12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

12.002 P-Set 6 Fall 2008 Solutions

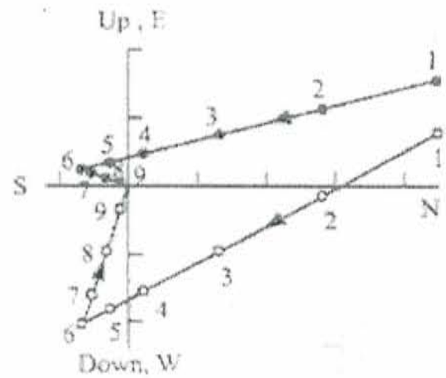
30p **1.**

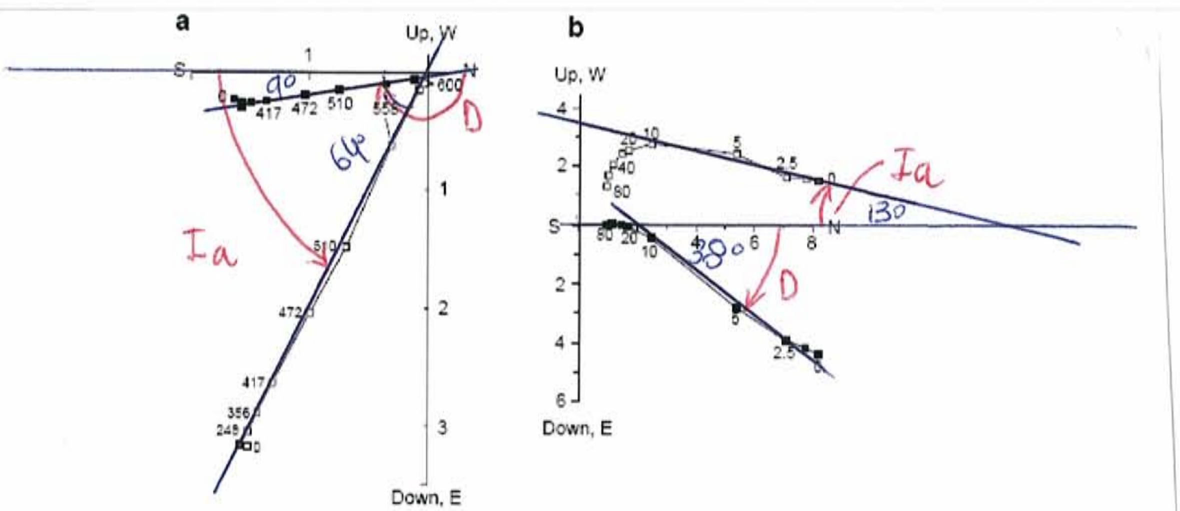
I found the following passage from "Paleomagnetism: Continents and Oceans" by M. W. McElhinny, Phillip L. McFadden, 2000, p .121 helpful:

The declination ($D$) of any component represented by a straight line segment on a Zijderveld diagram is readily determined from the direction of the line in the horizontal plane imagined to commence from the origin in each case. For example, the declination of component $A$ in Fig. 3.14a is determined by transposing the line 1–6 so that point 6 is at the origin and then measuring the angle the line makes with true north. In the vertical plane the angle between the corresponding line and the horizontal axis (transposed in the same way) is the *apparent inclination* ($I_a$), which is related to the true inclination ($I$) by

$$\tan I = \tan I_a\,|\cos \vartheta\,|, \qquad (3.4.1)$$

where $\vartheta$ is the angle between the line in the vertical plane and the common axis that lies in the horizontal plane. In Fig. 3.14 the common axis is NS, in which case $\vartheta = D$. However, if the common axis is EW (as might be chosen when $D$ is closer to 90° or 270° than 0° or 180°) then $\vartheta = D$-90°.




a) $\boxed{D = 171°}$

$I_a = 64°$

$I = \mathrm{atan}(\tan I_a\,|\cos D|)$

$\boxed{I = 63.7°}$

b) $\boxed{D = 38°}$

$I_a = 13°$

$\boxed{I = 10.3°}$

30p **2.**

i = -17.9°, so $\theta = \tan^{-1}\left(\frac{2}{\tan i}\right) = \tan^{-1}\left(\frac{2}{\tan(-17.9)}\right) = -80.83 + 180 = 99.2°$

then we use

$\sin \lambda_p = \sin \lambda_s \cos \theta + \cos \lambda_s \cos \theta \cos D = \sin(35)\cos(99.2) + \cos(35)\sin(99.2)\cos(232.6)$

$\sin \lambda_p = -0.583$ $\qquad\qquad$ $\lambda_p = -35.6° = 35.6°$ S

to find $\varphi_p$ we use: $\varphi_p = \varphi_s + \beta$ or $\varphi_p = \varphi_s + 180 - \beta$ where $\sin \beta = \sin \theta \sin D/\cos \lambda_p$

so $\beta = \sin^{-1}(\sin(99.2)\sin(232.6)/\cos(-35.6) = -74.7°$

$\sin(\lambda_s)\sin(\lambda_p) < \cos(\theta)$, so we use $\varphi_p = \varphi_s + \beta = 241.2° + -74.7° = 166.5°$

so the VGP is **35.6° S 166.5°E**

40p **3. a)**

We have the equations:

$\tan I = 2\cot \theta$
$\sin \lambda_p = \sin \lambda_s \cos \theta + \cos \lambda_s \sin \theta \cos D$

We also know that $\varphi_p = \varphi_s + \beta$ or $\varphi_p = \varphi_s + 180 - \beta$ where $\sin \beta = \sin \theta \sin D/\cos \lambda_p$. $\theta$ is the magnetic co-latitude – it's not the same thing as the geopgraphic co-latitude (ie. 90 - $\lambda_s$). I comes directly from $\theta$, so $\theta$ and D are the two unknowns we need to find. We have two equations involving our two unknowns, $\theta$ and D, but it's not straightforward (I'm not even sure if it's possible) to solve for either $\theta$ or D algebraically. However, we can find $\theta$ a different way. The magnetic co-latitude is basically the angle between the vectors pointing from the center of the earth towards the site and towards the magnetic pole. We can find that angle using the dot product: $A \cdot B = |A||B|\cos\theta$. We can do this by converting the vectors from spherical to cartesian coordinates using:

$x = r \sin(90 - \lambda) \cos \varphi$
$y = r \sin(90 - \lambda) \sin \varphi$
$z = r \cos(90 - \lambda)$

r = radius of the earth = R
so for Iceland:

$\lambda_s = 64$ $\qquad\qquad$ $\lambda_p = 81$
$\varphi_s = -22$ $\qquad\qquad$ $\varphi_p = 171$

$x_s = R \sin(90 - \lambda_s) \cos \varphi_s = R \sin(26)\cos(-22) = 0.4065R$
$y_s = R \sin(90 - \lambda_s) \sin \varphi_s = R \sin(26)\sin(-22) = -.1642R$
$z_s = R \cos(90 - \lambda_s) = R \cos(26) = 0.8989R$
$x_p = R \sin(90 - \lambda_p) \cos \varphi_p = R \sin(9)\cos(171) = -0.1545R$

$y_p = R \sin(90 - \lambda_p) \sin \varphi_p = R \sin(9)\sin(171) = -.0245R$
$z_p = R \cos(90 - \lambda_p) = R \cos(9) = 0.9877R$

so $A \cdot B = R^2(-0.4065*-0.1545 + -0.1642*-0.0245 + 0.8989*0.9877) = .8209\ R^2$
$|A| = |B| = R$
so $\cos\theta = .8209$ and $\theta = 34.8240°$

so $I = \tan^{-1}(2\cot(\theta)) = 70.8°$
and $D = \cos^{-1}(\sin\lambda_p - \sin\lambda_s \cos\theta)/(\cos\lambda_s \sin\theta) = (\sin 81 - \sin 64 \cos 34.8)/(\cos 64 \sin 34.8) = \cos^{-1}(.998) = -3.53°$

so **I = 70.8°** **D = -3.53°**

Doing the same thing for the Supai Shales and Springdale Sandstone (which is easy to do if you use Matlab or Excel) gives:
Supai:
$\lambda_s = 36$ $\lambda_p = 23$
$\varphi_s = -112$ $\varphi_p = 119$

**I = -26.2°**
**D = -47.6°**

Springdale:
$\lambda_s = 37$ $\lambda_p = 60$
$\varphi_s = -113$ $\varphi_p = 110$

**I = 25.2°**
**D = -20.5°**

You should check by plugging the I and D back in and seeing if you get the right coordinates for the pole. You should be careful with your result for D, as cos is an even function!

**b)**
Lots of you came with all sorts of valid reasons you might expect to see scatter in the data, and there is some scatter around each of the two main magnetization directions, but there are clearly two distinct, significantly different populations of measurements. This is not "scatter" and can't be explained by these various sources of error – if the effects of say, measurement errors and secular variation were big enough to get directions that far apart, you'd expect to see points scattered everywhere, not clustered into two groups. The two main directions are about **180° apart**, so the most likely explanation is that the **magnetic field reversed** during deposition of the formation. This is very common – people often date sedimentary rocks by comparing the pattern of reversals in the sedimentary sequence to the known magnetic polarity timescale.

```
% Iceland
lats = 64;
lons = -22;
latp = 81;
lonp = 171;

xs = sind(90-lats)*cosd(lons);
ys = sind(90-lats)*sind(lons);
zs = cosd(90-lats);

xp = sind(90-latp)*cosd(lonp);
yp = sind(90-latp)*sind(lonp);
zp = cosd(90-latp);
```

convert to cartesian

```
theta = acosd(xs*xp+ys*yp+zs*zp);

I = atand(2*cotd(theta));
D1 = acosd((sind(latp)-sind(lats)*cosd(theta))/(cosd(lats)*sind(theta)));
```

positive D

```
beta = asind(sind(theta)*sind(D1)/cosd(latp));
if (cosd(theta)<sind(lats)*sind(latp))
lonp1 = lons + 180 - beta;
else
lonp1 = lons + beta;
end
```

check the $\varphi_p$ for positive D

```
D2 = -D1;
beta = asind(sind(theta)*sind(D2)/cosd(latp));
if (cosd(theta)<sind(lats)*sind(latp))
lonp2 = lons + 180 - beta;
else
lonp2 = lons + beta;
end
```

check the $\varphi_p$ for negative D

```
if (lonp1 == lonp)
D = D1;
else
D = D2;
end
```

check which $\varphi_p$ from above is equal to the ~~[illegible]~~ original $\varphi_p$

```
Iceland_D = D
Iceland_I = I
```

take D for which $\varphi_p$ checks out

```
% Supai
lats = 36;
lons = -112;
latp = 23;
lonp = 119;

% Springdale
lats = 37;
lons = -113;
latp = 60;
lonp = 110;
```

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```
Iceland_D =  -3.5330

Iceland_I =   70.8213

Supai_D =  -47.4524

Supai_I =  -26.2086

Springdale_D =  -20.5069

Springdale_I =   25.2118
```