12.005 Lecture Notes 20

Plates

Rock rheology $\rightarrow$ function of T; T increases with depth.

Thin region near surface remains elastic on geologic times. Below this, mantle behaves as a viscous fluid.

Plate theory:

Assume plate thickness $h \ll L$, with L a characteristic length scale

3-D equations simplify

Applications:

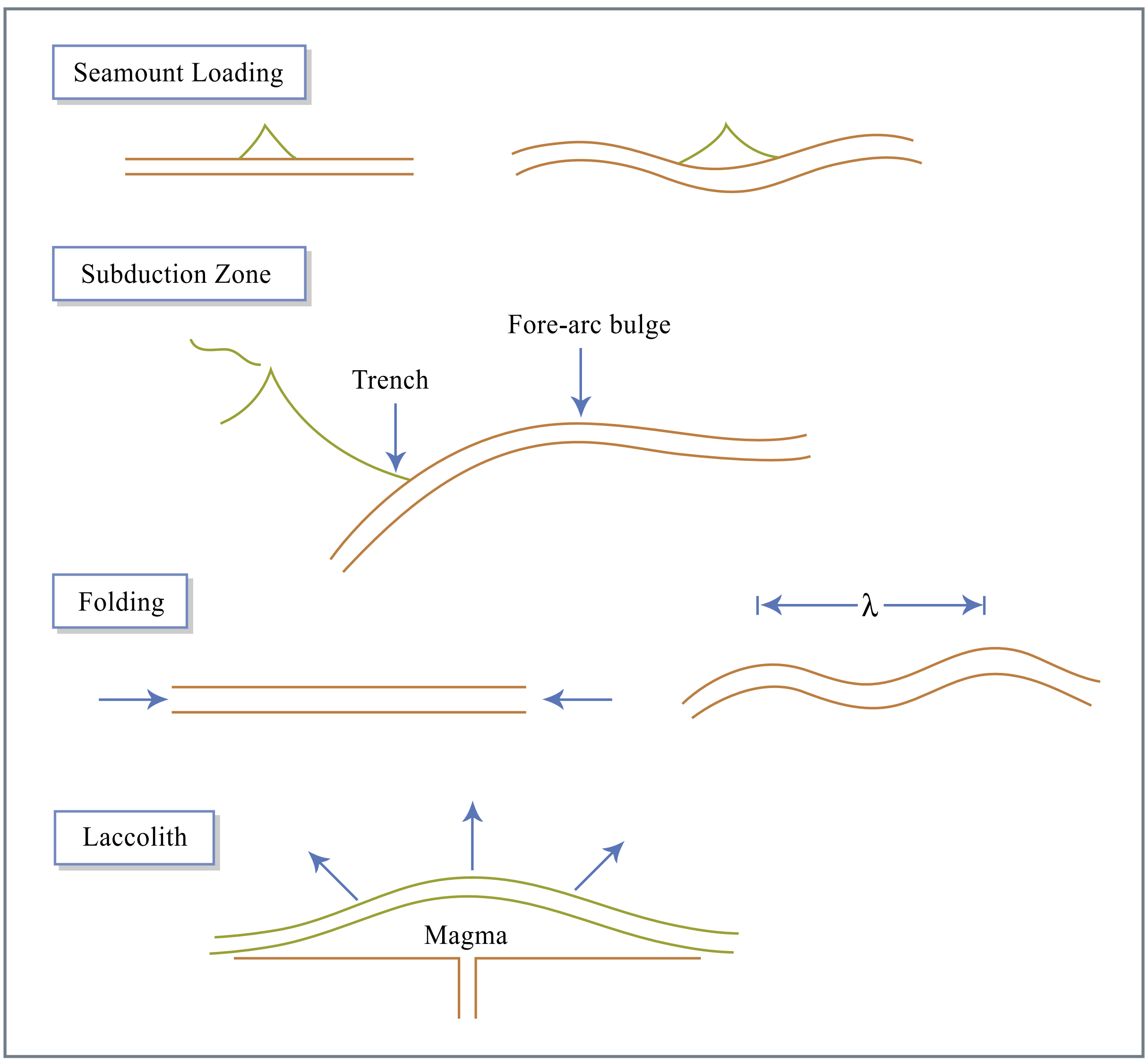

Figure 20.1
Figure by MIT OCW.

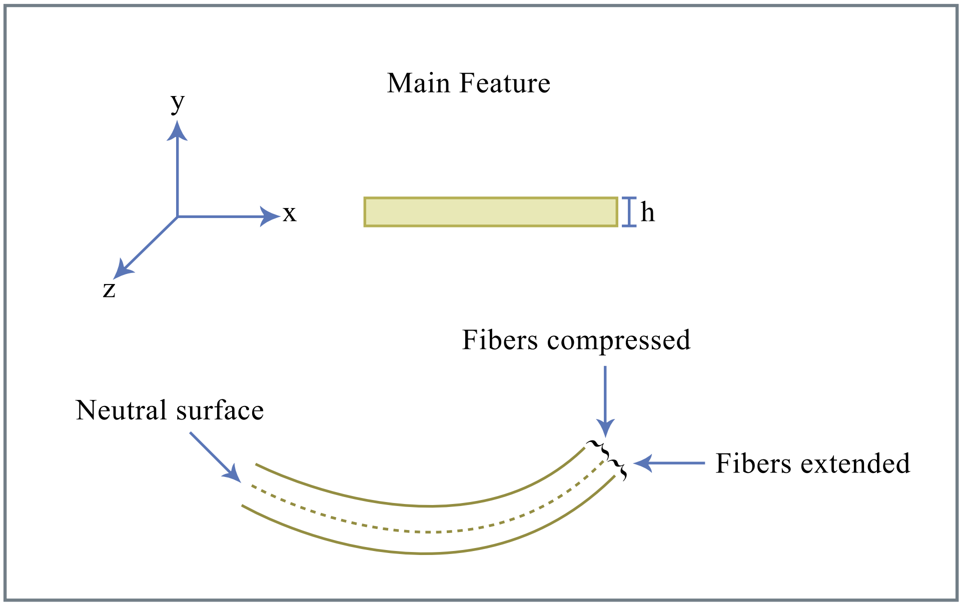

Figure 20.2

Figure by MIT OCW.

"Fiber stresses" large compare to tractions applied to surfaces.

Neutral surface – smooth and centered

Kirchoff's assumption – approximate, but useful.

$$\sigma_{yy} = \sigma_{yx} = \sigma_{yz} = 0$$

$$\sigma_{xx}, \sigma_{zz} \text{ linearly through } -\frac{h}{2} \leq y \leq \frac{h}{2}$$

or

every straight line originally perpendicular to neutral

plane remains straight and perpendicular

More accurate formulations lead to nonlinear coupled equations, not solvable analytically.

To derive plate equations – consider the following figures:

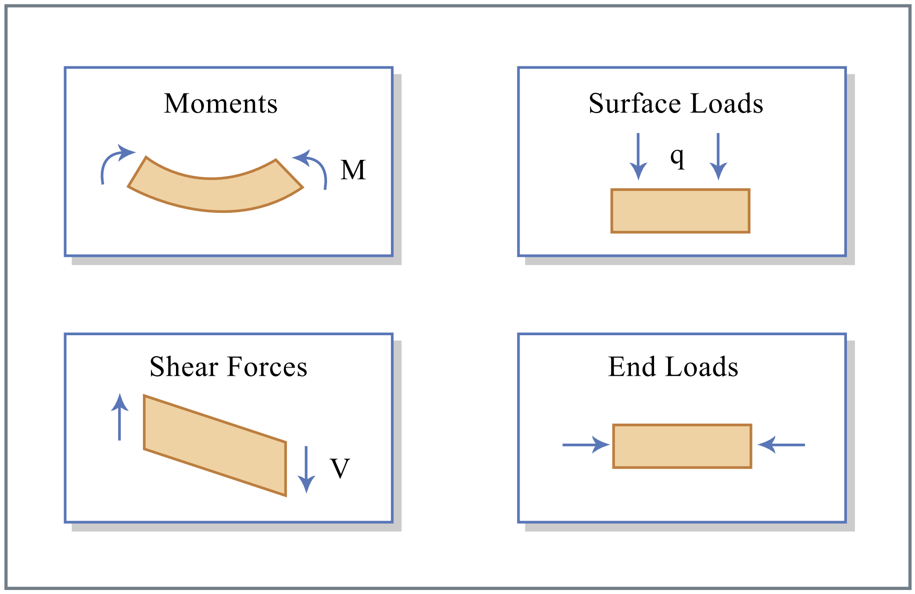

Figure 20.3
Figure by MIT OCW.

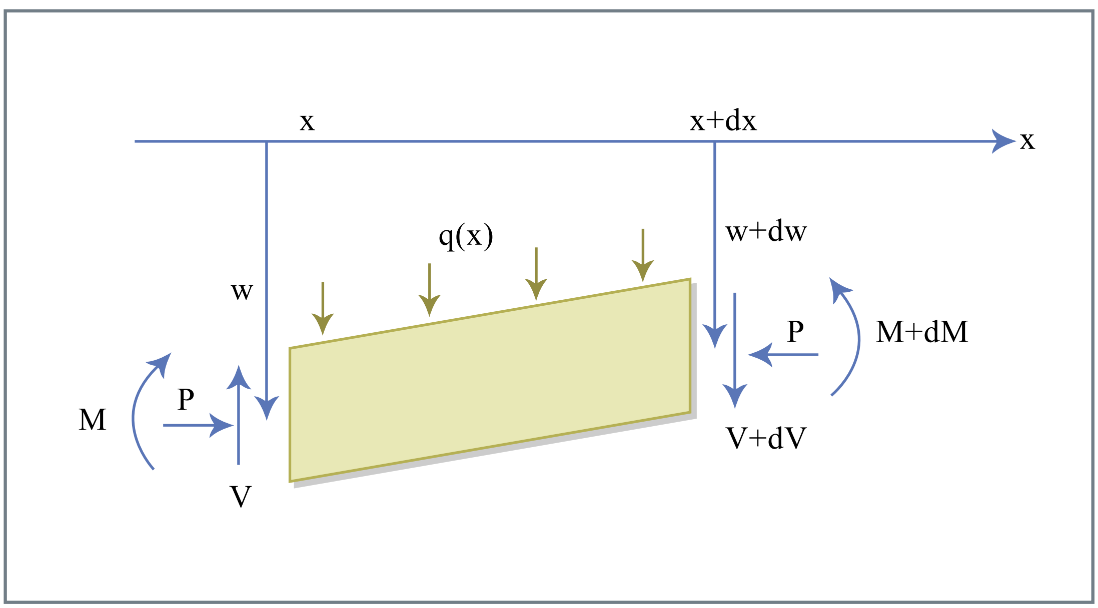

Figure 20.4
Figure by MIT OCW.

V – shear force / unit length

P – horizontal force / unit length

M – moment / unit length

q – force / unit area

Force balance – vertical

$$qdx + dV = 0 \quad \Rightarrow \quad \frac{dV}{dx} = -q$$

Torque balance (+ counterclockwise)

$$dM - Pdw - Vdx = 0$$
$$\frac{dM}{dx} = V + P\frac{dw}{dx}$$

Next – relate *M* to *w*.

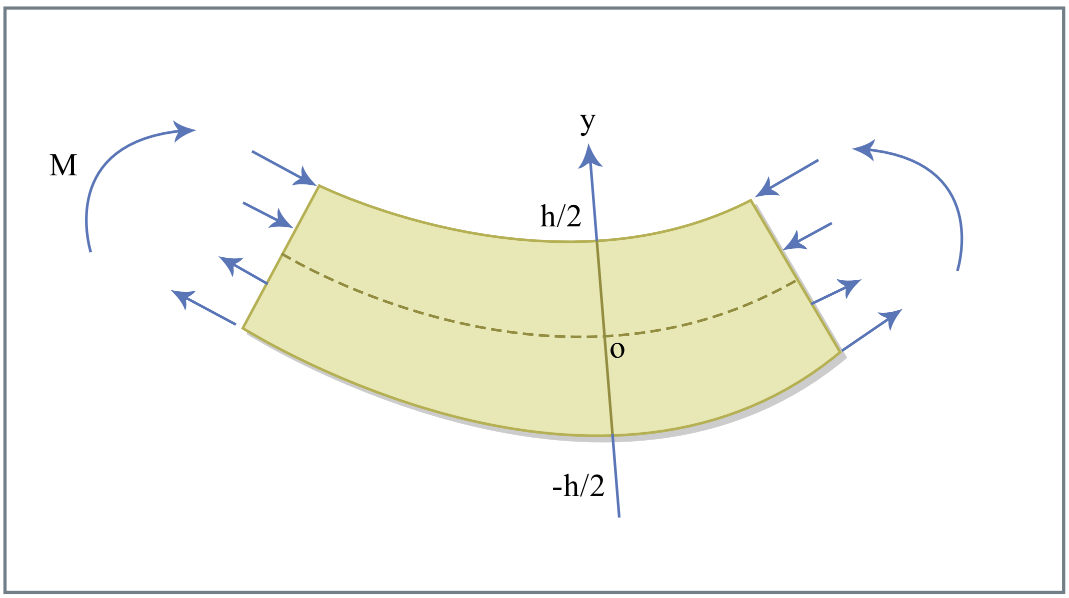

Figure 20.5

Figure by MIT OCW.

$$M = \int_{-h/2}^{h/2} \sigma_{xx} y dy$$

For plane stress,

$$\sigma_{xx} = \frac{E}{(1-\nu^2)} e_{xx}$$

Bending the plate causes fiber strains

$$e_{xx} = -\frac{\delta l}{l} = \frac{y}{R}$$

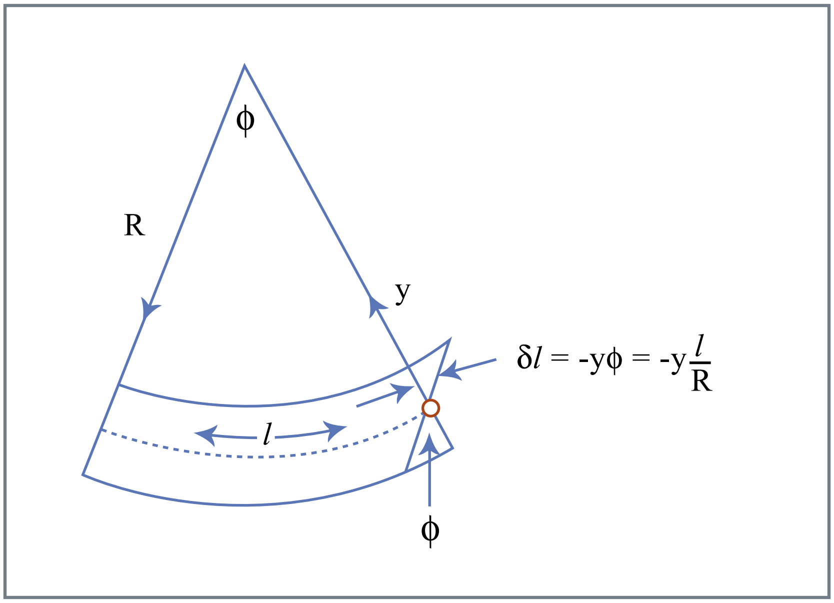

Figure 20.6

Figure by MIT OCW.

Now – let *l* get small

$$\varepsilon_{xx} = -y\frac{d^2 w}{dx^2}$$

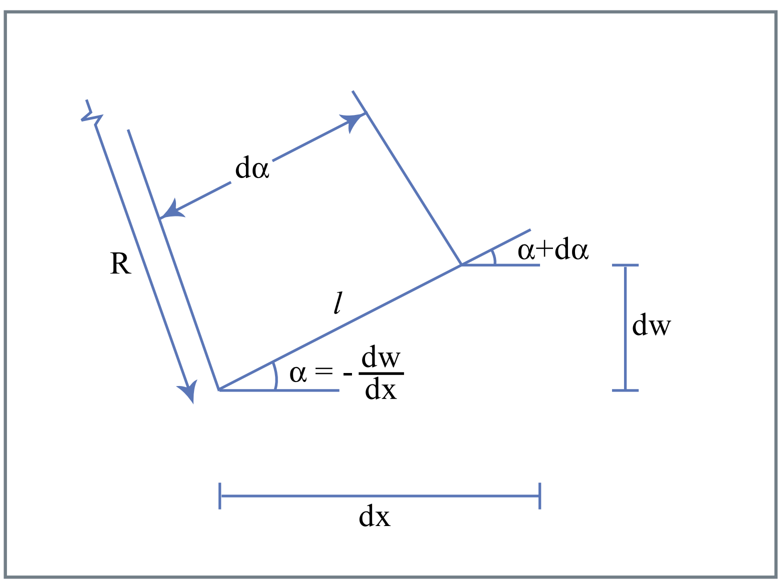

Figure 20.7

Figure by MIT OCW.

Then, substituting

$$M = -\frac{E}{(1-\nu^2)} \frac{d^2 w}{dx^2} \int_{-h/2}^{h/2} y^2 dy$$

$$M = -\frac{Eh^3}{12(1-\nu^2)} \frac{d^2 w}{dx^2}$$

or

$$M = -D \frac{d^2 w}{dx^2} = \frac{D}{R}$$

where $D \equiv \frac{Eh^3}{12(1-\nu^2)}$ (flexural rigidity)

Substituting

$$D \frac{d^4 w}{dx^4} = q(x) - P \frac{d^2 w}{dx^2}$$

This equation is called plate equation.