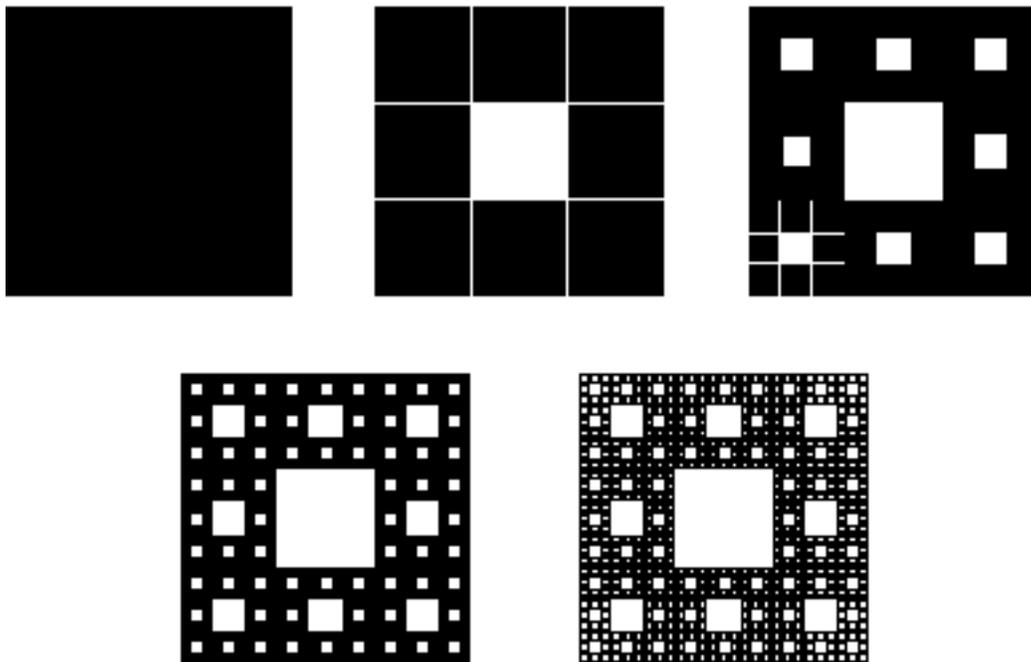


Problem Set 9

1. Fractal Dimensions

- (a) Construct a fractal that is similar to the Cantor set, but instead remove the middle $1/2$ from each previous section. Show analytically that its fractal dimension is $1/2$.
- (b) Calculate the fractal dimension of the Sierpinski carpet shown in the pictures.



2. The Hénon map is given by

$$\begin{aligned}X_{n+1} &= 1 + Y_n - \alpha X_n^2 \\ Y_{n+1} &= \beta X_n\end{aligned}$$

- (a) What are the fixed points? (Show analytically.)
- (b) Find (analytically) the value α_c such that for $\alpha > \alpha_c$ the fixed points are unstable.
- (c) `henon` is a numerical implementation of the Hénon map. Choose $\beta = 0.3$ and $\alpha = 1.4$, and initialize the iterates close to (but not exactly on) the unstable fixed point which lies close to the attractor. From your answers to a) and b) and some additional analysis, predict the slope of the attractor near the fixed points and then confirm your prediction from the numerical results.

3. The Lorenz model is given by

$$\begin{aligned}\dot{X} &= PY - PX \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ\end{aligned}$$

where P is the Prandtl number, b is a constant, and r is the ratio of the Rayleigh number to the critical Rayleigh number. Using $r = 28$, $P = 10$ and $b = 8/3$, generate time series in X and estimate the fractal dimension D using the correlation dimension ν .

As discussed in class, $\nu \leq D$. Here, the precise values of ν are dependent on lots of things, like integration timestep, total number of data points, and the embedding dimension. By experimenting with different combinations, see how close you can get to the correct value of $D \simeq 2.06$ ¹.

We've provided code to help you do this, but there a few extra steps required: let us know as soon as possible if there are any issues.

Python: `fracdim.py` runs the Lorenz model and calculates the correlation dimension using an implementation of the Grassberger-Procaccia algorithm given as part of the package `nolds`. This package isn't part of default installs and needs to be installed separately: generally, typing `pip3 install nolds` into the terminal should be sufficient.

Matlab: `fracdim.m` calculates correlation dimension using a Matlab-provided algorithm that's a bit better optimized. You'll need the Predictive Maintenance Toolbox to be installed.

¹From our own experiments, it's possible to get to at least $\nu \simeq 2.01$ in the Python code and $\nu \simeq 2.05$ in the Matlab code, but don't stress about it if you have trouble getting there.

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