Problem Solution

Just NE of Los Angeles, the San Andreas fault trends approximately N65°W - S65°E. To within observational error, the displacement gradient there is observed to be (each year):

$$\begin{bmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{bmatrix}$$

where x_1 is East, x_2 is North, and the units are 10^{-6} strain.

a) Write the (two dimensional), strain tensor, the rotation tensor, and the area dilatation.
b) What are the directions of maximum principal compression and extension?
c) Is this what you expect, if the San Andreas is a strike-slip fault?

Note: The SAF trends approximately N45°W - S45° everywhere, but around the LA area it slightly bends and has a direction approximately N65°W - S65°E

Solution:

The 2D displacement gradient tensor is $\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{bmatrix} \cdot 10^{-6}$

a)

The 2D strain tensor is the symmetric part of the displacement gradient tensor:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \text{ hence } \varepsilon = \begin{bmatrix} 0.15 & 0.12 \\ 0.12 & -0.15 \end{bmatrix} \cdot 10^{-6}$$

The 2D rotation tensor is the anti symmetric part of the displacement gradient tensor:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right), \text{ hence } \omega = \begin{bmatrix} 0 & 0.12 \\ -0.12 & 0 \end{bmatrix} \cdot 10^{-6}$$

and represents clockwise rotation if the ω_{12} component is positive.

The area dilatation is $\frac{\delta A}{A} = \varepsilon_{11} + \varepsilon_{22} = 0$

b)

The directions of maximum principal compression and extension:

The principal values for the strain tensor are (let's take $a = 0.15 \cdot 10^{-6}$ and $b = 0.12 \cdot 10^{-6}$):

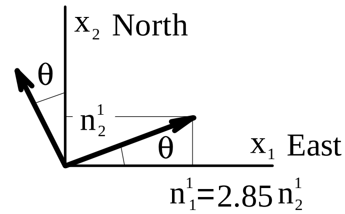
$$\begin{vmatrix} a-\lambda & b \\ b & -a-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda_{1,2} = \pm\sqrt{a^2+b^2} \approx \pm 1.9\cdot 10^{-7},$$

since we need to know just 2D principal directions, we can find one direction (here it will be for extension, i.e. positive λ)

$$\begin{bmatrix} 0.15-0.19 & 0.12 \\ 0.12 & -0.15-0.19 \end{bmatrix} \cdot \begin{bmatrix} n_1^{(1)} \\ n_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Rightarrow \quad n_1^{(1)} \approx 2.85 n_2^{(1)}$$

and from its component ratio determine the angle to the x_1 axis as :

$$\theta = \arctan\left(\frac{n_2^{(1)}}{n_1^{(1)}}\right) = \arctan\left(\frac{1}{2.85}\right) \approx 19.33^0$$



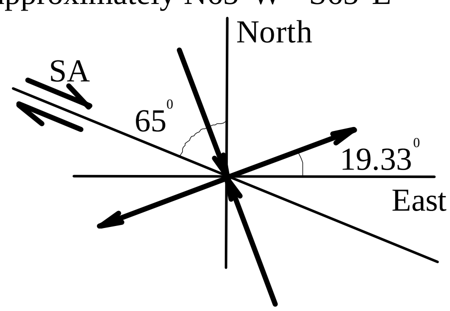
Then the other principal direction (for compression) will be perpendicular to the first one and have an angle θ to the x_2 axis.

A different way to find out these directions is to use equation 2.120 in T&S :
$\tan 2\theta = \dfrac{2\varepsilon_{12}}{\varepsilon_{11}-\varepsilon_{22}}$, that gives the same answer: $\tan 2\theta = \dfrac{2\cdot 0.12}{0.30} = 0.8 \quad \Rightarrow \quad \theta \approx 19.33^0$

So, for this fault the direction of maximum compression is θ≈19.33° from the N-S axis and the direction of maximum extension is θ≈19.33° from the W-E axis.

c)

If the San Andreas is a strike-slip fault trending approximately N65°W - S65°E then it looks like:



the angle between SA and the maximum extension direction is 65°-19.33° which is approximately 45°. That means that the principal directions are indeed the principal directions for San Andreas fault near LA as a strike-slip fault.

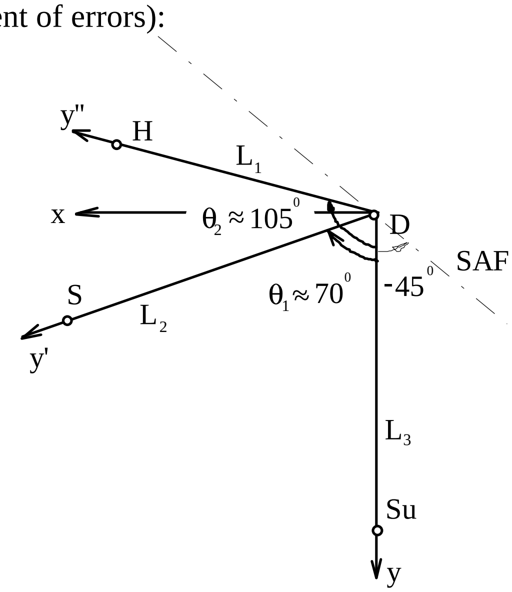
Problem Solution

Turcotte & Schubert, Problem 2-27.

Given in Figure 2-32 are the line lengths between the monument at Diablo and the monuments at Hills, Skyline, and Sunol obtained between 1970 and 1978 using a geodimeter. Assuming a uniform strain field, determine $\dot{\varepsilon}_{xx}$, $\dot{\varepsilon}_{yy}$, and $\dot{\varepsilon}_{xy}$. Take the Sunol-Diablo line to define the y coordinate. Discuss the results in terms of strain accumulation on San Andreas fault, which can be assumed to trend at 45° with respect to the Sunol-Diablo line.

(Don't work too hard getting a formal estimate of the slope of the lines through the data - an eyeball fit is OK. But do consider what a reasonable estimate of uncertainty might be.)

Solution (We do not carry out a rigorous treatment of errors):



Let's assume that line D-Su defines the y-coordinate, line D-S defines y' and line D-H - the y''-coordinate.

From the data on Fig. 2-32 of T&S we can estimate:

D-H: $dL_1/dt \approx +25\text{mm}/8\,\text{years} \approx 3$ mm/year

D-S: $dL_2/dt \approx -0\text{mm}/8\,\text{years} \approx 0$ mm/year

D-Su: $dL_3/dt \approx -50\text{mm}/8\,\text{years} \approx$ -6 mm/year

All the distances and angles we need are:

$L_1 = 19742$ m $\theta_1 \approx 70^0$

$L_2 = 23934$ m $\theta_2 \approx 105^0$

$L_3 = 29067$ m

the strain rates for all baselines are:

$$\dot{\varepsilon}_{yy} = \frac{dL_3/dt}{L_3} \approx \frac{-6}{2.9\cdot 10^7} \approx -2.0\cdot 10^{-7}\ \text{year}^{-1}$$

$$\dot{\varepsilon}_{y'y'} = \frac{dL_2/dt}{L_2} \approx \frac{0}{2.4\cdot 10^7} \approx 0\ \text{year}^{-1}$$

$$\dot{\varepsilon}_{y''y''} = \frac{dL_1/dt}{L_1} \approx \frac{3}{2.0\cdot 10^7} \approx 1.5\cdot 10^{-7}\ \text{year}^{-1}$$

using the equations (2.135-2.136) from T&S (or applying the rotation matrix, or applying the Mohr's circle technique):

$$\dot{\varepsilon}_{y'y'} = \dot{\varepsilon}_{yy}\cos^2\theta_1 + \dot{\varepsilon}_{xx}\sin^2\theta_1 + \dot{\varepsilon}_{xy}\sin 2\theta_1$$

$$\dot{\varepsilon}_{y''y''} = \dot{\varepsilon}_{yy}\cos^2\theta_2 + \dot{\varepsilon}_{xx}\sin^2\theta_2 + \dot{\varepsilon}_{xy}\sin 2\theta_2$$

plugging in values:

$$0 = -2\cdot 0.12 + \dot{\varepsilon}_{xx}\cdot 0.88 + \dot{\varepsilon}_{xy}\cdot 0.64$$

$$1.5 = -2\cdot 0.07 + \dot{\varepsilon}_{xx}\cdot 0.93 + \dot{\varepsilon}_{xy}\cdot(-0.5)$$

we find the strain rates:

$$\dot{\varepsilon}_{xx} \approx 1.1\cdot 10^{-7}\ \text{year}^{-1}$$

$$\dot{\varepsilon}_{yy} \approx -2.0\cdot 10^{-7}\ \text{year}^{-1}$$

$$\dot{\varepsilon}_{xy} \approx -1.2\cdot 10^{-7}\ \text{year}^{-1}$$

where positive values mean lengthening (area under extension in this direction) and negative - shortening (area under compression). Note the notation is opposite in T&S.

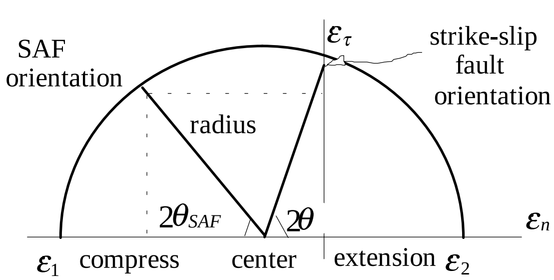
From the lecture notes we know that the San Andreas Fault (SAF) is a strike-slip fault trending approximately N45°W - S45°E . Let's see how it correlates with our results:

The strain rate tensor in the observed area is $\dot{\varepsilon} = \begin{bmatrix}\dot{\varepsilon}_{xx} & \dot{\varepsilon}_{xy}\\ \dot{\varepsilon}_{yx} & \dot{\varepsilon}_{yy}\end{bmatrix} = \begin{bmatrix}11 & -12\\ -12 & -20\end{bmatrix}\cdot 10^{-8}\ /\ year$

Its principal values are $\dot{\varepsilon}_1 \approx -24\cdot 10^{-8}$ and $\dot{\varepsilon}_2 \approx 15\cdot 10^{-8}$

The principal directions are $\begin{bmatrix}11-15 & -12\\ -12 & -20-15\end{bmatrix}\cdot\begin{bmatrix}n_x^{(2)}\\ n_y^{(2)}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix} \Rightarrow n_x^{(2)} \approx -3n_y^{(2)}$, hence the angle between the principal extensional axis ($\dot{\varepsilon}_2 \approx 15\cdot 10^{-8}$) and the x-axis is $\theta \approx 18°$.

Now let's draw a strain orientation picture and apply the Mohr's circle diagram approach to find out the strain rate state in the observed area:



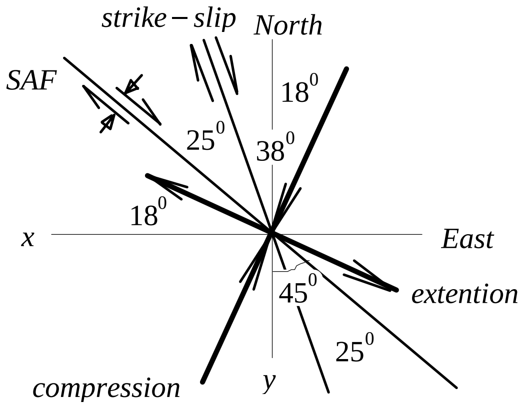
The center of the circle at

$$(\dot{\varepsilon}_1 + \dot{\varepsilon}_2)/2 \approx -4.4\cdot 10^{-8}$$

The radius is $(\dot{\varepsilon}_2 - \dot{\varepsilon}_1)/2 \approx 19.6\cdot 10^{-8}$

Since the orientation of the SAF is 45° from the EW direction or 45°-18°=27° from the extension axis, the double angle between the normal to the SAF and the compression axis is $2\theta_{SAF} = 54^0$.

From Mohr's circle one can define the value for the strain rate on the SAF:



$$\dot{\varepsilon}_\tau = rad \cdot \sin 2\theta_{SAF} \approx -15.9 \cdot 10^{-8} / yr$$
$$\dot{\varepsilon}_n = center + rad \cdot \cos 2\theta_{SAF} \approx -15.8 \cdot 10^{-8} / yr$$

Values for the strain rate on a strike-slip fault for this region are:

$$\dot{\varepsilon}_\tau = \sqrt{radius^2 + center^2} \approx -19.1 \cdot 10^{-8} / yr$$
$$\dot{\varepsilon}_n = 0$$

The result of this analysis based on the data given is:

- In the observed region the SAF is a combined strike-slip and thrust fault.
- The area dilatation observed in the region is negative.

Note that the value for "shear" strain rate on the strike-slip plane $\dot{\varepsilon}_\tau \approx -1.9 \cdot 10^{-7} / \text{year}$ is very close to the results obtained from solving problem 2) of the current homework (principal values of strain tensor are $\pm 1.9 \cdot 10^{-7} / \text{year}$). It means that the state of strain accumulation is almost the same for the San-Francisco area and LA, but rotated ~25°. It seems to be reasonable for plate-like horizontal scale for the driving forces and block-like horizontal scale for the geological material under stress.

Also note that the region discussed in this problem is where the destructive Loma Prieta earthquake occurred in 1989. This earthquake had a thrust component, as well as a strike-slip component.