Problem 2

Just NE of Los Angeles, the San Andreas fault trends approximately N65°W–S65°E. To within observational error, the displacement gradient there is observed to be (each year):

$$\begin{bmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{bmatrix}$$

where x_1 is East and x_2 is North, and the units are 10^{-6} strain.

a) Write the (two dimensional) strain tensor, the rotation tensor, and the areal dilation.

b) What are the directions of maximal principal compression and extension?

c) Is this what you would expect, if the San Andreas is a strike-slip fault?

Solution

a) The strain and rotation tensor are found directly from the displacement gradient tensor

$$\frac{du_i}{dx_j} = \begin{bmatrix} \frac{du_1}{dx_1} & \frac{du_1}{dx_2} \\ \frac{du_2}{dx_1} & \frac{du_2}{dx_2} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{bmatrix} \times 10^{-6}/\text{year}.$$

The strain tensor is

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{du_i}{dx_j} + \frac{du_j}{dx_i}\right) = \frac{1}{2}\begin{bmatrix} 2\frac{du_1}{dx_1} & \frac{du_1}{dx_2} + \frac{du_2}{dx_1} \\ \frac{du_2}{dx_1} + \frac{du_1}{dx_2} & 2\frac{du_2}{dx_2} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.12 \\ 0.12 & -0.15 \end{bmatrix} \times 10^{-6}/\text{year}.$$

The rotation tensor is

$$\omega_{ij} = \frac{1}{2}\left(\frac{du_i}{dx_j} - \frac{du_j}{dx_i}\right) = \frac{1}{2}\begin{bmatrix} 0 & \frac{du_1}{dx_2} - \frac{du_2}{dx_1} \\ \frac{du_2}{dx_1} - \frac{du_1}{dx_2} & 0 \end{bmatrix} = \begin{bmatrix} 0.00 & +0.12 \\ -0.12 & 0.00 \end{bmatrix} \times 10^{-6}/\text{year}.$$

The area dilation, $\frac{\delta A}{A}$, is the trace of the strain tensor

$$\frac{\delta A}{A} = \varepsilon_{ii} = \varepsilon_{11} + \varepsilon_{22} = (0.15 - 0.15)10^{-6}/\text{year} = 0/\text{year}$$

and the area is constant.

b) To find the principal strains and directions of the strain tensor, we follow the same procedure that we used to find the principal stresses and directions of the stress tensor (see the solutions for problem set 3 for details).

In brief, the principal strains, per year, are the solutions to the characteristic equation

$$\det(\varepsilon_{ij} - \lambda\delta_{ij}) = \begin{vmatrix} 0.15-\lambda & 0.12 \\ 0.12 & -0.15-\lambda \end{vmatrix} \times 10^{-6}$$

$$\Rightarrow \lambda = \pm\sqrt{(0.12)^2 + (0.15)^2}10^{-6} \approx \pm 0.19 \times 10^{-6}$$

where the positive and negative signs correspond to extension and contraction, respectively. The principal directions, $\hat{n}$, associated with each λ are found as the solutions to

$$\begin{pmatrix} 0.15 \mp 0.19 & 0.12 \\ 0.12 & -0.15 \mp 0.19 \end{pmatrix} \times 10^{-6} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

where the principal directions must form a right handed coordinate system. The solutions can be expressed as

$$\sigma_{nn}^{\text{principal}} = \begin{pmatrix} 0.19 & 0 \\ 0 & -0.19 \end{pmatrix} \times 10^{-6} \text{ and } \hat{n}_{j(n)} = \begin{pmatrix} +0.94 & -0.33 \\ +0.33 & +0.94 \end{pmatrix},$$

where the first matrix is the principal strain tensor, and the columns of the second matrix define the principal frame, given in terms of the original coordinate system. Note that the final solution to this problem is non-unique in terms of the signs of the eigenvectors, which is to say it is non-unique in terms of $\frac{\pi}{2}$ rotations about the axes.

c) Since the San Andreas is strike slip, the strain tensor will be simple shear in a coordinate system with one of the coordinates aligned along the fault trace, and the other normal to the trace. The principal directions of strain for a simple shear are 45° from the original coordinates, since rotating simple shear 45° results in pure shear (*i.e.* no off diagonal elements in the rotated strain tensor). Therefore, one would expect that the principal directions of the strain tensor for the San Andreas fault would be aligned $65^\circ - 45^\circ = 20^\circ$ from the coordinates given above. For the observed strain tensor, the principal directions are about 19.33° from the coordinates, taking care that the sense of slip on the San Andreas is right lateral, we see that the principal directions and strains for the strain tensor above are essentially coincident with what we would expect.