),
to the “equity after-tax cash flows” (EATCF). . .***

1) Property level (PBTCF):

- Net CF produced by property, before subtracting debt svc pmts (DS) and inc. taxes.
- CFs to Govt, Debt investors (mortgagees), equity owners.
- CFs due purely to underlying productive physical asset, not based on financing or income tax effects.
- Relatively easy to observe empirically.

2) Equity ownership after-tax level (EATCF):

- Net CF avail. to equity owner after DS & taxes.
- Determines value of equity only (not value to lenders).
- Sensitive to financing and income tax effects.
- Usually difficult to observe empirically (differs across investors).

3 MAJOR DIFFERENCES between PBTCF & EATCF levels:

- Depreciation: An expense that reduces income tax cash outflows, but not itself a cash outflow at the before-tax level. (IRS income tax rules for property income based on accrual accounting, not cash flow accounting.)
- Capital expenditures: Not an accrual “expense” (because adds to asset value, “asset” → life > 1 yr), hence not deducted from taxable income, even though they are a cash outflow.
- Debt principal amortization: Like capex, a cash outflow, but not deductible from taxable income.

Exhibit 14-1a: Equity After-Tax Cash Flows from Operations

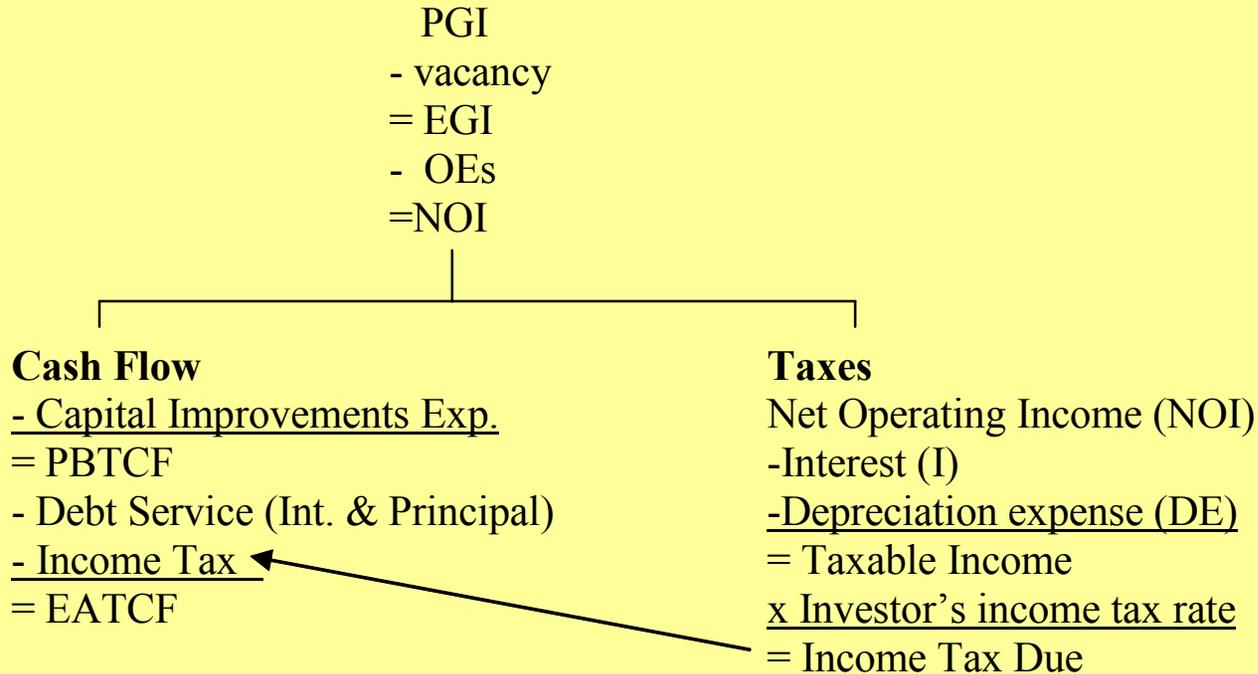


Exhibit 14-1b: Computation of CGT in Reversion Cash Flow

$$\begin{array}{r} \text{Net Sale Proceeds (NSP)} \\ - \text{ Adjusted Basis} \\ = \text{ Taxable Gain on Sale} \\ \times \text{ CGT Rate} \\ \hline = \text{ Taxes Due on Sale} \end{array}$$

where the *Adjusted Basis* or Net Book Value is calculated as:

$$\begin{array}{r} \text{Original Basis (Total Initial Cost)} \\ + \text{ Capital Improvement Expenditures} \\ - \text{ Accumulated Depreciation} \\ \hline = \text{ Adjusted Basis} \end{array}$$

Another perspective:

From PBTCF to EATCF. . .

Operating:

$$\begin{array}{r}
 \text{PBTCF} \\
 - \text{ DS} \leftarrow \left\{ \begin{array}{l} \text{IE} \\ +\text{PP} \end{array} \right. \\
 \hline
 \text{EBTCF} \\
 - \text{ tax} \leftarrow \left\{ \begin{array}{l} \tau(\text{NOI}) \\ -\tau(\text{DE}) \leftarrow \text{"Tax Shield" (DTS)} \\ -\tau(\text{IE}) \leftarrow \text{"Tax Shield" (ITS) offset} \end{array} \right. \\
 \hline
 \text{EATCF}
 \end{array}$$

by tax expense to lender

Reversion:

$$\begin{array}{r}
 \text{PBTCF} \\
 - \text{ OLB} \\
 \hline
 \text{EBTCF} \\
 - \text{ CGT} = \tau_G[V_T - \text{SE} - (V_0 + \text{AccCI})] + \tau_R(\text{AccDE}) \\
 \hline
 \text{EATCF}
 \end{array}$$

Depreciation Expense:

- Straight-line
 - 39 years, commercial
 - 27.5 years, residential (apts)
- Land not depreciable:
 - (typic. 20% in Midwest, South)
 - (often 50% in big E. & W. Coast cities)

Exhibit 14-2

Exhibit 14-2: Example After-Tax Income & Cash Flow Proformas . . .

Property Purchase Price (Year 0):	\$1,000,000	Unlevered:	Levered:
Depreciable Cost Basis:	\$800,000	Before-tax IRR:	6.04%
Ordinary Income Tax Rate:	35.00%	After-tax IRR:	4.34%
Capital Gains Tax Rate:	15.00%	Ratio AT/BT:	0.719
Depreciation Recapture:	25.00%		0.870

Operating:	Year:	1	2	3	4	5	6	7	8	9	Oper. Yr.10	Reversion Item:	Rever. Yr.10	Total Yr.10
Accrual Items:														
NOI		\$60,000	\$60,600	\$61,206	\$61,818	\$62,436	\$63,061	\$63,691	\$64,328	\$64,971	\$65,621	Sale Price	\$1,104,622	
- Depr.Exp.		\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	- Book Val	\$809,091	
- Int.Exp.		\$41,250	\$41,140	\$41,030	\$40,920	\$40,810	\$40,700	\$40,590	\$40,480	\$40,370	\$40,260			
=Net Income (BT)		(\$10,341)	(\$9,631)	(\$8,915)	(\$8,193)	(\$7,465)	(\$6,730)	(\$5,990)	(\$5,243)	(\$4,490)	(\$3,730)	=Book Gain	\$295,531	\$291,801
- IncTax		(\$3,619)	(\$3,371)	(\$3,120)	(\$2,867)	(\$2,613)	(\$2,356)	(\$2,096)	(\$1,835)	(\$1,571)	(\$1,305)	- CGT	\$73,421	
=Net Income (AT)		(\$6,722)	(\$6,260)	(\$5,795)	(\$5,325)	(\$4,852)	(\$4,375)	(\$3,893)	(\$3,408)	(\$2,918)	(\$2,424)	=Gain (AT)	\$222,111	\$219,686

Adjusting Accrual to Reflect Cash Flow:

- Cap. Imprv. Expdtr.		\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0			
+ Depr.Exp.		\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	+ Book Val	\$809,091	
-DebtAmort		\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	-LoanBal	\$730,000	
=EATCF		\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	=EATCF	\$301,202	\$325,868
+ IncTax		(\$3,619)	(\$3,371)	(\$3,120)	(\$2,867)	(\$2,613)	(\$2,356)	(\$2,096)	(\$1,835)	(\$1,571)	(\$1,305)	+ CGT	\$73,421	
=EBTCF		\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF	\$374,622	\$397,983

CASH FLOW COMPONENTS FORMAT

Operating:	Year:	1	2	3	4	5	6	7	8	9	Oper. Yr.10	Reversion Item	Rever. Yr.10	Total Yr.10
Accrual Items:														
NOI		\$60,000	\$60,600	\$61,206	\$61,818	\$62,436	\$63,061	\$63,691	\$64,328	\$64,971	\$65,621	Sale Price	\$1,104,622	
- Cap. Imprv. Expdtr.		\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0			
=PBTFCF		\$60,000	\$60,600	\$11,206	\$61,818	\$62,436	\$63,061	\$63,691	\$14,328	\$64,971	\$65,621	=PBTFCF	\$1,104,622	\$1,170,243
- Debt Svc		\$43,250	\$43,140	\$43,030	\$42,920	\$42,810	\$42,700	\$42,590	\$42,480	\$42,370	\$42,260	- LoanBal	\$730,000	
=EBTCF		\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF	\$374,622	\$397,983
-taxNOI		\$21,000	\$21,210	\$21,422	\$21,636	\$21,853	\$22,071	\$22,292	\$22,515	\$22,740	\$22,967	taxMktGain	\$693	\$23,661
+ DTS		\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	- AccDTS	(\$72,727)	(\$62,545)
+ ITS		\$14,438	\$14,399	\$14,361	\$14,322	\$14,284	\$14,245	\$14,207	\$14,168	\$14,130	\$14,091			\$14,091
=EATCF		\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	EATCF	\$301,202	\$325,868

Year 1 projection, Operating Cash Flow (details):

NOI	= \$60,000, 1 st yr.
- Depr.Exp.	= \$800,000/27.5 = \$29,091, ea. yr.
<u>- Int.Exp.</u>	= <u>\$750,000*5.5%</u> = \$41,250, 1 st yr.
=Net Income (BT)	= 60000 - 29091 - 41250 = -\$10,341.
<u>- IncTax</u>	= <u>(.35)(-10341)</u> = - \$3,619, 1 st yr.
=Net Income (AT)	= -10341 - (-3619) = - \$6,722, 1 st yr.
Adjusting Accrual to Reflect Cash Flow:	
- Cap. Imprv. Expdtr.	= - \$0, 1 st yr.
+ Depr.Exp.	= + \$29,091, ea. yr.
<u>-DebtAmort</u>	= <u>- \$2,000</u> , ea. yr (this loan).
=EATCF	= (-6722-0+29091-2000) = \$20,369, 1 st yr.
<u>+ IncTax</u>	= <u>+(-\$3,619)</u> = -\$3,619, 1 st y r.
=EBTCF	= 20369 - 3619 = \$16,750, 1 st yr.

Reversion Cash Flow, Year 10 (details):

Sale Price	$= V_T - SE$ $= NOI_{11}/.06 - SE = 1.01*\$65,621/0.06 - 0 = \$1,104,620$
<u>- Book Val</u>	$= - (V_0 + AccCI - AccDE)$ $= - (1000000 + 100000 - 290910) = - \$809,091$
=Book Gain	$= 1104620 - 809091 = \$295,531$ Inclu 1104620 - (1000000+100000) = 4620 Gain, + 290910 Recapture
<u>- CGT</u>	$= (.15)(4620) + (.25)(290910) = -\$73,421$
=Gain (AT)	$= 295531 - 73421 = \$222,111$
Adjusting Accrual to Reflect Cash Flow:	
+ Book Val	$= + \$809,091$
<u>-LoanBal</u>	$= - (750000 - 10*2000) = -\$730,000$
=EATCF	$= 222111 + 809091 - 730000 = \$301,202$
<u>+ CGT</u>	$= + \$73,421$
=EBTCF	$= 301202 + 73421 = \$374,622$

Cash Flow Components Format...

Exercise: For 2005 numbers, replace: \$90000 with \$60000; 10% with 5.5% & \$75000 with \$41250; \$1280085 with \$1104620; .4 with .35; .20 with .15, . . .

Operating:

PBTCF			$= \text{NOI} - \text{CI} = \$90,000 - \$0 = \$90,000, 1^{\text{st}} \text{ yr.}$
- DS ←----	{	IE	$= \$750,000 * 10\% = \$75,000, 1^{\text{st}} \text{ yr.}$
		+PP	$= + \$2,000 = \$77,000, 1^{\text{st}} \text{ yr.}$
EBTCF			$= \$90,000 - \$77,000 = \$13,000$

- tax ←----	{	$\tau(\text{NOI})$	$= - (.4)\$90,000 = \$36,000, 1^{\text{st}} \text{ yr.}$
		$-\tau(\text{DE}) \leftarrow \text{---}(\text{"DTS"})$	$= + (.4)\$29,091 = \$11,636, \text{ ea. yr.}$
		$-\tau(\text{IE}) \leftarrow \text{---}(\text{"ITS"})$	$= + (.4)\$75,000 = \$30,000, 1^{\text{st}} \text{ yr.}$
EATCF			$= \$13,000 - \$36,000 + \$11,636 + \$30,000$ $= \$18,636, 1^{\text{st}} \text{ yr.}$

Reversion (Yr.10):

PBTCF	$= 1.025 * \$112,398 / 0.09 = \$1,280,085.$
- OLB	$= \$750,000 - (10 * \$2,000) = \$730,000.$

EBTCF	$= 1280085 - 730000 = \$550,085$
- CGT Mkt Gain Component	$= \tau_G [V_T - \text{SE} - (V_0 + \text{AccCI})]$ $= - (0.20)(1280085 - 0 - (1000000 + 100000))$ $= - (0.20)(1280085 - 1100000) = \$36,017.$
- CGT DTS Recapture Comp.	$= - \tau_R (\text{AccDE})$ $= - (0.25)(\$290,910) = - \$72,728.$

EATCF	$= 550085 - (36017 + 72728)$ $= 550085 - 108744 = \$441,340.$
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You should get:
EATCF=\$20,369 (oper)
& \$301,202 (rever)...

14.2.5: Cash Flow Components:

Recall our previous apt property investment...

Exhibit 14-2: Example After-Tax Income & Cash Flow Proformas . . .

Property Purchase Price (Year 0):	\$1,000,000	Unlevered:	Levered:
Depreciable Cost Basis:	\$800,000	Before-tax IRR:	6.04%
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- Int.Exp.		\$41,250	\$41,140	\$41,030	\$40,920	\$40,810	\$40,700	\$40,590	\$40,480	\$40,370	\$40,260			
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Adjusting Accrual to Reflect Cash Flow:														
- Cap. Imprv. Expdtr.		\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0			
+ Depr.Exp.		\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	\$29,091	+ Book Val	\$809,091	
-DebtAmort		\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	\$2,000	-LoanBal	\$730,000	
=EATCF		\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	=EATCF	\$301,202	\$325,868
+ IncTax		(\$3,619)	(\$3,371)	(\$3,120)	(\$2,867)	(\$2,613)	(\$2,356)	(\$2,096)	(\$1,835)	(\$1,571)	(\$1,305)	+ CGT	\$73,421	
=EBTCF		\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF	\$374,622	\$397,983

CASH FLOW COMPONENTS FORMAT

Operating:	Year:	1	2	3	4	5	6	7	8	9	Oper. Yr.10	Reversion Item	Rever. Yr.10	Total Yr.10
Accrual Items:														
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- Cap. Imprv. Expdtr.		\$0	\$0	\$50,000	\$0	\$0	\$0	\$0	\$50,000	\$0	\$0			
=PBTCF		\$60,000	\$60,600	\$11,206	\$61,818	\$62,436	\$63,061	\$63,691	\$14,328	\$64,971	\$65,621	=PBTCF	\$1,104,622	\$1,170,243
- Debt Svc		\$43,250	\$43,140	\$43,030	\$42,920	\$42,810	\$42,700	\$42,590	\$42,480	\$42,370	\$42,260	- LoanBal	\$730,000	
=EBTCF		\$16,750	\$17,460	(\$31,824)	\$18,898	\$19,626	\$20,361	\$21,101	(\$28,152)	\$22,601	\$23,361	=EBTCF	\$374,622	\$397,983
-taxNOI		\$21,000	\$21,210	\$21,422	\$21,636	\$21,853	\$22,071	\$22,292	\$22,515	\$22,740	\$22,967	taxMktGain	\$693	\$23,661
+ DTS		\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	\$10,182	- AccDTS	(\$72,727)	(\$62,545)
+ ITS		\$14,438	\$14,399	\$14,361	\$14,322	\$14,284	\$14,245	\$14,207	\$14,168	\$14,130	\$14,091			\$14,091
=EATCF		\$20,369	\$20,831	(\$28,704)	\$21,766	\$22,239	\$22,716	\$23,198	(\$26,317)	\$24,173	\$24,667	EATCF	\$301,202	\$325,868

Here are the example property's cash flows by component...

Exhibit 14-3:

Apprec.Rate =	1.00%	Bldg.Val/Prop.Val=	80.00%	Loan=	\$750,000
Yield =	6.00%	Depreciable Life=	27.5 years	Int=	5.50%
Income Tax Rate =	35.00%	CGTax Rate =	15.00%	Amort/yr	\$2,000
		DepRecapture Rate=	25.00%		

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Year	Prop.Val	NOI	CI	PBTCF	tax w/out shields	DTS	(4)-(-5)+(6) PATCF	LoanBal	Loan DS	ITS	(4)-(-9) EBTCF	(7)-(-9)+(10) EATCF	(9)-(-10) LoanATCFs
0	\$1,000,000				(\$1,000,000)		(\$1,000,000)	\$750,000	(\$750,000)		(\$250,000)	(\$250,000)	(\$750,000)
1	\$1,010,000	\$60,000	\$0	\$60,000	\$21,000	\$10,182	\$49,182	\$748,000	\$43,250	\$14,438	\$16,750	\$20,369	\$28,813
2	\$1,020,100	\$60,600	\$0	\$60,600	\$21,210	\$10,182	\$49,572	\$746,000	\$43,140	\$14,399	\$17,460	\$20,831	\$28,741
3	\$1,030,301	\$61,206	\$50,000	\$11,206	\$21,422	\$10,182	(\$34)	\$744,000	\$43,030	\$14,361	(\$31,824)	(\$28,704)	\$28,670
4	\$1,040,604	\$61,818	\$0	\$61,818	\$21,636	\$10,182	\$50,364	\$742,000	\$42,920	\$14,322	\$18,898	\$21,766	\$28,598
5	\$1,051,010	\$62,436	\$0	\$62,436	\$21,853	\$10,182	\$50,765	\$740,000	\$42,810	\$14,284	\$19,626	\$22,239	\$28,527
6	\$1,061,520	\$63,061	\$0	\$63,061	\$22,071	\$10,182	\$51,171	\$738,000	\$42,700	\$14,245	\$20,361	\$22,716	\$28,455
7	\$1,072,135	\$63,691	\$0	\$63,691	\$22,292	\$10,182	\$51,581	\$736,000	\$42,590	\$14,207	\$21,101	\$23,198	\$28,384
8	\$1,082,857	\$64,328	\$50,000	\$14,328	\$22,515	\$10,182	\$1,995	\$734,000	\$42,480	\$14,168	(\$28,152)	(\$26,317)	\$28,312
9	\$1,093,685	\$64,971	\$0	\$64,971	\$22,740	\$10,182	\$52,413	\$732,000	\$42,370	\$14,130	\$22,601	\$24,173	\$28,241
10	\$1,104,622	\$65,621	\$0	\$1,170,243	\$23,661	(\$62,545)	\$1,084,037	\$730,000	\$772,260	\$14,091	\$397,983	\$325,868	\$758,169

IRR of above CF Stream =	6.04%	4.34%	5.50%	7.40%	6.44%	3.58%
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Projected Total Return Calculations:

(Based on \$1,000,000 price...)

10-yr Going-in IRR:		
	Property (Unlvd)	Equity (Levd)
Before-tax	6.04%	7.40%
After-tax	4.34%	6.44%
AT/BT	$434/604 = 72\%$	$644/740 = 87\%$
→ Effective Tax Rate With ord inc=35%, CGT=15%, Recapt=25%.	$100\% - 72\% = 28\%$	$100\% - 87\% = 13\%$

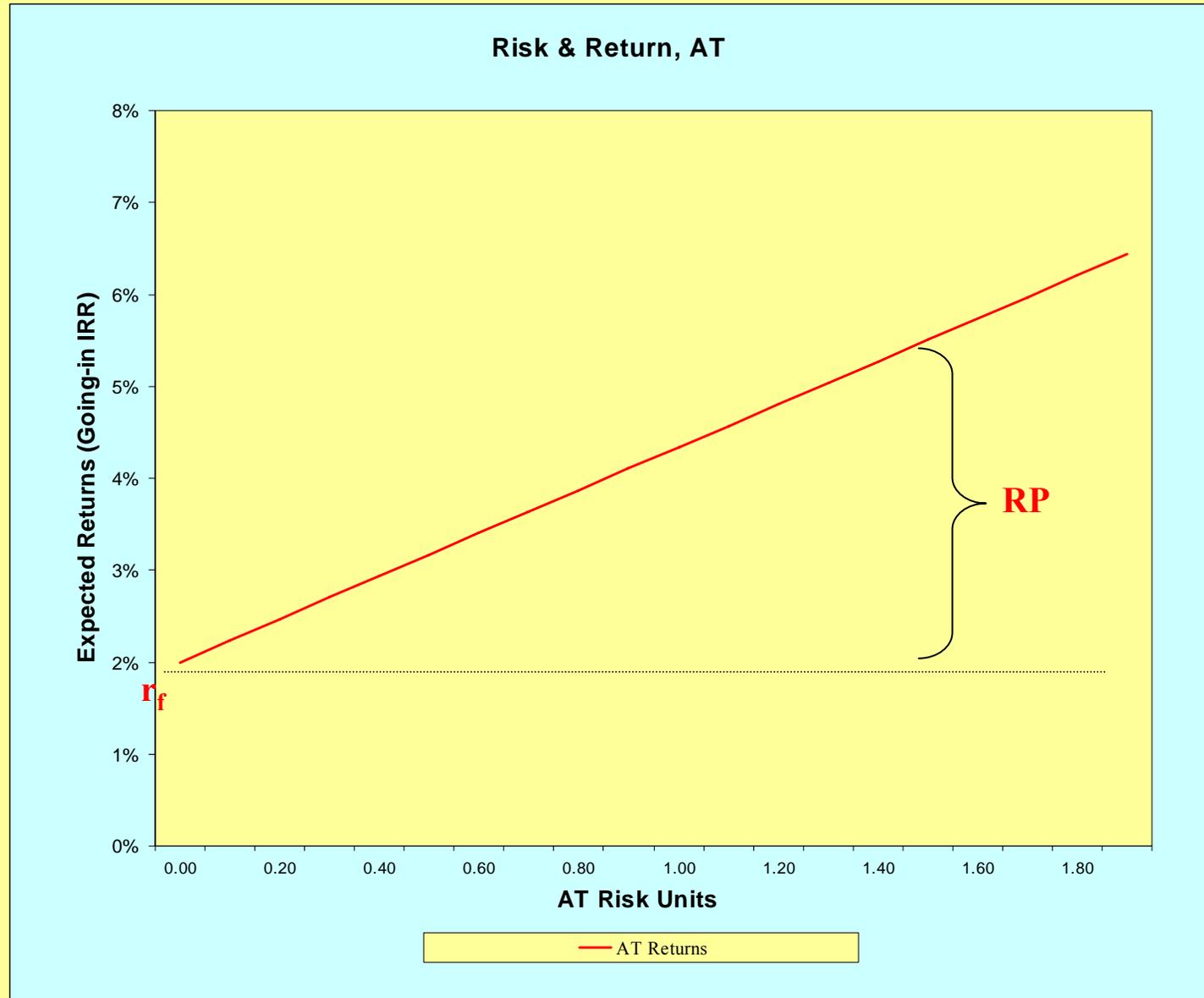
Lower effective tax rate on levered equity does not imply any “free lunch”

In equilibrium,
the linear
relationship (RP
proportional to
risk)



must hold after-tax for marginal
investors in the
relevant asset
market.

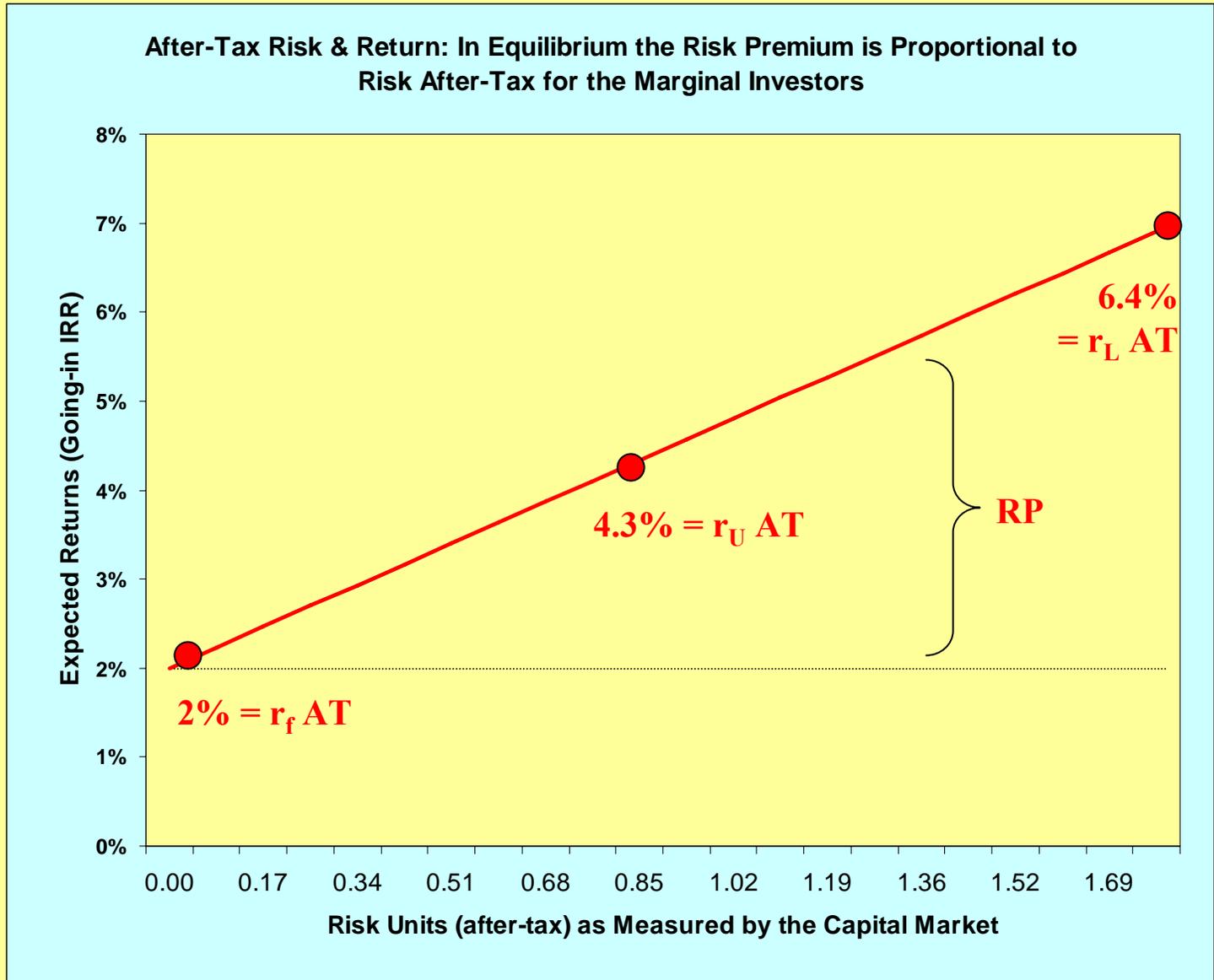
(See Ch.12, sect.12.1.)



Using these returns from our apartment bldg example, we could have this linear relationship



Lower effective tax rate in levered return does not imply that risk premium per unit of risk is greater with leverage than without.



14.3 After-Tax Equity Valuation & Capital Budgeting

After-Tax (AT) analysis generally applies to “Investment Value” (IV), while before-tax (BT) analysis applies to “Market Value” (MV).

14.3.1 After-Tax DCF in General:

- Discount Equity After-Tax Cash Flows (EATCF),
- At Equity After-Tax Discount Rate (Levered),
- To arrive at PV of Equity.
- Add PV of Loan (cash borrower obtains),
- To arrive at Investment Value (IV) of property:
 - Max price investor should pay (if they must):
 - Don't forget injunction against paying more than MV (regardless of IV):
 - ➔ *Always consider MV, not just IV.*

Although it makes theoretical sense, there is a major practical problem with the EATCF/EATOCC approach described above:

It is very difficult to empirically observe or to accurately estimate the appropriate levered equity after-tax opportunity cost of capital, $E[r]$, the appropriate hurdle going-in IRR.

- **Can observe market-based unlevered (property) going-in IRR, and adjust for leverage using WACC, but**
- **WACC not accurate for long-term IRRs,**
- **And we still must account for the effective tax rate on the marginal investor (recall that discount rate OCC even for investment value is market OCC, reflecting marginal investor's after-tax OCC).**
- **This is a function not only of tax rates, but holding period and degree of leverage (recall Exh.14-2 apartment example: Effective tax rate went from 28% to 13% in that case, with that particular bldg, holding period, & loan).**

e.g., In apartment building example,

If we did not already (somehow) know that \$1,000,000 was the market value of the property,

What would be the meaning of the 6.44% equity after-tax IRR expectation that we calculated?

But if we already know the property market value, then what does the 6.44% equity after-tax IRR tell us, that matters? . . .

How can we compare it to that offered by other alternative investments unless we know they are of the same risk (or we can quantify the risk difference in a way that can be meaningfully related to the required going-in IRR risk premium)?

Have we gone to a lot of trouble to calculate a number (the equity after-tax going-in IRR) that has no rigorous use in decision making? . . .

Fasten your seatbelts...

The latter part of Chapter 14 is an attempt to bring some rigor into micro-level real estate investment valuation based on fundamental economic principles.

14.3.2 Shortcut for Marginal Investors (& for Market Value):

Ignore the equity after-tax level!

Work with the Property Before-Tax (PBT) level cash flows and OCC.

This always works for Market Value (MV) analysis,

And also works for Investment Value (IV) analysis for marginal investors (those who are typically about equally on both the buy and sell side of the relevant asset market – recall Ch.12).

In the above circumstances,

The PBT approach is not only simpler,

It is more accurate (fewer parameters to be estimated → less chance of estimation error).

Example:

Recall the apartment property with the 6.04% PBTCF-based going-in IRR...

The 6.04% PBT going-in IRR would presumably be empirically observable in the manner described in Section 11.2 of Chapter 11:

- **Based on analysis of ex post return performance (e.g., NCREIF Index);**
- **Based on current market survey information (&/or brokers' knowledge);**
- **Backed out from observable recent transaction prices in the market (cap rates + realistic growth, accounting for CI).**

Then apply the 6.04% market PBT discount rate to derive the \$1,000,000 MV of the property (based on the PBTCF).

Recognize that this MV also equals IV for marginal investors, those typical on both the buy-side and sell-side of the market.

As noted in Ch.12, it's probably best to assume you are a marginal investor unless you can clearly document how you differ (e.g., in tax status) from typical investors in the market.

Example (cont.):

As a next step (after you've estimated the \$1,000,000 market value from market evidence), you can develop an EATCF for a typical marginal investor in the market for this type of property, and derive the levered equity after-tax OCC for the property, based on the typical EATCFs and the \$1,000,000 MV and the fact that $MV = IV$ for marginal investors.

In our previous example, if the investor subject to the 35% ordinary income tax rate (& 15% capital gains & 25% recapture rates) were typical, then we would derive the 6.44% rate noted previously, for the given amount of leverage. (In most markets, such taxable investors are probably typical marginal investors.)

This 6.44% after-tax, levered equity market OCC could then be used as the discount rate in an analysis of your investment value (based on *your own* EATCF projection), assuming a similar holding period and similar degree of leverage. (Don't forget to add the loan amount to the equity value.) (See Section 14.3.4.)

Example (cont.):

Obviously, if you are similar to the typical marginal investor in the market, you will get the same \$1,000,000 PV again, as you should (when you discount the same EATCFs @ the 6.44% IRR that was based on that price and those EATCFs, and add the loan amount). Your IV will equal MV, by construction (defining your EATCFs as typical of marginal investors). Hence, the validity of the PBT shortcut.

But if you are not similar (i.e., your EATCFs differ from the typical investor's), then (using the 6.44% discount rate) you may get a personal IV for yourself that differs from the MV (and marginal IV) of the property (i.e., different from \$1,000,000).

But still, remember that you should not generally pay more than MV (or sell for less than MV), even if your IV differs from MV. (Recall Section 12.1 in Chapter 12.)

Use the PBT shortcut to estimate MV (based on as much market price evidence as you can get).

Corollary: If you are not a marginal investor, then the after-tax levered equity IRR you calculate based on the property's current market value and your CFs will not equal the opportunity cost of capital relevant for evaluating your investment value of that levered equity, and will therefore not be relevant for quantifying the risk in that position.

Suppose \$1,000,000 = MV of property = \$750,000 loan val + \$250,000 eq. val.

Then we know that \$1,000,000 = IV_M = IV of property for marginal investor.

Suppose the modeled investor (tax rates = 35%, 15%, 25%) is typical of marginal investors in mkt for this type of property.

Suppose modeled leverage & holding period (75% LTV, 10-yr hold) is typical of marginal investors in mkt for this type of property.

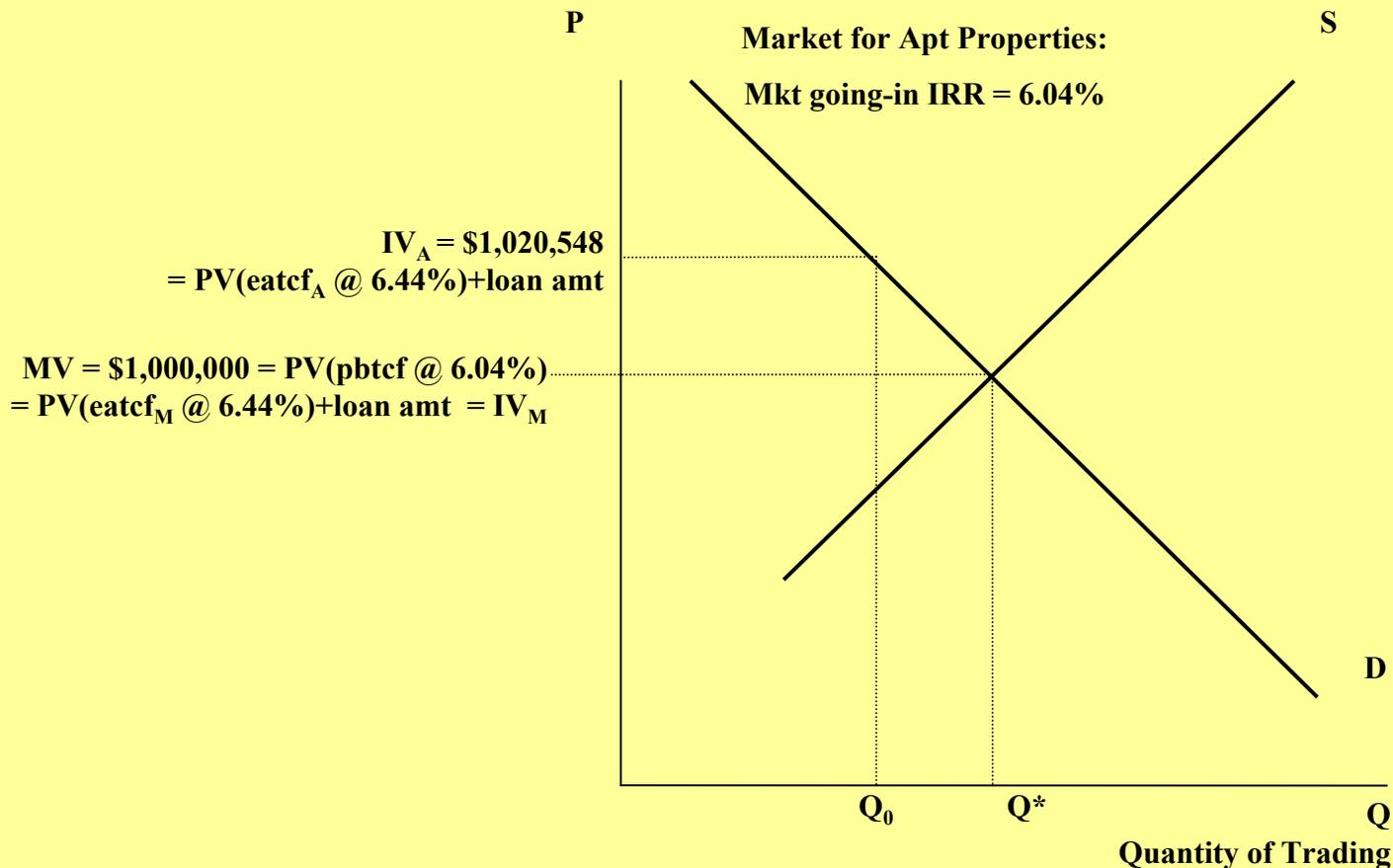
Then 6.44% = mkt after-tax levered OCC for this type of investment.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Year	Prop.Val	NOI	CI	PBTCF	tax w/out shields	DTS	PATCF	LoanBal	Loan DS	ITS	EBTCF	EATCF	LoanATCFs
0	\$1,000,000			(\$1,000,000)			(\$1,000,000)	\$750,000	(\$750,000)		(\$250,000)	(\$250,000)	(\$750,000)
1	\$1,010,000	\$60,000	\$0	\$60,000	\$21,000	\$10,182	\$49,182	\$748,000	\$43,250	\$14,438	\$16,750	\$20,369	\$28,813
2	\$1,020,100	\$60,600	\$0	\$60,600	\$21,210	\$10,182	\$49,572	\$746,000	\$43,140	\$14,399	\$17,460	\$20,831	\$28,741
3	\$1,030,301	\$61,206	\$50,000	\$11,206	\$21,422	\$10,182	(\$34)	\$744,000	\$43,030	\$14,361	(\$31,824)	(\$28,704)	\$28,670
4	\$1,040,604	\$61,818	\$0	\$61,818	\$21,636	\$10,182	\$50,364	\$742,000	\$42,920	\$14,322	\$18,898	\$21,766	\$28,598
5	\$1,051,010	\$62,436	\$0	\$62,436	\$21,853	\$10,182	\$50,765	\$740,000	\$42,810	\$14,284	\$19,626	\$22,239	\$28,527
6	\$1,061,520	\$63,061	\$0	\$63,061	\$22,071	\$10,182	\$51,171	\$738,000	\$42,700	\$14,245	\$20,361	\$22,716	\$28,455
7	\$1,072,135	\$63,691	\$0	\$63,691	\$22,292	\$10,182	\$51,581	\$736,000	\$42,590	\$14,207	\$21,101	\$23,198	\$28,384
8	\$1,082,857	\$64,328	\$50,000	\$14,328	\$22,515	\$10,182	\$1,995	\$734,000	\$42,480	\$14,168	(\$28,152)	(\$26,317)	\$28,312
9	\$1,093,685	\$64,971	\$0	\$64,971	\$22,740	\$10,182	\$52,413	\$732,000	\$42,370	\$14,130	\$22,601	\$24,173	\$28,241
10	\$1,104,622	\$65,621	\$0	\$1,170,243	\$23,661	(\$62,545)	\$1,084,037	\$730,000	\$772,260	\$14,091	\$397,983	\$325,868	\$758,169
IRR of above CF Stream =				6.04%			4.34%		5.50%		7.40%	6.44%	3.58%

IRR(EATCF) = 6.44%

Representing this as in Ch.12 market model (*see Exh.12-1*) ,
 we have . . .

Exhibit 14-6:



Pension fund is an *intra-marginal* buyer.

This approach can be refined by breaking the investment cash flows into components of different risk categories, and computing the PV of each component using a discount rate appropriate for the risk in each component.

(Recall Ch.10.)

This can be viewed as motivating a useful analytical procedure known as “Adjusted Present Value (APV), or “Value Additivity” . . .

14.3.4 Value Additivity & the APV Decision Rule

The PBT approach is an example of use of the principle of “*Value Additivity*”:

Value Additivity \leftrightarrow

“The value of the whole equals the sum of the values of the parts, i.e., the sum of the values of all the marketable (private sector) claims on the asset.”

$$\text{Prop. Val} = \text{Equity Val} + \text{Dbt Val}$$

$$\mathbf{V = E + D}$$

Where: V = Value of the property,
 E = Value of the equity,
 D = Value of the debt.

Therefore: $\mathbf{E = V - D}$

“D” is usually straightforward to compute (unless loan is subsidized, MV of loan equals loan amount).

So, use PBT to compute MV of “V”, then subtract MV of “D” to arrive at MV of “E”, rather than trying to estimate “E” directly.

“Adjusted Present Value” (APV) Decision Rule...

Like NPV, only accounts for financing...

$$\mathbf{APV(\text{equity}) = NPV(\text{property}) + NPV(\text{financing})}$$

Based on the Value Additivity Principle:

$$\textit{Prop.Val} = \textit{Equity Val} + \textit{Dbt Val}$$

$$\mathbf{V = E + D}$$

Where: V = Value of the property,

E = Value of the equity,

D = Value of the debt.

Define: P = Price paid for the property,

L = Amount of the loan...

$$V - P = E + D - P$$

$$= E + D - ((P - L) + L)$$

$$= E - (P - L) + D - L$$

$$= E - (P - L) - (L - D)$$

Thus: $E - (P - L) = (V - P) + (L - D)$

Or: $APV(\text{Equity}) = NPV(\text{Prop}) + NPV(\text{Fin})$

Note: Arbitrage basis of Value Additivity applies to MV, but the common sense of Value Additivity can be applied to IV as well.

“Adjusted Present Value” (APV)...

APV Investment Decision Rule:

Analogous to that of NPV...

- 1) Maximize APV over mutually exclusive alternatives;**
- 2) Never do a deal with $APV < 0$.**

(APV = 0 is OK.)

(APV = 0 is normal from a MV perspective.)

APV procedure can be expanded to any number of additive components of a complex deal structure. e.g.,

$$\text{APV}(\text{equity}) = \text{NPV}(\text{property}) + \text{NPV}(\text{preferred}) + \text{NPV}(\text{debt}) + \text{NPV}(\text{tax credits})$$

Try to use fundamental economic principles to help evaluate the deal:

- **Market equilibrium (competition),**
- **Rational behavior (Max NPV).**

For example...

- **If the deal structure is typical (e.g., of a “class”), and sufficiently common that there is a functioning market for these types of deals, then *market equilibrium* will tend to make it reasonable to expect: $\text{APV}(\text{equity})=0$. (You can use this as a working assumption to help ascertain the MV of the individual deal components and positions.)**
- **Even if the overall deal structure is unique (such that there is not a market for the equity as structured), many (or even all) of the individual components and positions may be typical enough that there is a market for them. It should then be assumed that *rational behavior* on the part of all the parties to the deal would tend to drive NPV toward zero for each component of the deal. If all the components of the APV are zero, then so is the APV.**

*Note that the above refers to market value based analysis (MV), not IV.
Do you recall why MV is important to the investor? . . .*

$$\text{APV}(\text{equity}) = \text{NPV}(\text{property}) + \text{NPV}(\text{financing})$$

NPV(property) = NPV of unlevered (all equity) investment in the property (as if no debt).

For MV based NPV, this can (should) be computed using the PBT approach.

NPV(financing) = NPV of loan transaction.

Debt market is relatively efficient & competitive (debt products are relatively homogeneous, often securitized in 2ndary mkt, relatively transparent and straightforward to evaluate).

Hence, in the absence of *subsidized* (below market interest rate) financing:

NPV(financing) = 0, normally (on an MV basis).

Recall: $NPV_{MV}(\text{buyer}) = -NPV_{MV}(\text{seller})$.

So, for the loan transaction: $NPV_{MV}(\text{borrower}) = -NPV_{MV}(\text{lender})$

In this (normal) situation (from an MV perspective):

APV(equity) = NPV(property).

i.e., Evaluate the deal without the loan. (Use the PBT shortcut).

14.3.5 After-Tax Valuation of Debt Financing

Let's now delve into more depth in the valuation of the debt component of the deal...

Suppose a certain investor in bond mkt (call her Mary) faces effective income tax rate of 25% on her bond returns.

And faces an after-tax OCC (from the capital mkt) of 3%.

What will be the IV to this investor of a \$100 (par) 4% perpetual bond (pays \$4/yr forever)?

Answer:

Discount Mary's after-tax cash flows, at the mkt-based after-tax OCC. . .

$$IV_M = \frac{(1-.25)4}{1.03} + \frac{(1-.25)4}{(1.03)^2} + \frac{(1-.25)4}{(1.03)^3} + \dots = \frac{3}{0.03} = \$100$$

Suppose Mary typifies the *marginal* investor in the bond mkt.

What will be the MV of this bond?

Answer:

$$MV = IV \text{ (marginal investor)} = IV_M = \$100.$$

What will be the observed “mkt yield” in the bond mkt?

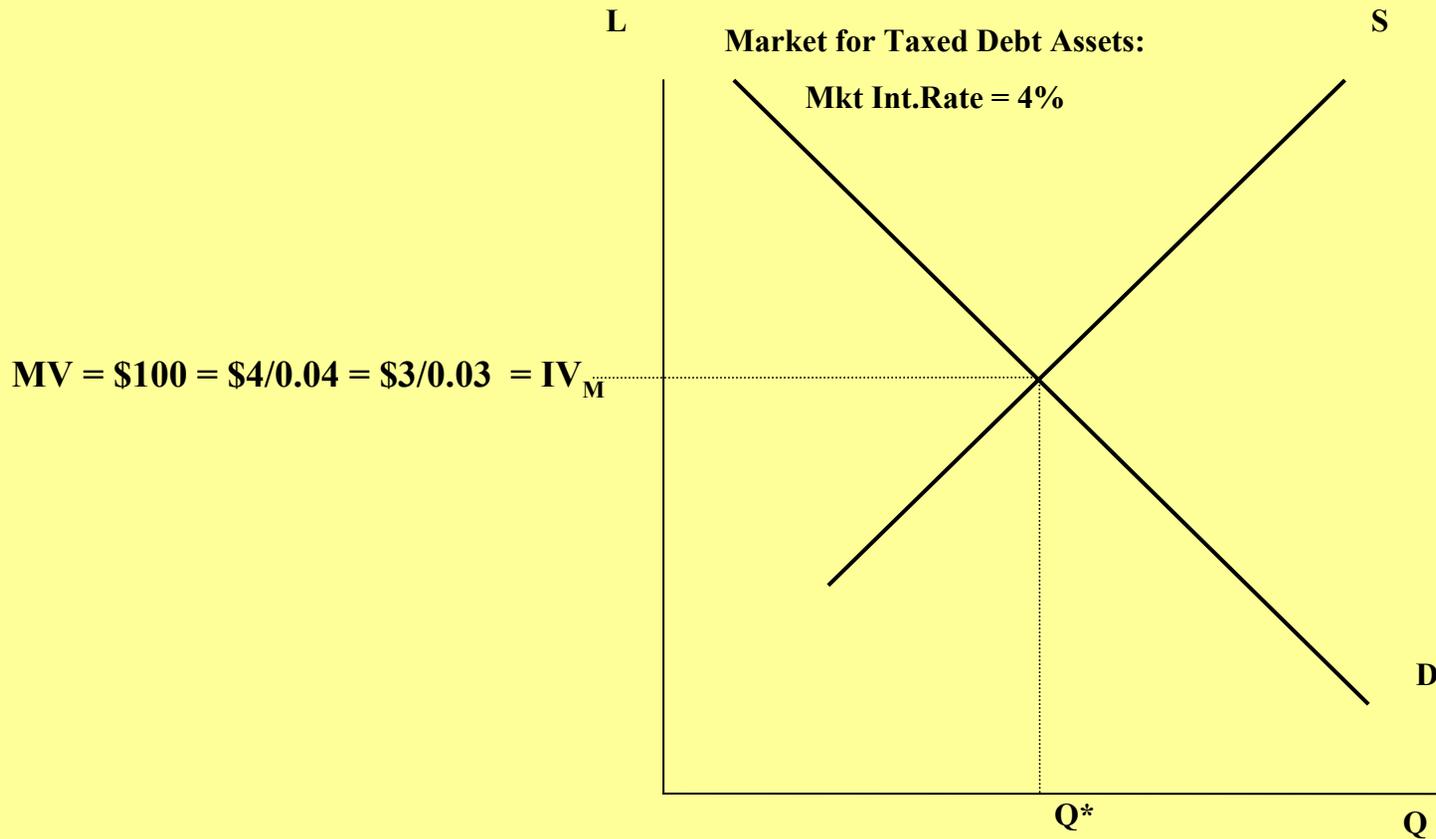
Answer:

Observable mkt returns are *pre-tax*, so:

Mkt yield = going-in IRR @ observed mkt price:

$$= \text{IRR}(-100, 4, 4, 4, \dots) = 4\%.$$

Here is a picture of what we have just discovered about the bond market . . .



L = PV of a Loan (Debt Asset).

Note: We started out with the assumption that Mary was the marginal investor.

Suppose we didn't know that she was the marginal investor.

Or suppose that we didn't know that the marginal investor faced an effective tax rate of 25%.

How could we derive (from empirically observable data) that the market's after-tax OCC is 3%?

Answer:

If there is a simultaneous market for tax-exempt bonds, and investors can trade between the two markets, then we can directly observe the market's after-tax OCC as the yield on tax-exempt (municipal) bonds.

We can also compare the yields across the two types of bond mkt (taxed & tax-exempt) to derive what must be the effective tax rate of the marginal investor.

In the previously described situation (4% yield in bond mkt, marginal investors face 25% tax rate), what will be the observed mkt yield in the *municipal* (tax-exempt) bond mkt?

Answer: 3%, because $3\% = (1-.25)4\% =$ after-tax yield for marginal investors in bond mkt.

Otherwise, what?...

If muni yield were $> 3\%$, marginal investors in taxed bond mkt would sell bonds and buy munis, driving up muni price (driving down muni yield) until the 3/4 relationship in yields existed.

If muni yield were $< 3\%$, marginal investors in muni bond mkt would sell munis and buy bonds, driving down muni price (driving up muni yield) until the 3/4 relationship in yields existed.

Thus, ratio:

muni yld / taxable bond yld

= $1 - \text{effective tax rate on margl investor in bond mkt.}$

→ Margl tax rate = $1 - \text{muniYld/bondYld}$

Suppose Abner is a tax-advantaged investor compared to the marginal investor (Mary).

Abner faces only 20% tax on bond returns.

What is the IV of the 4% perpetual bond to Abner?

Answer:

Discount Abner's after-tax cash flows, at the mkt-based after-tax OCC. . .

$$IV_A = \frac{(1-.20)4}{1.03} + \frac{(1-.20)4}{(1.03)^2} + \frac{(1-.20)4}{(1.03)^3} + \dots = \frac{3.20}{0.03} = \$107$$

Thus: $IV_A = \$107 > \$100 = MV = IV_M$.

Which makes sense, to reflect Abner's tax advantage compared to the market's marginal investor who determines the price in the mkt.

Suppose Clarence is a tax-disadvantaged investor compared to the marginal investor (Mary).

Clarence faces 30% tax on bond returns.

What is the IV of the 4% perpetual bond to Clarence?

Answer:

Discount Clarence's after-tax cash flows, at the mkt-based after-tax OCC. . .

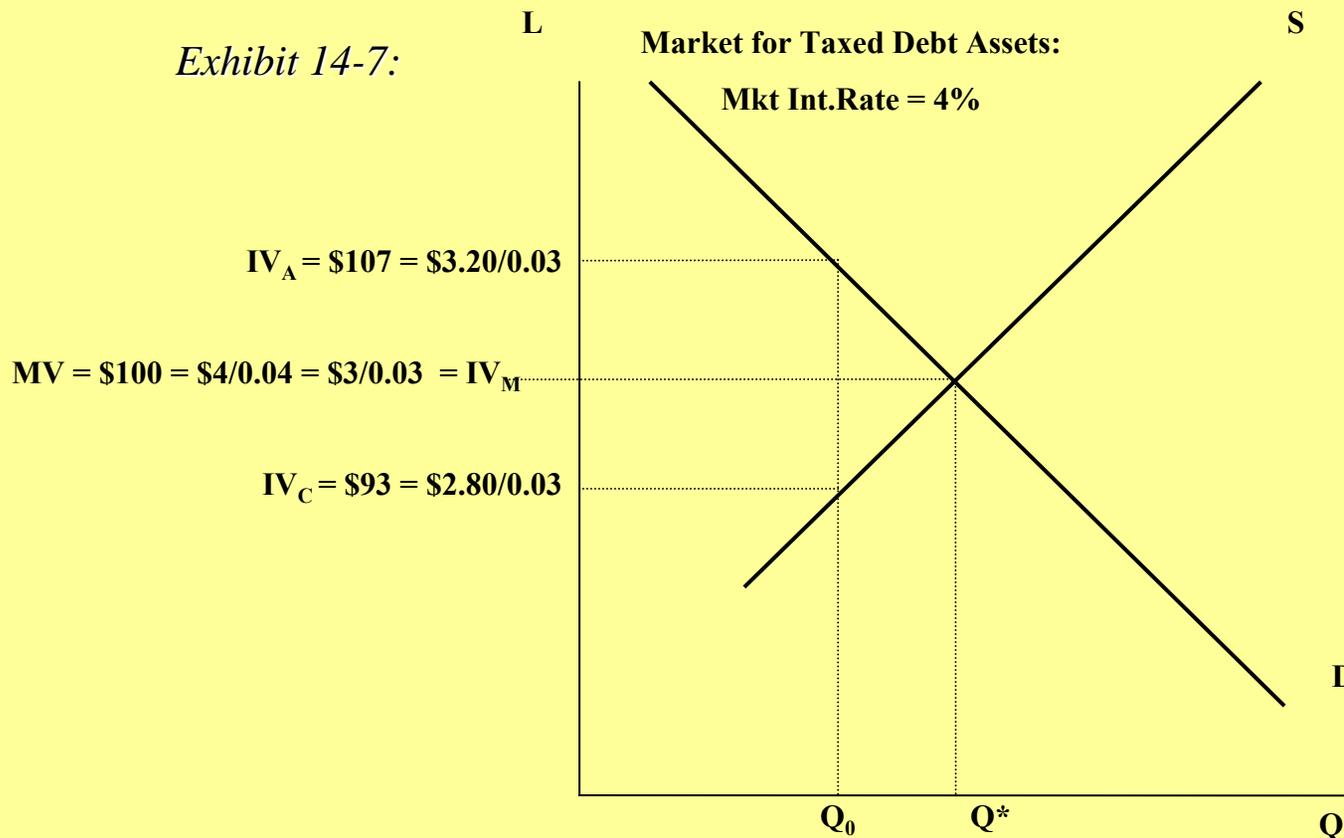
$$IV_C = \frac{(1-.30)4}{1.03} + \frac{(1-.30)4}{(1.03)^2} + \frac{(1-.30)4}{(1.03)^3} + \dots = \frac{2.80}{0.03} = \$93$$

Thus: $IV_C = \$93 < \$100 = MV = IV_M$.

Which makes sense, to reflect Clarence's tax disadvantage compared to the market's marginal investor who determines the price in the mkt.

Abner & Clarence are *intra-marginal* market participants in the bond mkt.

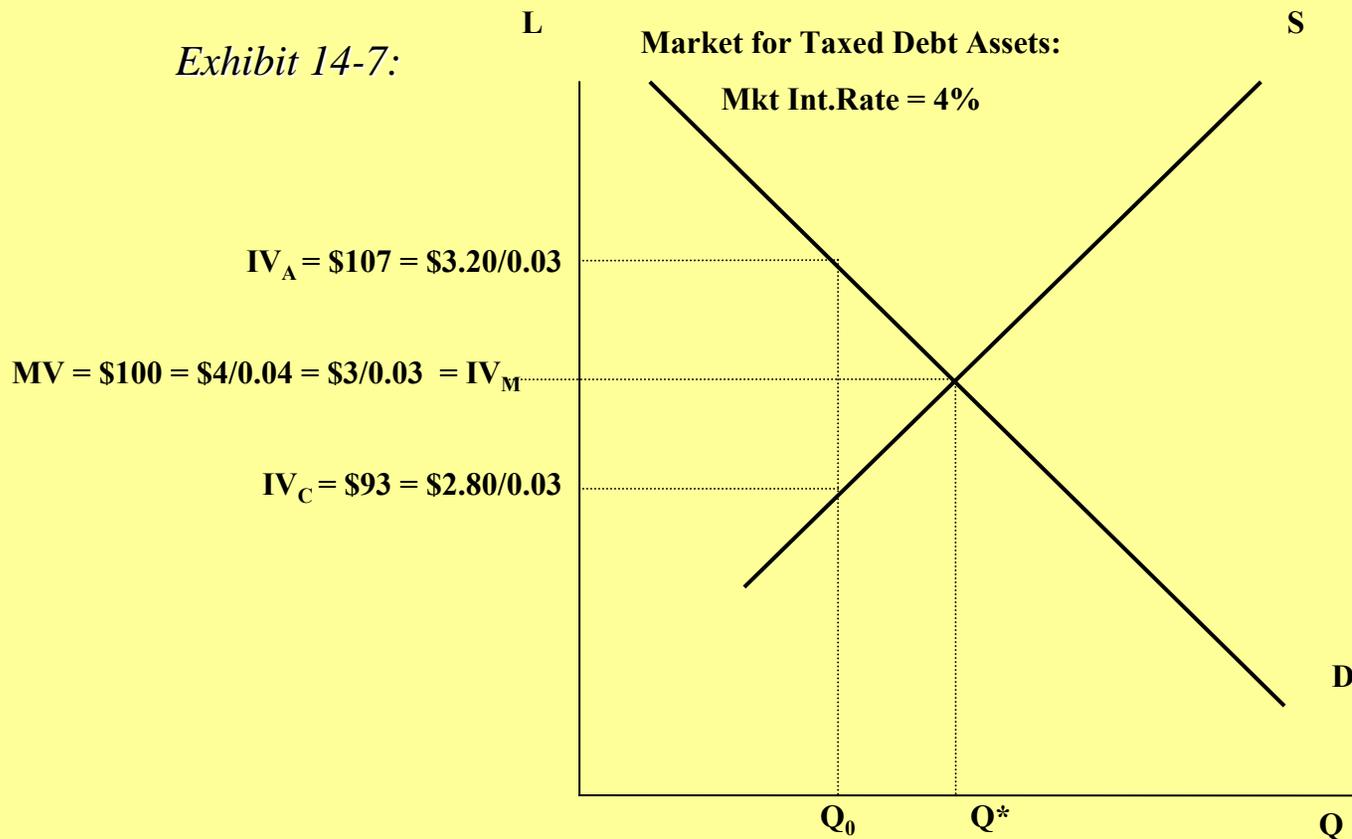
Abner is an intra-marginal buyer. Clarence is an intra-marginal seller.



Trading occurs at the mkt price of \$100 (4% yield).

NPV(for Abner buying, based on his IV) = \$107 - \$100 = +\$7.

NPV(for Clarence selling, based on his IV) = \$100 - \$93 = +\$7.



Relationship to value of “*interest tax shields*” (ITS) in borrowing money to finance real estate investment . . .

Borrowing money is like selling bonds (receive cash up front, pay back contractual periodic cash flows over time).

Mary, Abner, & Clarence are all considering making real estate investments, and taking out a mortgage to finance that investment.

The mkt interest rate on the mortgage is 4%, and the loan is perpetual, interest only.

What is the value of the ITS to Mary, Abner, & Clarence, in this mortgage?

Value of the ITS:

The amount of the ITS each year equals the income tax savings that year: the investor's tax rate times the interest expense. The PV is found by discounting at the OCC...

$$PV(ITS)_M = \frac{(.25)4}{1.03} + \frac{(.25)4}{(1.03)^2} + \frac{(.25)4}{(1.03)^3} + \dots = \frac{1}{0.03} = \$33.$$

$$PV(ITS)_A = \frac{(.20)4}{1.03} + \frac{(.20)4}{(1.03)^2} + \frac{(.20)4}{(1.03)^3} + \dots = \frac{0.80}{0.03} = \$27.$$

$$PV(ITS)_C = \frac{(.30)4}{1.03} + \frac{(.30)4}{(1.03)^2} + \frac{(.30)4}{(1.03)^3} + \dots = \frac{1.20}{0.03} = \$40.$$

The ITS are worth more to the more heavily-taxed investor (Clarence), and for everyone they are worth a large fraction of the loan amount (\$100).

But what is the NPV of the borrowing transaction, to Mary, Abner, & Clarence? . . .

NPV_{IV} of borrowing to finance real estate investment
(@ mkt interest rate, no subsidy):

$$NPV_M = 100 - \left(\frac{(1-.25)^4}{1.03} + \frac{(1-.25)^4}{(1.03)^2} + \frac{(1-.25)^4}{(1.03)^3} + \dots \right) = 100 - \frac{3}{0.03} = 100 - 100 = 0.$$

$$NPV_A = 100 - \left(\frac{(1-.20)^4}{1.03} + \frac{(1-.20)^4}{(1.03)^2} + \frac{(1-.20)^4}{(1.03)^3} + \dots \right) = 100 - \frac{3.20}{0.03} = 100 - 107 = -\$7.$$

$$NPV_C = 100 - \left(\frac{(1-.30)^4}{1.03} + \frac{(1-.30)^4}{(1.03)^2} + \frac{(1-.30)^4}{(1.03)^3} + \dots \right) = 100 - \frac{2.80}{0.03} = 100 - 93 = +\$7.$$

Even though the ITS have substantial value to all three investors,

Borrowing is zero NPV for the marginal investor (Mary).

Borrowing is negative NPV for the tax-advantaged investor (Abner).

Borrowing is only positive NPV for the tax-disadvantaged investor (relative to the marginal investor in the bond mkt),

And even for him (Clarence) the NPV of borrowing is much smaller than the PV of his ITS (\$7 vs \$40).

(Note: This is NPV based on IV. NPV based on MV is zero by defn in non-subsidized loan.)

Why might each of them (Clarence, Mary, Abner) borrow to finance their real estate investment?...

- **Clarence (alone) can justify the loan purely for the positive NPV in its tax shelter (with the caveat that it is not a large positive NPV).**
- **Clarence and Mary can justify the loan if they want leverage (to magnify risk & return in their R.E. investment), but Abner can't use this justification due to his $NPV < 0$ (he should look for other ways to increase risk & return).**
- **All three investors can justify the loan if they are capital constrained and the R.E. investment has a sufficient positive NPV. (For Clarence, the R.E. NPV can even be a bit negative.) For Abner, the positive R.E. NPV may result from his tax advantage (if he is also tax advantaged relative to the marginal investor in the R.E. asset mkt, not just relative to the marginal investor in the bond mkt).**

Now suppose the seller of the real estate offers *below-market* financing (subsidized loan), an interest rate of 3% instead of the mkt rate of 4%, on the same (perpetual) loan.

What would be the MV of this loan (if the seller sold it in the 2ndary mkt)?

Answer:

Discount the loan's *before-tax* cash flows at the market interest rate . . .

$$MV = \frac{3}{1.04} + \frac{3}{(1.04)^2} + \frac{3}{(1.04)^3} + \dots = \frac{3}{0.04} = \$75$$

Or (more fundamentally), discount the loan's *after-tax* cash flows to the marginal investor in the bond mkt (Mary), at the mkt-based *after-tax* OCC. . .

$$MV = IV_M = \frac{(1-.25)3}{1.03} + \frac{(1-.25)3}{(1.03)^2} + \frac{(1-.25)3}{(1.03)^3} + \dots = \frac{2.25}{0.03} = \$75$$

What is the IV based NPV of the subsidized loan offer to each of our investors? . . .

$$NPV(loan)_M = 100 - \left(\frac{(1-.25)^3}{1.03} + \frac{(1-.25)^3}{(1.03)^2} + \frac{(1-.25)^3}{(1.03)^3} + \dots \right) = 100 - \frac{2.25}{0.03} = 100 - 75 = +\$25.$$

$$NPV(loan)_A = 100 - \left(\frac{(1-.20)^3}{1.03} + \frac{(1-.20)^3}{(1.03)^2} + \frac{(1-.20)^3}{(1.03)^3} + \dots \right) = 100 - \frac{2.40}{0.03} = 100 - 80 = +\$20.$$

$$NPV(loan)_C = 100 - \left(\frac{(1-.30)^3}{1.03} + \frac{(1-.30)^3}{(1.03)^2} + \frac{(1-.30)^3}{(1.03)^3} + \dots \right) = 100 - \frac{2.10}{0.03} = 100 - 70 = +\$30.$$

How much more should each of the investors be willing to pay for the property (more than they think it is otherwise worth), as a result of the subsidized loan offer from the seller? . . .

Mary → \$25 more.

Abner → \$20 more.

Clarence → \$30 more.

(Note: These IV-based NPV effects of loan rate subsidies are reduced the shorter the loan term.)

Preceding used perpetuity example.

Suppose finite loan (extreme case 1-period).

Compare for subsidized loan NPV(loan @ MV after-tax for margl investor: 25% tax rate) vs NPV(MV before-tax)...

Perpetuity: NPV(loan @ MV) =

$$100 - \left(\frac{3}{1.04} + \frac{3}{(1.04)^2} + \frac{3}{(1.04)^3} + \dots \right) = 100 - \frac{3}{0.04} = 100 - 75 = +\$25.$$

Perpetuity: NPV(loan @ IV_M) =

$$100 - \left(\frac{(1-.25)3}{1.03} + \frac{(1-.25)3}{(1.03)^2} + \frac{(1-.25)3}{(1.03)^3} + \dots \right) = 100 - \frac{2.25}{0.03} = 100 - 75 = +\$25.$$

It's the same (as you would expect, for margl investor).

1-period: NPV(loan @ MV) =

$$100 - \frac{3+100}{1.04} = 100 - \frac{103}{1.04} = 100 - 99.04 = +\$0.96.$$

1-period: NPV(loan @ IV_M) =

$$100 - \frac{(1-.25)3+100}{1.03} = 100 - \frac{102.25}{1.03} = 100 - 99.27 = +\$0.73.$$

It's different! NPV(loan @ IV_M) \approx (1 - T)NPV(loan @ MV):

$$\$0.73 \approx (1 - 0.25)\$0.96.$$

Recall Ch.12 (sect.12.1) rules about what to do when NPV differs from MV and IV perspectives:

Use common sense!

Summarizing NPV(loan) from borrower's perspective:

- **Unsubsidized (mkt rate) loans:**
 - NPV(loan) = 0 from **MV** (before-tax) perspective.
 - NPV(loan) > 0 (but much less than PV(ITS)) for **high** tax-bracket taxable investors from **IV** perspective.
 - NPV(loan) < 0 for **low** tax-bracket taxable investors from **IV** perspective.
- **Subsidized (below mkt rate) loans:**
 - NPV(loan) > 0 from **MV** perspective (by definition).
 - NPV(loan) > 0 from **IV** perspective for **high** or **average** tax bracket investors, but
 - NPV(loan) IV (after-tax) \leq NPV(loan) MV (before-tax):
 - NPV(loan) IV (after-tax) converges to $(1-T)$ NPV(loan) MV (before-tax) as loan-term approaches zero (where T is borrower's tax rate);
 - NPV(loan) IV (after-tax) converges to exactly equal NPV(loan) MV (before-tax) as loan-term approaches infinity (perpetual debt).

Let's now apply APV to dissect the valuation of our previous apartment property . . .

14.3.6 Example Application of APV to a Marginal Investor

Suppose \$1,000,000 = MV of property, & 25% is tax rate of marginal investor in debt mkt,
 → ATOCC dbt = $(1-.25)*5.5\% = 4.13\%$

Suppose the modeled investor (tax rates = 35%, 15%, 25%) is typical of marginal investors in mkt for this type of property, & modeled leverage & holding period (75% LTV, 10-yr hold) is typical of marginal investors in mkt for this type of property. → IV-based APV = 0 for whole deal (inclu debt).

Then we know margl invstr in prop mkt is *intra-marginal* in debt mkt on the sell (borrow) side: Debt part is pos-NPV, thus:

APV = 0 (mkt equilibr) → Property (unlevrd) is neg-NPV.

Apprec.Rate =		1.00%		Bldg.Val/Prop.Val=		80.00%		Loan=		\$750,000							
Yield =		6.00%		Depreciable Life=		27.5 years		Int=		5.50%							
Income Tax Rate =		35.00%		CGTax Rate =		15.00%		Amort/yr		\$2,000							
Debt Mkt Margl Tax Rate=		25.00%		DepRecapture Rate=		25.00%											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)				
Year	Prop.Val	NOI	CI	PBTCF	tax w/out shields	DTS	(4)-(5)+(6)	PATCF	LoanBal	Loan DS	ITS	EBTCF	(7)-(9)+(10)	EATCF	LoanATCF&Val	(9)-(10)	
0	\$1,000,000				(\$1,000,000)			(\$967,119)	\$750,000	(\$750,000)		(\$250,000)	(\$250,000)		(\$717,119)		
1	\$1,010,000	\$60,000	\$0	\$60,000	\$21,000	\$10,182	\$49,182	\$748,000	\$43,250	\$14,438	\$16,750	\$20,369	\$28,813				
2	\$1,020,100	\$60,600	\$0	\$60,600	\$21,210	\$10,182	\$49,572	\$746,000	\$43,140	\$14,399	\$17,460	\$20,831	\$28,741				
3	\$1,030,301	\$61,206	\$50,000	\$11,206	\$21,422	\$10,182	(\$34)	\$744,000	\$43,030	\$14,361	(\$31,824)	(\$28,704)	\$28,670				
4	\$1,040,604	\$61,818	\$0	\$61,818	\$21,636	\$10,182	\$50,364	\$742,000	\$42,920	\$14,322	\$18,898	\$21,766	\$28,598				
5	\$1,051,010	\$62,436	\$0	\$62,436	\$21,853	\$10,182	\$50,765	\$740,000	\$42,810	\$14,284	\$19,626	\$22,239	\$28,527				
6	\$1,061,520	\$63,061	\$0	\$63,061	\$22,071	\$10,182	\$51,171	\$738,000	\$42,700	\$14,245	\$20,361	\$22,716	\$28,455				
7	\$1,072,135	\$63,691	\$0	\$63,691	\$22,292	\$10,182	\$51,581	\$736,000	\$42,590	\$14,207	\$21,101	\$23,198	\$28,384				
8	\$1,082,857	\$64,328	\$50,000	\$14,328	\$22,515	\$10,182	\$1,995	\$734,000	\$42,480	\$14,168	(\$28,152)	(\$26,317)	\$28,312				
9	\$1,093,685	\$64,971	\$0	\$64,971	\$22,740	\$10,182	\$52,413	\$732,000	\$42,370	\$14,130	\$22,601	\$24,173	\$28,241				
10	\$1,104,622	\$65,621	\$0	\$1,170,243	\$23,661	(\$62,545)	\$1,084,037	\$730,000	\$772,260	\$14,091	\$397,983	\$325,868	\$758,169				

IRR of above CF Stream = 6.04% 4.76% 5.50% 7.40% 6.44% 4.13%

Suppose \$1,000,000 = MV of property, & 25% is tax rate of marginal investor in debt mkt,
 → ATOCC dbt = (1-.25)*5.5% = 4.13%

Suppose the modeled investor (tax rates = 35%, 15%, 25%) is typical of marginal investors in mkt for this type of property, & modeled leverage & holding period (75% LTV, 10-yr hold) is typical of marginal investors in mkt for this type of property. → IV-based APV = 0 for whole deal (inclu debt).

Then we know margl invstr in prop mkt is *intra-marginal* in debt mkt on the sell (borrow) side: Debt part is pos-NPV, thus:

APV = 0 (mkt equilibr) → Property (unlevrd) is neg-NPV.

Exhibit 14-8:

Apprec. Rate =		1.00%		Bldg.Val/Prop.Val=		80.00%		Loan=		\$750,000							
Yield =		6.00%		Depreciable Life=		27.5 years		Int=		5.50%							
Income Tax Rate =		35.00%		CGTax Rate =		15.00%		Amort/yr		\$2,000							
Debt Mkt Margl Tax Rate=		25.00%		DepRecapture Rate=		25.00%											
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(9)-(10)				
Year	Prop.Val	NOI	CI	PBTCF	tax w/out shields	DTS	(4)-(5)+(6) PATCF	LoanBal	Loan DS	ITS	EBTCF	(7)-(9)+(10) EATCF	LoanATCF&Val				
0	\$1,000,000			(\$1,000,000)			(\$967,119)	\$750,000	(\$750,000)		(\$250,000)	(\$250,000)	(\$717,119)				
1	\$1,010,000	\$60,000	\$0	\$60,000	\$21,000	\$10,182	\$49,182	\$748,000	\$43,250	\$14,438	\$16,750	\$20,369	\$28,813				
2	\$1,020,100	\$60,600	\$0	\$60,600	\$21,210	\$10,182	\$49,572	\$746,000	\$43,140	\$14,399	\$17,460	\$20,831	\$28,741				
3	\$1,030,301	\$61,206	\$50,000	\$11,206	\$21,422	\$10,182	(\$34)	\$744,000	\$43,030	\$14,361	(\$31,824)	(\$28,704)	\$28,670				
4	\$1,040,604	\$61,818	\$0	\$61,818	\$21,636	\$10,182	\$50,364	\$742,000	\$42,920	\$14,322	\$18,898	\$21,766	\$28,598				
5	\$1,051,010	\$62,436	\$0	\$62,436	\$21,853	\$10,182	\$50,765	\$740,000	\$42,810	\$14,284	\$19,626	\$22,239	\$28,527				
6	\$1,061,520	\$63,061	\$0	\$63,061	\$22,071	\$10,182	\$51,171	\$738,000	\$42,700	\$14,245	\$20,361	\$22,716	\$28,455				
7	\$1,072,135	\$63,691	\$0	\$63,691	\$22,292	\$10,182	\$51,581	\$736,000	\$42,590	\$14,207	\$21,101	\$23,198	\$28,384				
8	\$1,082,857	\$64,328	\$50,000	\$14,328	\$22,515	\$10,182	\$1,995	\$734,000	\$42,480	\$14,168	(\$28,152)	(\$26,317)	\$28,312				
9	\$1,093,685	\$64,971	\$0	\$64,971	\$22,740	\$10,182	\$52,413	\$732,000	\$42,370	\$14,130	\$22,601	\$24,173	\$28,241				
10	\$1,104,622	\$65,621	\$0	\$1,170,243	\$23,661	(\$62,545)	\$1,084,037	\$730,000	\$772,260	\$14,091	\$397,983	\$325,868	\$758,169				

IV(loan liab) @ 4.13% = -\$717,119

IRR of above CF Stream =

6.04%

4.76%

5.50%

7.40%

6.44%

4.13%

$$0 = APV = NPV(prop) + NPV(loan) = (X - \$1,000,000) + (\$750,000 - \$717,119);$$

$$\rightarrow X = \$967,119 = IV(prop); \rightarrow AT(unlevrd)OCC = 4.76%;$$

$$\text{Therefore: } NPV(prop) = \$967,119 - \$1,000,000 = -\$32,881; \quad NPV(loan) = \$750,000 - \$717,119 = +\$32,881.$$

Here's another way to use Value Additivity to dissect the apartment deal...

Consider the investment by a typical (taxed) marginal investor again (for whom $IV_M = MV$, not now the intra-marginal P.F. for whom $IV_A > MV$).

Break the cash flows into components by risk class...

Exhibit 14-9a:

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
		Prop.Val	NOI	CI	PBTCF	tax w/out shields	DTS	(4)-(5)+(6) PATCF	LoanBal	Loan DS	ITS	EBTCF	(7)-(9)+(10) EATCF	(9)-(10) LoanATCF&Val
Apprec.Rate =	1.00%													
Yield =	6.00%													
Income Tax Rate =	35.00%													
Debt Mkt Margl Tax Rate=	25.00%													
Bldg.Val/Prop.Val=	80.00%													
Depreciable Life=	27.5 years													
CGTax Rate =	15.00%													
DepRecapture Rate=	25.00%													
Loan=	\$750,000													
Int=	5.50%													
Amort/yr	\$2,000													
Year														
0	\$1,000,000				(\$1,000,000)			(\$967,119)	\$750,000	(\$750,000)		(\$250,000)	(\$250,000)	(\$717,119)
1	\$1,010,000	\$60,000	\$0	\$0	\$60,000	\$21,000	\$10,182	\$49,182	\$748,000	\$43,250	\$14,438	\$16,750	\$20,369	\$28,813
2	\$1,020,100	\$60,600	\$0	\$0	\$60,600	\$21,210	\$10,182	\$49,572	\$746,000	\$43,140	\$14,399	\$17,460	\$20,831	\$28,741
3	\$1,030,301	\$61,206	\$50,000	\$0	\$11,206	\$21,422	\$10,182	(\$34)	\$744,000	\$43,030	\$14,361	(\$31,824)	(\$28,704)	\$28,670
4	\$1,040,604	\$61,818	\$0	\$0	\$61,818	\$21,636	\$10,182	\$50,364	\$742,000	\$42,920	\$14,322	\$18,898	\$21,766	\$28,598
5	\$1,051,010	\$62,436	\$0	\$0	\$62,436	\$21,853	\$10,182	\$50,765	\$740,000	\$42,810	\$14,284	\$19,626	\$22,239	\$28,527
6	\$1,061,520	\$63,061	\$0	\$0	\$63,061	\$22,071	\$10,182	\$51,171	\$738,000	\$42,700	\$14,245	\$20,361	\$22,716	\$28,455
7	\$1,072,135	\$63,691	\$0	\$0	\$63,691	\$22,292	\$10,182	\$51,581	\$736,000	\$42,590	\$14,207	\$21,101	\$23,198	\$28,384
8	\$1,082,857	\$64,328	\$50,000	\$0	\$14,328	\$22,515	\$10,182	\$1,995	\$734,000	\$42,480	\$14,168	(\$28,152)	(\$26,317)	\$28,312
9	\$1,093,685	\$64,971	\$0	\$0	\$64,971	\$22,740	\$10,182	\$52,413	\$732,000	\$42,370	\$14,130	\$22,601	\$24,173	\$28,241
10	\$1,104,622	\$65,621	\$0	\$0	\$1,170,243	\$23,661	(\$62,545)	\$1,084,037	\$730,000	\$772,260	\$14,091	\$397,983	\$325,868	\$758,169
IRR of above CF Stream =					6.04%			4.76%		5.50%		7.40%	6.44%	4.13%

DTS & loan ATCF are relatively low risk (legally fixed): OCC = 4.13%.

PBTCF & tax w/out shields are relatively high risk: OCC = 4.76%.

PBTCF - Tax w/out Shlds = Tot.Risky

DTS - loan ATCF = Tot.Fixed

DTS	DS - ITS	Fixed
\$10,182	\$28,813	-\$18,631
\$10,182	\$28,741	-\$18,559
\$10,182	\$28,670	-\$18,488
\$10,182	\$28,598	-\$18,416
\$10,182	\$28,527	-\$18,345
\$10,182	\$28,455	-\$18,273
\$10,182	\$28,384	-\$18,202
\$10,182	\$28,312	-\$18,130
\$10,182	\$28,241	-\$18,059
(\$62,545)	\$758,169	-\$820,714

PBTCF	tax w/out shields	Risky
\$60,000	\$21,000	\$39,000
\$60,600	\$21,210	\$39,390
\$11,206	\$21,422	-\$10,216
\$61,818	\$21,636	\$40,182
\$62,436	\$21,853	\$40,584
\$63,061	\$22,071	\$40,989
\$63,691	\$22,292	\$41,399
\$14,328	\$22,515	-\$8,187
\$64,971	\$22,740	\$42,231
\$1,170,243	\$23,661	\$1,146,583

PV @ 4.13% = - \$683,592

PV @ 4.76% = \$933,257

Fixed + Risky = EATCF

Fixed	Risky	EATCF
-\$18,631	\$39,000	\$20,369
-\$18,559	\$39,390	\$20,831
-\$18,488	-\$10,216	-\$28,704
-\$18,416	\$40,182	\$21,766
-\$18,345	\$40,584	\$22,239
-\$18,273	\$40,989	\$22,716
-\$18,202	\$41,399	\$23,198
-\$18,130	-\$8,187	-\$26,317
-\$18,059	\$42,231	\$24,173
-\$820,714	\$1,146,583	\$325,868

Risky Val: \$933,257
+ Fixed Val: -683,592
= Eq. Val: \$249,664

→ IRR(risky for PV \$933,257) = 4.75%
= Risky OCC

PV @ 6.44% = \$250,000 → PV(risky) = \$250,000 - (-\$683,592) = \$933,592 ≈ \$933,257

MV prop = IV_M equity + loan amt = \$250,000 + \$750,000 = \$1,000,000 = \$966,473 val w/out tax shields + \$33,000 IV_M tax shields to margl investor .

$$\text{MV prop} = \text{IV}_M \text{equity} + \text{loan amt} = \$250,000 + \$750,000 = \$1,000,000 = \$966,473 \text{ val w/out tax shields} + \$33,527 \text{ IV}_M \text{ tax shields to margl investor} .$$

This is consistent with PV of depreciation tax shields:

$$\text{PV(DTS @ 4.13\%)} = \$33,527:$$

DTS
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
\$10,182
(\$62,545)

And with there being no further component of MV (= IV_M) attributable to tax shields, given that we are here assuming that the loan is zero-NPV to the marginal investor: $\text{NPV}_M(\text{loan}) = \$750,000 - \$750,000 = 0$.

For a more in-depth perspective on the effect debt financing can have given income tax considerations, see the following slides ⁵⁶.

Does the lower effective tax rate on levered equity imply that borrowing is profitable (in the sense of $NPV > 0$)?

(Do you believe in a “free lunch”?)...

Recall from Chapter 13:

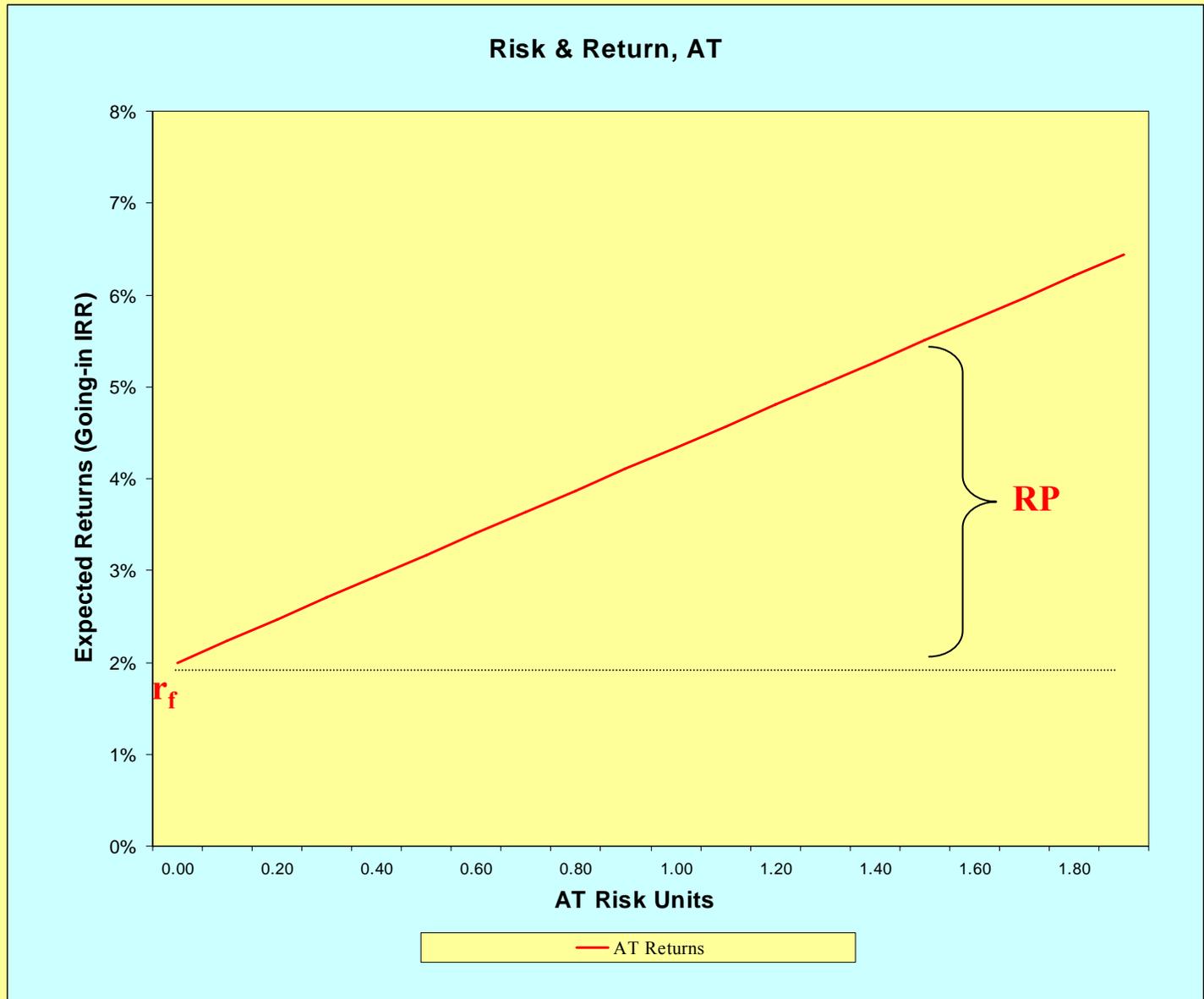
- **Leverage increases expected total return,**
- **But it also increases risk.**
- **Risk increases proportionately to *risk premium* in $E[r]$.**
- **Hence $E[RP] / \text{Unit of Risk}$ remains constant.**
- **Hence, $NPV(\text{borrowing})=0$ (No “free lunch”).**
- **This holds true after-tax as well as before-tax (at least for marginal investors – those with tax rates typical of marginal investors in the debt market).**

In equilibrium,
the linear
relationship (RP
proportional to
risk)



must hold after-tax for marginal
investors in the
relevant asset
market.

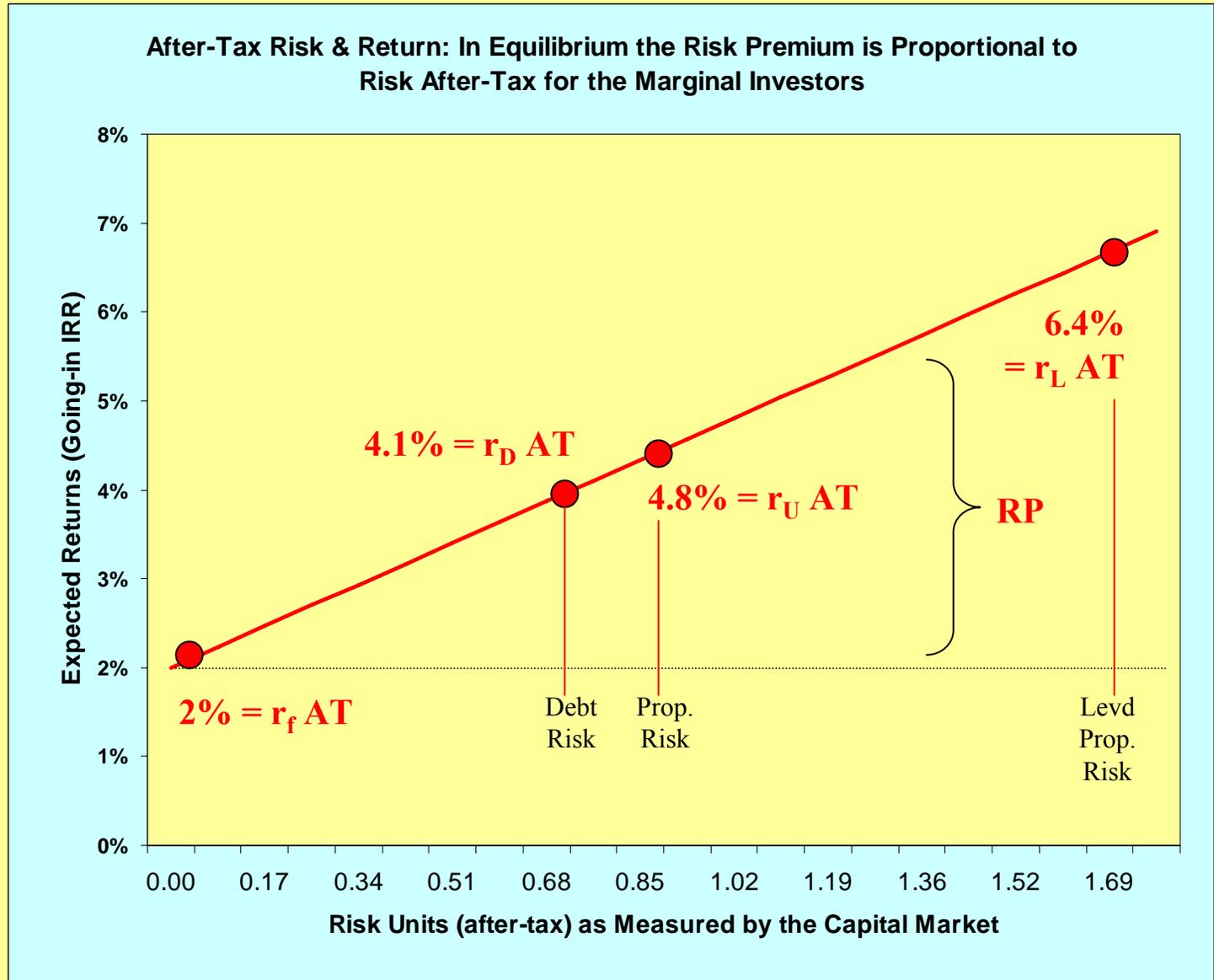
(See Ch.12, sect.12.1.)



Using these returns from our apartment bldg example, we could have this linear relationship



Lower effective tax rate in levered return does not imply that risk premium per unit of risk is greater with leverage than without.



Now let's apply the APV approach to dissect the deal...

$$\text{APV}(\text{equity}) = \text{NPV}(\text{property}) + \text{NPV}(\text{financing})$$

From an IV perspective, the NPV(property) for the P.F. is:

$$\text{NPV} = \text{IV}(\text{property}) - \text{Prop.Price}$$

$$= \text{PV}(\text{patcf}_A @ 4.76\%) - \$1,000,000$$

= mkt unlevd AT OCC, from marginal investor's AT IRR without leverage (@ MV = \$1,000,000).

$$= \text{PV}(\text{pbtcf} @ 4.76\%) - \$1,000,000$$

Because P.F. tax-exempt: PATCF = PBTCF.

$$= \$1,104,714 - \$1,000,000 = +\$104,714.$$

From an IV perspective, the NPV(financing) for the P.F. is:

$$\text{NPV} = \text{Loan Amt} - \text{IV}(\text{loan})$$

$$= \$750,000 - \text{PV}(\text{loan atcf}_A @ \text{muni yld}\%)$$

$$= \$750,000 - \text{PV}(\text{loan btcf} @ 4.13\%)$$

Suppose muni yld = 4.13% = (1 - .25)5.5%.

$$= \$750,000 - \$832,202 = -\$82,202.$$

$$\text{APV}(\text{equity}) = +104,714 - \$82,202 = +\$22,512 \approx 20,548^*.$$

*In principle this equation should be exactly equal, otherwise there is some sort of "arbitrage" opportunity between the markets. But in reality valuations are not that precise.

Tax-exempt investor (e.g., pension fund) “Value Additivity” valuation of apartment investment by components . . .

Exhibit 14-10:

Year	(1) Prop.Val	(2) NOI	(3) CI	(4) PBTFCF	(5) tax w/out shields	(6) DTS	(7) PATCF	(8) LoanBal	(9) Loan DS	(10) ITS	(11) EBTCF	(12) EATCF	(13) LoanATCF&Val
0	\$1,000,000			(\$1,000,000)			(\$1,104,714)	\$750,000	(\$750,000)		(\$250,000)	(\$270,548)	(\$832,202)
1	\$1,010,000	\$60,000	\$0	\$60,000	\$0	\$0	\$60,000	\$748,000	\$43,250	\$0	\$16,750	\$16,750	\$43,250
2	\$1,020,100	\$60,600	\$0	\$60,600	\$0	\$0	\$60,600	\$746,000	\$43,140	\$0	\$17,460	\$17,460	\$43,140
3	\$1,030,301	\$61,206	\$50,000	\$11,206	\$0	\$0	\$11,206	\$744,000	\$43,030	\$0	(\$31,824)	(\$31,824)	\$43,030
4	\$1,040,604	\$61,818	\$0	\$61,818	\$0	\$0	\$61,818	\$742,000	\$42,920	\$0	\$18,898	\$18,898	\$42,920
5	\$1,051,010	\$62,436	\$0	\$62,436	\$0	\$0	\$62,436	\$740,000	\$42,810	\$0	\$19,626	\$19,626	\$42,810
6	\$1,061,520	\$63,061	\$0	\$63,061	\$0	\$0	\$63,061	\$738,000	\$42,700	\$0	\$20,361	\$20,361	\$42,700
7	\$1,072,135	\$63,691	\$0	\$63,691	\$0	\$0	\$63,691	\$736,000	\$42,590	\$0	\$21,101	\$21,101	\$42,590
8	\$1,082,857	\$64,328	\$50,000	\$14,328	\$0	\$0	\$14,328	\$734,000	\$42,480	\$0	(\$28,152)	(\$28,152)	\$42,480
9	\$1,093,685	\$64,971	\$0	\$64,971	\$0	\$0	\$64,971	\$732,000	\$42,370	\$0	\$22,601	\$22,601	\$42,370
10	\$1,104,622	\$65,621	\$0	\$1,170,243	\$0	\$0	\$1,170,243	\$730,000	\$772,260	\$0	\$397,983	\$397,983	\$772,260

IRR of above CF Stream = 6.04%

PV @ 6.44% = \$270,548.

PV @ 4.76% = \$1,104,714

PV @ 4.13% = \$832,202

Value Additivity:

$$E = V - D$$

$$IV(\text{equity}) = \$1,104,714 - \$832,202 = \$272,512 \approx \$270,548^*.$$

*In principle this equation should be exactly equal, otherwise there is some sort of “arbitrage” opportunity between the markets. But in reality valuations are not that precise.

$$\text{APV}(\text{equity}) = +104,714 - \$82,202 = +\$22,512.$$

From property investment

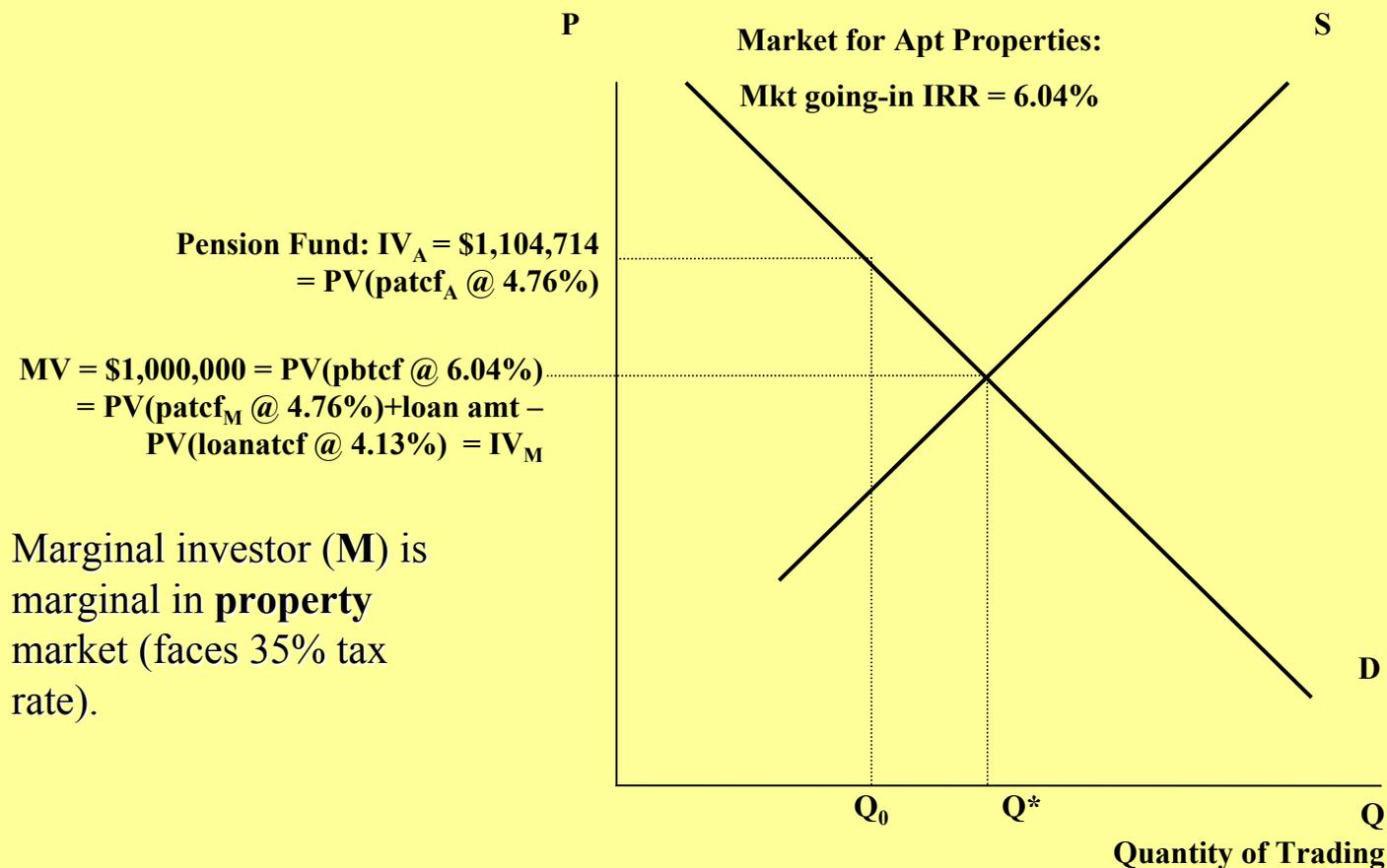
From loan

P.F. has:

- Positive NPV (+104,714) in property investment as intra-marginal buyer,
- Negative NPV (-82,202) in loan borrowing transaction as its tax-exempt status makes it an intra-marginal lender (not borrower).
- Net is still slightly positive (APV).

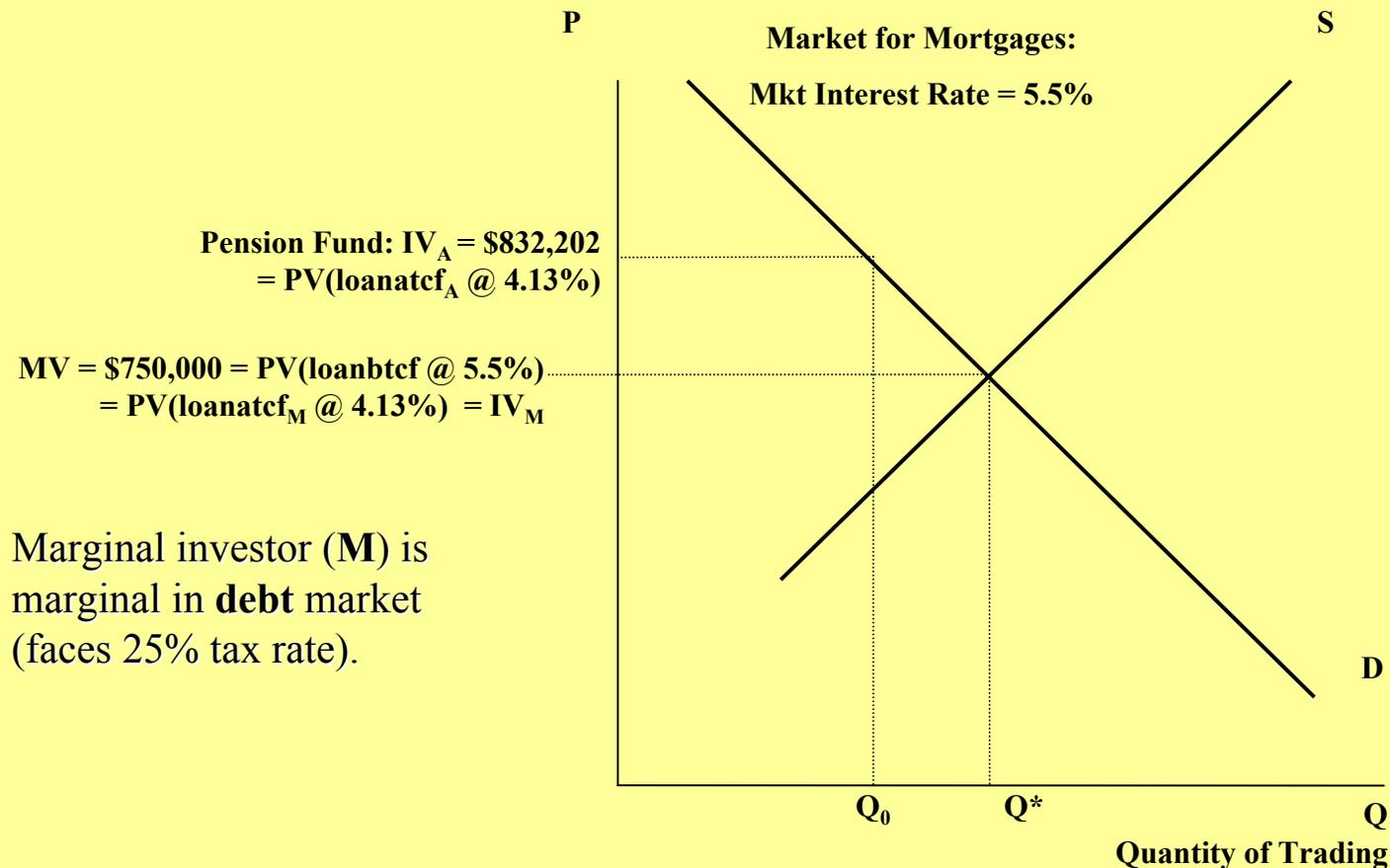
Would be better off not using debt to finance the investment (unless capital constrained).

Representing this as in Ch.12 market model, we have for the property market . . .



Pension fund is an *intra-marginal* buyer.

Representing this as in Ch.12 market model, we have for the mortgage market . . .



Pension fund is an *intra-marginal* buyer (lender, not borrower).