12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

(1)

Use the formula from the notes:

$$\Delta g_{line} = \frac{2bG\Delta\rho\, dx_0\, dz_0}{(x_0 - x)^2 + b^2}$$

In our case we set $x_0 = 0$ (center our graph at $x=0$) and the cross-section area of the linear mass is $dx_0\, dz_0 = \pi R^2$, where $R = 1$ km

So the formula becomes:

$$\Delta g_{line} = \frac{2bG\Delta\rho\pi R^2}{x^2 + b^2}$$ → see matlab code and graphs

Observations: for b = 10 km the amplitude is greater but for x = 100 km the width is greater so that the area under the graph is the same in both cases.

(2)

a) Using the same formula as above we have:

$$\Delta g_{big} = \frac{2bG\Delta\rho\pi R^2}{x^2 + b^2}$$

$$\Delta g_{small} = \frac{2bG\Delta\rho\pi r^2}{(a-x)^2 + b^2} + \frac{2bG\Delta\rho\pi r^2}{(-a-x)^2 + b^2}$$

Where $2\pi r^2 = \pi R^2 \Rightarrow r = \frac{R}{\sqrt{2}}$

i) $0.99\,\Delta g_{big} = \Delta g_{small}$

$$0.99\,\frac{2bG\Delta\rho\pi R^2}{x^2 + b^2} = \frac{2bG\Delta\rho\pi \frac{R^2}{2}}{(a-x)^2 + b^2} + \frac{2bG\Delta\rho\pi \frac{R^2}{2}}{(-a-x)^2 + b^2}$$

$x = 0$

$$0.99\,\frac{2bG\Delta\rho\pi R^2}{b^2} = \frac{bG\Delta\rho\pi R^2}{a^2 + b^2} + \frac{bG\Delta\rho\pi R^2}{a^2 + b^2}$$

$$0.99(a^2 + b^2) = b^2$$

$$b = \sqrt{99}\, a$$

ii) analogically we have

$$b = \sqrt{\frac{0.95}{0.05}}\, a = \sqrt{19}\, a$$

iii) $b = \sqrt{\frac{0.75}{0.25}}\, a = \sqrt{3}\, a$

b) → see graphs

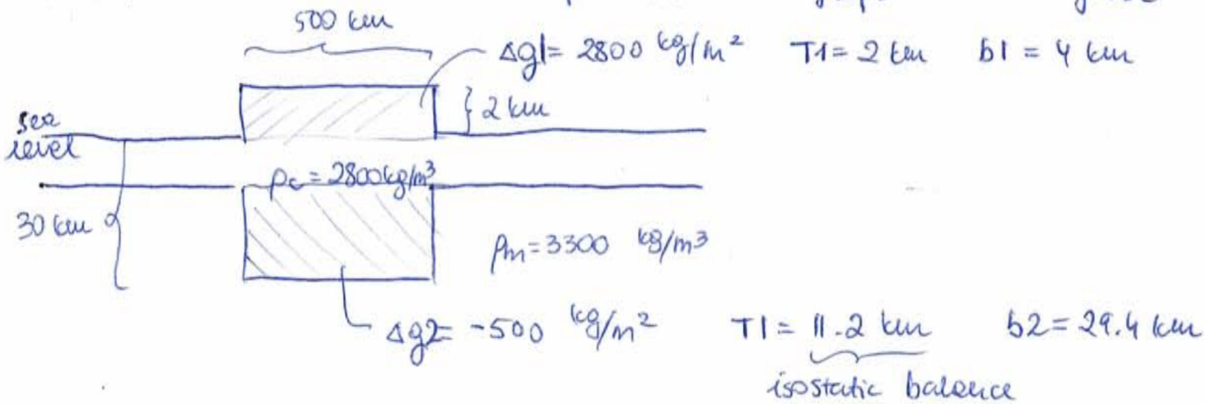
c) Flying at $b = 300$ km we will be able to distinguish between the 2 cases to within 1% error if the distance between the smaller masses is at least 60.3 km.
We should be aware of the limitations of our measuring apparatus so that not to make a false interpretation

(3)

a) Using the formula from the notes:

$$\Delta g_{strip} = 2G\Delta\rho\, d_{20} \left(\tan^{-1}\frac{x+a}{b} + \tan^{-1}\frac{x-a}{b}\right)$$

In our case we have 2 components Δg_{topo} and Δg_{root}

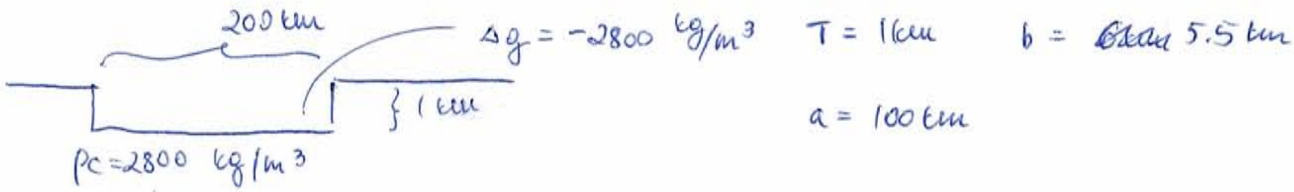


$\Delta g_{total} = \Delta g_{topo} + \Delta g_{root}$ → see matlab code and graph

b) do the same but for $b1 = 149$ km and $b2 = 174.4$ km

(4)

a)



$$\Delta g = 2G\Delta\rho T \left(\tan^{-1}\frac{x+a}{b} + \tan^{-1}\frac{x-a}{b}\right)$$

b) do the same for $b = 150.5$ km

(5)

Venus – highlands exhibit positive gravity anomaly thus they are uncompensated

Moon – highlands exhibit generally neutral gravity (on a big scale) so they are compensated.
Craters are not compensated (or overcompensated). They are low in topography but they have positive gravity signature possibly related to post-impact mantle upwelling.

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% Problem 1

delta_rho = 500;
R = 1000;                   % [m]
b1 = 10000;                 % [m]
b2 = 100000;                % [m]
G = 6.673*10^-11;           % [m^3*kg^-1*s^-2]
x = -100000:100:100000; % [m]

delta_g1 = 2*pi*G*R^2*delta_rho*b1./(x.^2+b1^2);          % [m/s^2]
delta_g2 = 2*pi*G*R^2*delta_rho*b2./(x.^2+b2^2);          % [m/s^2]

subplot(221)
plot(x/1000,delta_g1/10^-5);
xlabel('x in km');
ylabel('gravity anomaly in mgal');
title('Probem 1, b=10 km [cont] and b=100 km [discont]');
hold on
plot(x/1000,delta_g2/10^-5,'--');


% Problem 2

b = sqrt(19)
x = -15:0.01:15;

delta_g1 = b./(x.^2+b^2);
delta_g2 = 0.5.*(b./((x+1).^2+b^2) + b./((x-1).^2+b^2));

subplot(222)
plot(x,delta_g1);
xlabel('x');
ylabel('gravity');
title('Probem 2, big mass [cont] and small [discont]');
hold on
plot(x,delta_g2,'--');

% Problem 3

delta_rho1 = 2800;              % [kg/m^3]
delta_rho2 = -500;              % [kg/m^3]

T1 = 2000;                      % [m]
T2 = 11200;                     % [m]
x = -500000:100:500000;         % [m]
a = 250000;                     % [m]
b1 = 4000;                      % [m]
b2 = 29400;                     % [m]

delta_g1 = 2*G*delta_rho1*T1.*(atan((x+a)/b1) - atan((x-a)/b1)) + 2*G*delta_rho2*T2.*(atan
((x+a)/b2) - atan((x-a)/b2));               % [m/s^2]

b1 = 4000 + 145000;             % [m]
b2 = 29400 + 145000;            % [m]

delta_g2 = 2*G*delta_rho1*T1.*(atan((x+a)/b1) - atan((x-a)/b1)) + 2*G*delta_rho2*T2.*(atan
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((x+a)/b2) - atan((x-a)/b2));           % [m/s^2]

subplot(223)
plot(x/1000,delta_g1/10^-5);
xlabel('x in km');
ylabel('gravity anomaly in mgal');
title('Probem 3, b=5 km [cont] and b=150 km [discont]');
hold on
plot(x/1000,delta_g2/10^-5,'--');
hold on
plot(x/1000,0)

% Problem 4

delta_rho = -2800;                      % [kg/m^3]
T = 1000;                               % [m]
x = -500000:100:500000;                 % [m]
a = 100000;                             % [m]
b = 5500;                               % [m]

delta_g1 = 2*G*delta_rho*T.*(atan((x+a)/b) - atan((x-a)/b));    % [m/s^2]

b = 150500;                             % [m]
delta_g2 = 2*G*delta_rho*T.*(atan((x+a)/b) - atan((x-a)/b));    % [m/s^2]

subplot(224)
plot(x/1000,delta_g1/10^-5);
xlabel('x in km');
ylabel('gravity anomaly in mgal');
title('Probem 4, b=5 km [cont] and b=150 km [discont]');
hold on
plot(x/1000,delta_g2/10^-5,'--');
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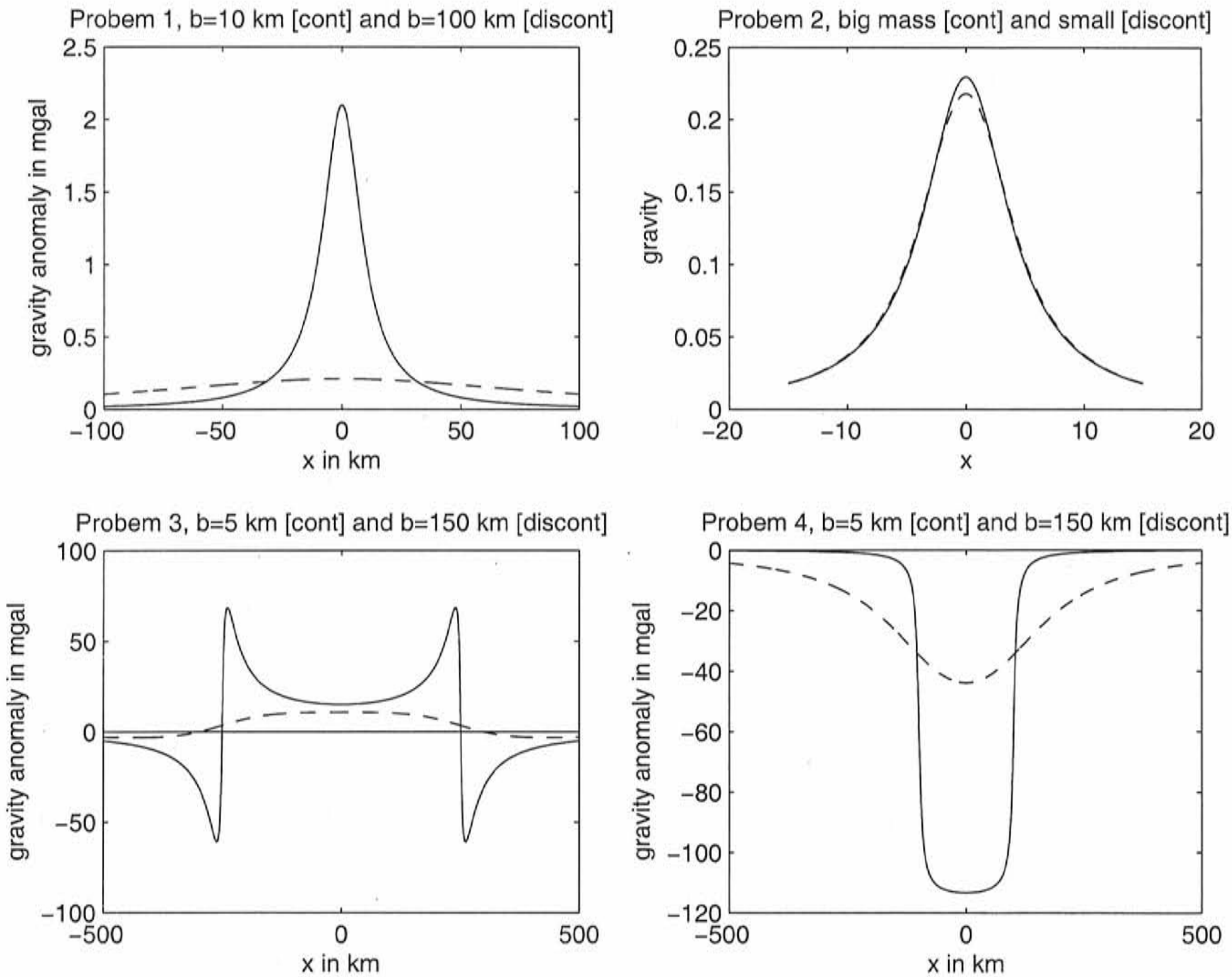
Probem 1, b=10 km [cont] and b=100 km [discont]
gravity anomaly in mgal
x in km
Probem 2, big mass [cont] and small [discont]
gravity
x
Probem 3, b=5 km [cont] and b=150 km [discont]
gravity anomaly in mgal
x in km
Probem 4, b=5 km [cont] and b=150 km [discont]
gravity anomaly in mgal
x in km