12.002 Physics and Chemistry of the Earth and Terrestrial Planets
Fall 2008

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Professors Leigh Royden and Benjamin Weiss

Problem Set #3: Seismology
due Friday Oct 10 in class

Problem 1. Snell's Law

You are running an active seismic experiment using dynamite explosions as the source of the seismic waves. Using Snell's law, trace by hand the ray paths for two rays that leave the source at angles of 45° and 30° from the vertical. Both rays traverse a horizontally layered crust. The top layer has a P-wave velocity of 4 km/sec and is 10 km thick. The next layer has a thickness of 20 km and a P-wave velocity of 6 km/sec. The deepest crustal layer has a thickness of 5 km and a P-wave velocity of 7 km/sec. Be accurate – use a protractor!

Problem 2. Ray Paths in the Mantle

Explain clearly, in one to two well-written paragraphs, why the P, PKP and PKiKP paths have the shape shown in Fig. 7 on p. 139 and Fig. 3 p. 141 of the Lehman handout. You will probably want to refer to the velocity-depth diagram shown in Fig. 5 on p. 142 of the same handout. (Ignore the phases PcP, PP, and PKIIKP).

Problem 3. Core Radius and Seismic Velocity from Seismology

You are appointed to plan the next expeditions to Mercury and Mars. Having learned about the importance of seismology during your undergraduate years at MIT, you decide that the missions should include a dense deployment of seismometers that cover the planetary surfaces. (Fortunately seismometers now weigh only 0.25 km and can be dropped onto the surface from a spacecraft, making this plan realistic.) When the data from the first year of seismic deployments comes in, you do some quick data reduction to find:

	v_p (mantle)	v_p (core)	core radius	planet radius
Mercury	8 km/s	6 km/s	1800 km	2440 km
Mars	10 km/s	11 km/s	1200 km	3400 km

Plot the P and PKP travel times for:

(a) Mars

(b) Mercury

Use time in seconds on the vertical axis and Δ from 0° to 150° on the horizontal axis. How are the two planets different? What do the results suggest about the state of the planetary interiors?

Problem 4. Phase transitions in Planetary Mantles

The phase transition from olivine through spinel to perovskite occurs over the depth range of 410-660 km depth in the Earth. The most important, and last, of these is the transformation to perovskite at 660 km depth. This corresponds to a pressure of ~24 GPa (= $24 \cdot 10^9$ Pa = $2.4 \cdot 10^{10}$ Pa). If this phase transformation is mainly a function of pressure, estimate the depths at which this pressure will be reached inside Mars and the Moon. Will it be in the mantle? What does this imply about the possible presence of perovskite in these two planetary bodies?

	radius (km)	average density (kg/m^3)	approximate core radius (km)
Mars	3400	3930	1200 km ?
Moon	1740	3350	400 km ?

You will need to remember that pressure due to overburden is equal to the integral over depth of ρg. You will also want to remember that g changes with depth, or radial distance, inside the planet and is given by $g = M_{inside}G/r^2 = (4\pi/3)\rho G r$ where r is distance from the center of the planet to the point at which g is being computed and M_{inside} is the mass of the material with radial distance less than or equal to r. For simplicity, use a density equal to the average density of each planet (this is not correct since the core is denser than the mantle, but makes calculating easier.)

Problem 5. Error Function

Show that the error function, as given in the notes, is a solution to the heat flow equation.

Problem 6. Thermal Conduction and Lithospheric Thickness

The mission to Mercury also brings back information that the total lithospheric thickness on Mercury is only 50 km.

(a) Assuming that the thermal diffusivity of Mercury's mantle is similar to Earth's (10^{-6} m^2/s), and that the lithosphere formed by cooling conductively from a mantle at 1500°C, compute the approximate age of Mercury's lithosphere using the error function solution to the heat flow equation.

(b) Six month later you receive the results of isotopic dating of Mercury's surface, which shows that the surface age is approximately 4.0 Ga. Obviously you were wrong about the age of the lithosphere (if you did your computation in (a) correctly!). How can you reconcile this new information with the old age for Mercury's surface?