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1 Plate tectonics: the volcanic source

Earth's biological carbon cycle derives its energy from the sun, which fuels photosynthesis.

A small portion of the organic carbon that is fixed is buried as rock. Likewise, some inorganic carbon is buried as carbonate.

If this burial were to continue without replacement, all the carbon in the atmosphere and oceans would be gone within about 10^6 yr.

There must therefore be a resupply of CO₂ from geologic sources.

This resupply of CO₂ comes via volcanism. The relative magnitude of its flux is small [1]:

source	flux (Gt C/yr)
volcanism	0.1
other degassing	0.1
combustion	8.7
respiration	100

Volcanism is nevertheless hugely significant at evolutionary time scales: without it, the carbon cycle would have long ago grinded to a halt, and life as we know it would not exist!

In this way the biological carbon cycle is inextricably tied to the geological carbon cycle, or “rock cycle.”

The rock cycle is fueled by the heat flux that comes out of the Earth. We proceed to describe an important manifestation of this heat flux: *plate tectonics*.

We then use mathematical and physical reasoning to quantitatively evaluate predictions of plate tectonics.

1.1 Thermal convection and plate tectonics

There are two internal sources of heat:

- Heat accumulated during the process of planetary accretion, i.e., the gravitational energy dissipated by the formation of the Earth 4.5 Ga.
- Heat generated by the radioactive decay of uranium, thorium, and potassium.

To first approximation, then, we can think of the Earth as being heated from within and cooled at its surface.

We seek an understanding of how the resulting heat flux generates volcanism and crustal motion at the surface.

1.2 Conductive vs. convective heat flow

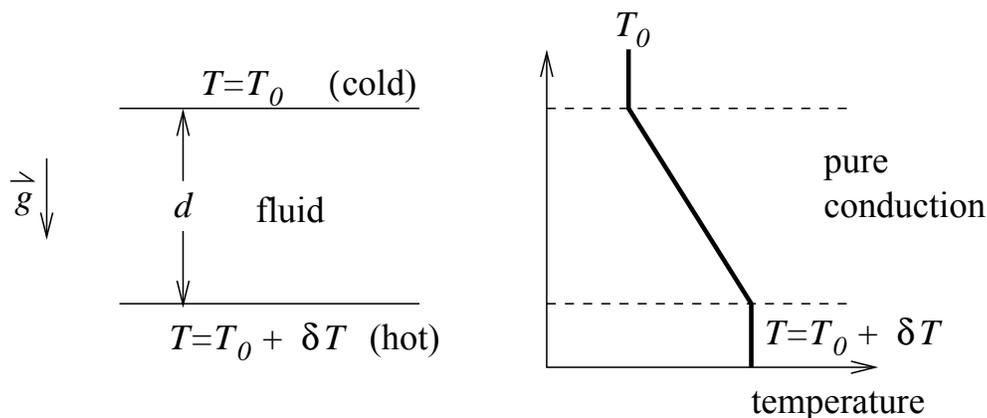
Measurements of temperature within the Earth’s crust show that heat “flows” upward out of the Earth.

In general, there are two types of heat flux: conductive and convective.

- Conductive heat flux: only heat is transported, but not the material being heated.
- Convective heat flux: material is transported along with heat.

Which one characterizes the geophysical heat flux?

To investigate this problem, we consider a simpler problem in which a fluid confined between two parallel heat-conducting plates is heated from below.



In the absence of convection—the transport of hot fluid *up* and cold fluid *down*—the temperature gradient is constant.

Recall, however, that hot fluid rises in a thermally expansive fluid.

There are thus two cases of interest:

- δT small: no convective motion, due to stabilizing effects of viscous friction.
- δT large: convective motion occurs.

The convective case corresponds to the plate tectonic motions that interest us.

1.3 Rayleigh number

Reference: [2].

How large, then, is a “large δT ” ? We seek a non-dimensional formulation.

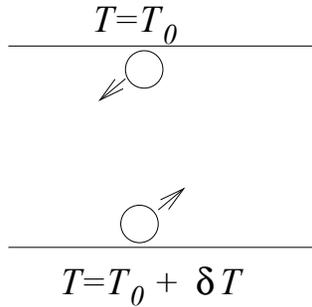
The following fluid properties are important:

- viscosity
- density
- thermal expansivity
- thermal diffusivity (heat conductivity)

Convection is also determined by

- d , the box size
- δT (of course)

Consider a small displacement of a cold blob downwards and a hot blob upwards:



Left undisturbed, buoyancy forces would allow the hot blob to continue rising and the cold blob to continue falling.

There are however damping (dissipation) mechanisms:

- diffusion of heat
- viscous friction

Let D = thermal diffusivity, which has units

$$[D] = \frac{\text{length}^2}{\text{time}}$$

The temperature difference between the two blobs can therefore be maintained at a characteristic time scale

$$\tau_{\text{th}} \sim \frac{d^2}{D}$$

We also seek a characteristic time scale for buoyant displacement over the length scale d .

Let

$$\begin{aligned} \rho_0 &= \text{mean density} \\ \Delta\rho &= -\alpha\rho_0\Delta T, \quad \alpha = \text{expansion coefficient} \end{aligned}$$

Setting $\Delta T = \delta T$,

$$\begin{aligned} \text{buoyancy force density} &= |\vec{g}\Delta\rho| \\ &= g\alpha\rho_0\delta T. \end{aligned}$$

Note units:

$$[g\alpha\rho_0\delta T] = \frac{\text{mass}}{(\text{length})^2(\text{time})^2}$$

The buoyancy force is resisted by viscous friction between the two blobs separated by $\sim d$.

Viscosity is analogous to heat diffusivity, but it instead refers to the diffusion of momentum, so that fast-moving fluid causes nearby slow-moving fluid to move faster, and vice-versa.

Because viscous stresses \propto velocity gradients, the viscous friction between the two blobs diminishes like $1/d$.

We represent the viscosity with the coefficient μ , which has dimensions

$$[\mu] = \left(\frac{\text{mass}}{\text{volume}} \right) \times \text{diffusivity} = \frac{M L^2}{L^3 T} = \frac{M}{LT}.$$

The rescaled viscosity has units

$$\left[\frac{\mu}{d} \right] = \frac{\text{mass}}{(\text{length})^2(\text{time})}.$$

Dividing the rescaled viscosity by the buoyancy force, we obtain the characteristic time τ_m for convective motion:

$$\tau_m \sim \frac{\mu/d}{\text{buoyancy force}} = \frac{\mu}{g\alpha\rho_0 d \delta T}.$$

Convection (sustained motion) occurs if

time for motion $<$ diffusion time for temperature difference

$$\tau_m < \tau_{\text{th}}$$

Thus convection requires

$$\frac{\tau_{\text{th}}}{\tau_m} > \text{constant}$$

or

$$\frac{\rho_0 g \alpha d^3}{\mu D} \delta T \equiv \text{Ra} > \text{constant}$$

Ra is called the *Rayleigh number*. (A detailed stability calculation reveals that the critical constant is 1708.)

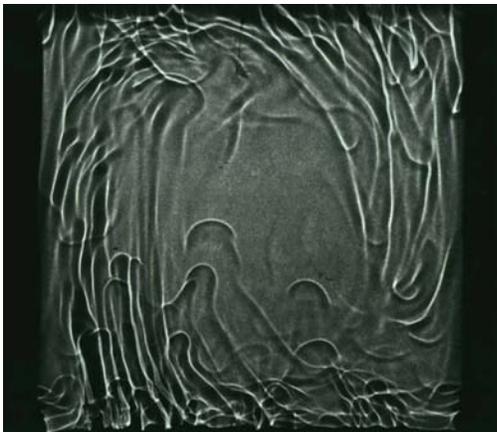
Our derivation of the Rayleigh number shows that the convective instability is favored by

- large δT , α , d , ρ_0 .
- small μ , D .

In other words, convection occurs when the buoyancy force $\rho_0 g \alpha d^3 \delta T$ exceeds the dissipative effects of viscous drag and heat diffusion.

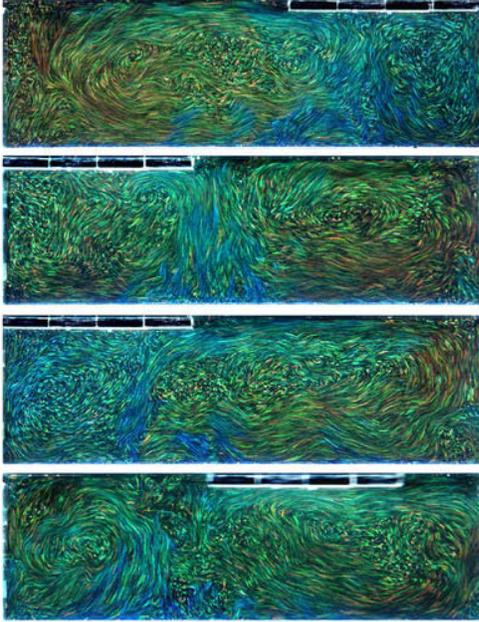
Note that box height enters Ra as d^3 . This means that small increases in box size can have a dramatic effect on Ra.

For Ra sufficiently large, the flow becomes turbulent. Some examples (from Prof. Jun Zhang, NYU, <http://www.physics.nyu.edu/~jz11/>):

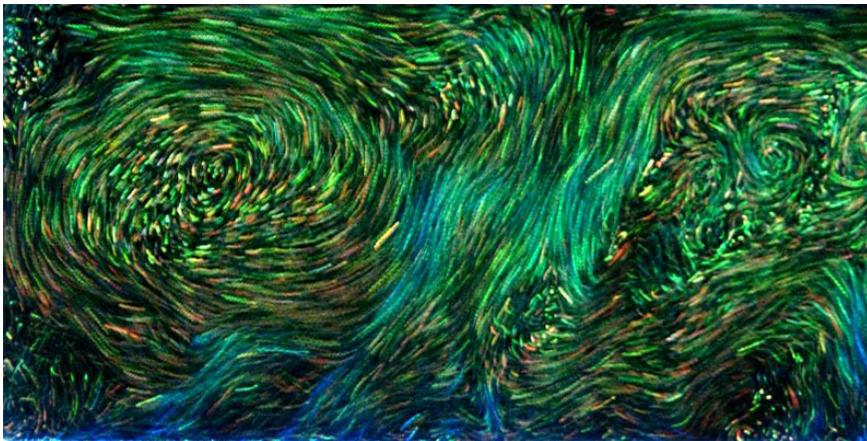


Here the gray scale is related to the thermal gradient.

Image courtesy of Prof. Jun Zhang, New York University. Used with permission.



Here the viscous flow moves the floating boundary and the the boundary affects the flow, an interplay roughly analogous to fluid motions beneath tectonic plates.



A close-up (red is cool, blue is warm).

Image courtesy of Prof. Jun Zhang, New York University. Used with permission.

1.4 Convection in the Earth

The Earth's radius is about 6378 km. It is layered, with the main divisions being the inner core, outer core, mantle, and crust.

The Earth's crust—the outermost layer—is about 30 km thick.

The mantle ranges from about 30–2900 km.

The mantle is widely thought to be in a state of thermal convection. The source of heat is thought to be the radioactive decay of isotopes of uranium,

thorium, and potassium. Another heat source is related to the heat deriving from the gravitational energy dissipated by the formation of the Earth roughly 4.5 Ga.

At long time scales mantle rock is thought to flow like a fluid. However its effective viscosity is the subject of much debate.

One might naively think that the huge viscosity would make the Rayleigh number quite small. Recall, however, that Ra scales like d^3 , where d is the “box size”. For the mantle, d is nearly 3000 km!!!

Consequently Ra is probably quite high. Estimates suggest that

$$3 \times 10^6 \lesssim Ra_{\text{mantle}} \lesssim 10^9$$

which corresponds to roughly

$$10^3 \times Ra_c \lesssim Ra_{\text{mantle}} \lesssim 10^6 Ra_c$$

The uncertainty derives principally from the viscosity, and its presumed variation by a factor of about 300 with depth.

Regardless of the uncertainty, we can conclude that Ra for the mantle is more than sufficient for convection, and therefore that convection is likely the driving force of plate tectonics.

1.5 Mid-ocean ridges, seafloor age, and volcanism

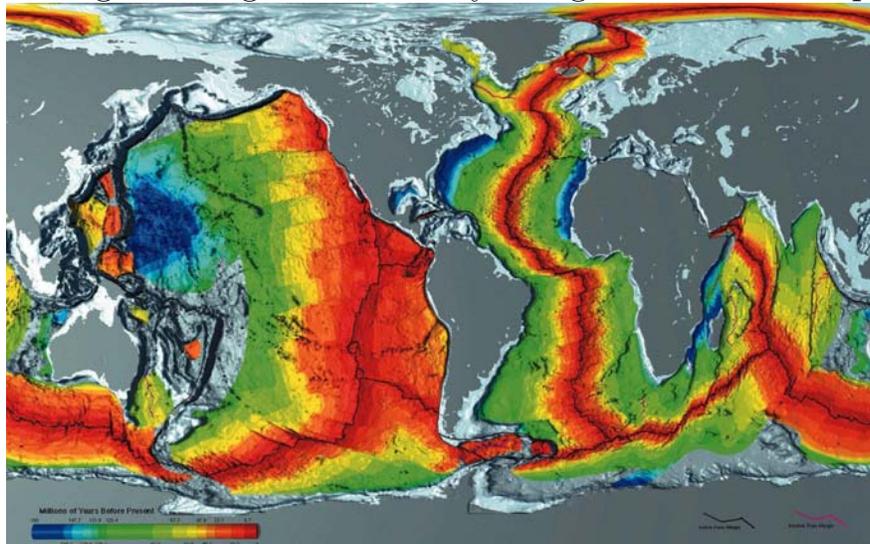
Reference: [1, 3].

Two kinds of plate motion caused by plate tectonics are of interest to the carbon cycle:

- Divergent (constructive) margins, typically at mid-ocean ridges, where new seafloor is created by volcanic processes.
- Convergent (destructive) margins, where plates collide and, typically, one plate is subducted beneath the other. The process forms island

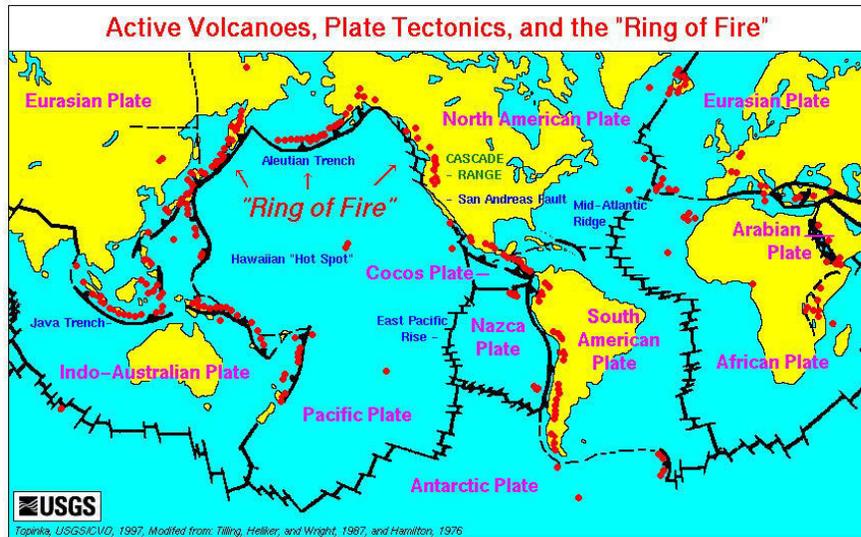
chains and mountain ranges, and results in volcanism about 250 km into the over-riding plate.

Divergent margins are readily recognizable from maps of seafloor age:



Seafloor age. Young seafloor is red, old is blue (NOAA, Wikipedia).

The volcanic “Ring of Fire” occurs along convergent margins:



Volcanism ● (USGS).

Total CO₂ outgassing at convergent and divergent margins is roughly equal, within a factor of two or so.

1.6 Seafloor heat flux and topography

Reference: [4].

Aspects of the qualitative discussion above can be quantitatively expressed and compared to observations.

In the following, we calculate the heat flux out of the seafloor as a function of its age, or distance from the ridge, and compare it to measurements.

As the seafloor cools with age, it becomes more dense and effectively sinks more deeply into the underlying mantle. We predict how this change occurs and also compare it to measurements.

All this derives from the diffusive transport of heat, to which we now turn.

1.6.1 Heat equation

To keep things simple, we consider spatial variations of temperature T only with respect to a single spatial coordinate z (i.e., depth beneath the seafloor).

Define

j = flux of heat through a unit area,

which has units

$$[j] = \frac{\text{watt}}{\text{m}^2} = \frac{\text{joule/s}}{\text{m}^2}.$$

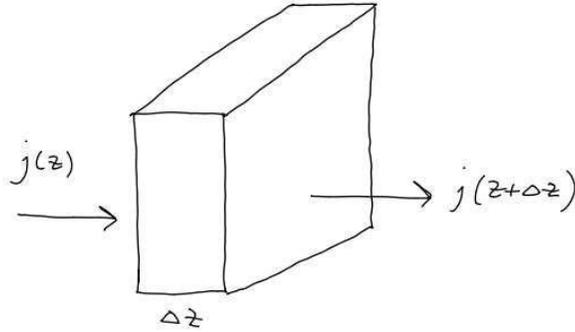
Fourier's law states that the flux j is proportional to the temperature gradient:

$$j = -k \frac{\partial T}{\partial z}, \tag{1}$$

where k is the *thermal conductivity*, with units

$$[k] = \frac{\text{watt}}{\text{°K} \cdot \text{m}}.$$

Consider a slab with an incoming heat flux $j(z)$ and an outgoing heat flux $j(z + \Delta z)$:



If there are neither sources nor sinks of heat, then a net heat flow into the slab must change its temperature (i.e., energy is conserved).

Suppose the temperature of the slab increases by an amount ΔT during a short time Δt .

Define

$$\begin{aligned} c &= \text{heat required for a } 1^\circ\text{K change in a unit mass of the slab} \\ &= \textit{specific heat capacity}. \end{aligned}$$

which has units

$$[c] = \frac{\text{joule}}{\text{kg } ^\circ\text{K}}.$$

Then

$$\begin{aligned} [j(z) - j(z + \Delta z)]\Delta t &= (\text{heat absorbed by slab over time } \Delta t)/\text{area} \\ &= c\Delta T \left(\frac{\text{slab mass}}{\text{area of slab face}} \right) \\ &= c\Delta T\rho\Delta z \end{aligned}$$

where ρ is the density of the slab.

Rearranging, we have

$$\rho c \frac{\Delta T}{\Delta t} = \frac{j(z) - j(z + \Delta z)}{\Delta z}$$

In the limit as $\Delta t \rightarrow 0$ and $\Delta z \rightarrow 0$,

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial j}{\partial z}.$$

Substitution of Fourier's law (1) for j then gives

$$\boxed{\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial z^2}}, \quad (2)$$

the one-dimensional *heat equation* or *diffusion equation*, where the *thermal diffusivity*

$$D = \frac{k}{\rho c}, \quad [D] = \frac{\text{length}^2}{\text{time}}.$$

1.6.2 Heating of a semi-infinite medium

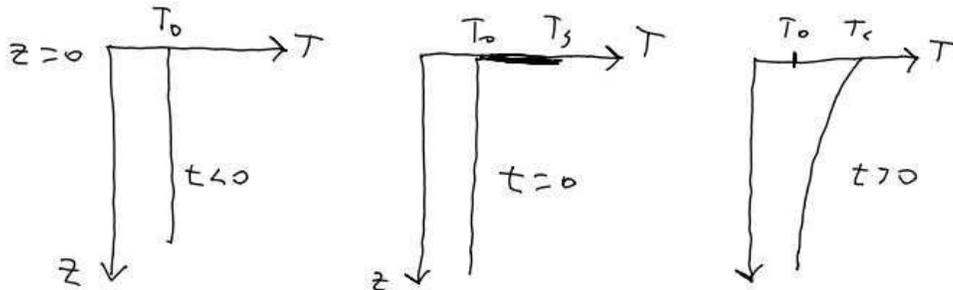
We next consider a classic problem. Suppose that the temperature is a constant (say, $T = T_0$) for all $z > 0$, but at time $t = 0$, the surface $z = 0$ is held at a higher or lower temperature T_s .

How does the temperature $T(z, t)$ of this half-space, or semi-infinite medium, change in time and space?

In other words, we seek the time-dependent solution of the heat equation subject to the conditions

$$\begin{aligned} T &= T_0, & z > 0, & t = 0 \\ T &= T_s, & z = 0, & t > 0 \\ T &\rightarrow T_0, & z \rightarrow \infty, & t > 0. \end{aligned}$$

Conceptually, we envision the transfer of heat from the surface $z = 0$ inwards.



(Note that in the seafloor problem, the half-space will be cooled from above, so that $T_s < T_0$, but the mathematical development is unchanged.)

To make matters simpler, we consider only the difference $T - T_0$ of temperature with respect to the amount $T_s - T_0$ by which the surface is heated. We therefore define the dimensionless variable

$$\theta = \frac{T - T_0}{T_s - T_0}.$$

The diffusion equation (2) then becomes

$$\frac{\partial \theta}{\partial t} = D \frac{\partial^2 \theta}{\partial z^2} \quad (3)$$

and our boundary conditions are now

$$\begin{aligned} \theta(z, 0) &= 0 \\ \theta(0, t) &= 1 \\ \theta(\infty, t) &= 0. \end{aligned}$$

Obviously θ varies with both time and space. However, because no characteristic length scale exists (i.e., the medium is semi-infinite, and the heating is at the boundary), the only quantity other than z with dimensions of length is

$$(Dt)^{1/2}.$$

We therefore suspect that

$$\theta(z, t) = \theta(\eta),$$

i.e., that θ is a function of the single dimensionless variable

$$\eta = \frac{z}{2\sqrt{Dt}},$$

where the factor of two is chosen for convenience.

η is called a *similarity variable*, because solutions at different points z and different times t are equivalent so long as η is unchanged. The solutions are then said to be *self-similar*.

As a practical matter, expressing the diffusion equation in terms of $\theta(\eta)$ rather than $\theta(z, t)$ allows us to transform a partial-differential equation to an ordinary differential equation. This is a classic trick in such problems, which in this case is known as the *Boltzmann transformation*.

We proceed as follows. Using the chain rule, we transform the LHS of the diffusion equation:

$$\begin{aligned}\frac{\partial \theta}{\partial t} &= \frac{d\theta}{d\eta} \frac{\partial \eta}{\partial t} \\ &= \frac{d\theta}{d\eta} \left(-\frac{1}{4t} \frac{z}{\sqrt{Dt}} \right) \\ &= -\frac{\eta}{2t} \frac{d\theta}{d\eta}.\end{aligned}$$

To obtain the RHS, we first write

$$\begin{aligned}\frac{\partial \theta}{\partial z} &= \frac{d\theta}{d\eta} \frac{\partial \eta}{\partial z} \\ &= \frac{1}{2\sqrt{Dt}} \frac{d\theta}{d\eta}.\end{aligned}$$

Using this result and again invoking the chain rule,

$$\begin{aligned}\frac{\partial}{\partial z} \frac{\partial \theta}{\partial z} &= \frac{\partial}{\partial z} \left(\frac{1}{2\sqrt{Dt}} \frac{d}{d\eta} \theta[\eta(z, t)] \right) \\ &= \frac{1}{2\sqrt{Dt}} \frac{\partial \eta}{\partial z} \frac{d^2 \theta}{d\eta^2} \\ &= \frac{1}{4Dt} \frac{d^2 \theta}{d\eta^2}.\end{aligned}$$

Substitution of these results into the diffusion equation (3) then yields

$$-\frac{\eta}{2t} \frac{d\theta}{d\eta} = D \frac{1}{4Dt} \frac{d^2 \theta}{d\eta^2}$$

or more simply

$$-\eta \frac{d\theta}{d\eta} = \frac{1}{2} \frac{d^2 \theta}{d\eta^2} \tag{4}$$

Note that all terms are now dimensionless.

To obtain the boundary conditions, note that

$$\begin{aligned}z = 0 &\rightarrow \eta = 0, \\ z = \infty &\rightarrow \eta = \infty, \\ t = 0 &\rightarrow \eta = \infty.\end{aligned}$$

The new boundary conditions are then

$$\begin{aligned}\theta(0) &= 1 \\ \theta(\infty) &= 0\end{aligned}$$

To solve equation (4) with these boundary conditions, define

$$\phi = \frac{d\theta}{d\eta} \tag{5}$$

and substitute into (4) to obtain

$$-\eta\phi = \frac{1}{2} \frac{d\phi}{d\eta}$$

or, equivalently,

$$-2\eta d\eta = \frac{d\phi}{\phi}.$$

Integrating both sides, we obtain

$$-\eta^2 = \ln \phi - \ln C_1,$$

where $-\ln C_1$ is an integration constant. We thus have that

$$\phi = C_1 e^{-\eta^2}.$$

Combining this result with equation (5), we find

$$\frac{d\theta}{d\eta} = C_1 e^{-\eta^2}.$$

which upon integration becomes

$$\theta(\eta) = C_1 \int_0^\eta e^{-\eta'^2} d\eta' + C_2. \tag{6}$$

Using the boundary condition $\theta(0) = 1$, we find that the integration constant

$$C_2 = 1.$$

Thus the second b.c., $\theta(\infty) = 0$, requires that

$$0 = C_1 \int_0^\infty e^{-\eta'^2} d\eta' + 1$$

and therefore

$$\begin{aligned}\frac{-1}{C_1} &= \int_0^\infty e^{-\eta'^2} d\eta' \\ &= \frac{\sqrt{\pi}}{2},\end{aligned}$$

which may be found in a table of integrals. Inserting the constants C_1 and C_2 into (6), we find

$$\theta(\eta) = \frac{-2}{\sqrt{\pi}} \int_0^\eta e^{-\eta'^2} d\eta' + 1,$$

or more succinctly

$$\theta = 1 - \operatorname{erf}(\eta)$$

where the *error function*

$$\operatorname{erf}(\eta) = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta'^2} d\eta'.$$

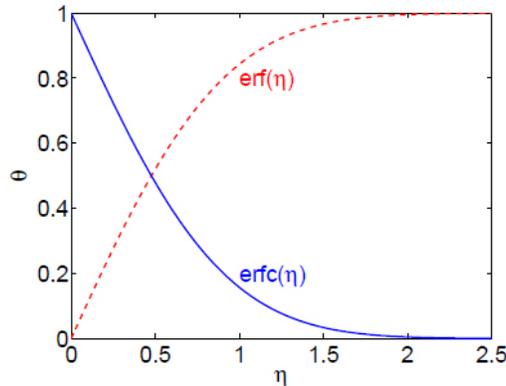
Not only does erf occur commonly in mathematics, physics, and probability theory, but so too does the *complementary error function*

$$\operatorname{erfc}(\eta) = 1 - \operatorname{erf}(\eta).$$

Using this definition, the rescaled temperature θ is simply

$$\theta = \operatorname{erfc}(\eta),$$

which is plotted below as the solid blue line:



Reverting to dimensional variables, we have

$$\frac{T - T_0}{T_s - T_0} = \operatorname{erfc}\left(\frac{z}{2\sqrt{Dt}}\right) \quad (7)$$

Looking at the plot, we see that there is a characteristic *diffusion length*

$$\eta \simeq 1 \quad \Rightarrow \quad z \simeq \sqrt{Dt}$$

over which heat extends from the boundary into the medium.

The essential result is that the thickness of this *thermal boundary layer* grows like $t^{1/2}$, which is typical in problems of diffusion.

1.6.3 Heat flux

The heat flux at $z = 0$ is straightforwardly obtained by computing the flux $j(0)$ from Fourier's law:

$$\begin{aligned} j(0) &= -k \left. \frac{\partial T}{\partial z} \right|_{z=0} \\ &= -k(T_s - T_0) \left. \frac{\partial}{\partial z} \operatorname{erfc} \left(\frac{z}{2\sqrt{Dt}} \right) \right|_{z=0} \\ &= k(T_s - T_0) \left. \frac{\partial}{\partial z} \operatorname{erf} \left(\frac{z}{2\sqrt{Dt}} \right) \right|_{z=0} \end{aligned}$$

Noting that $\frac{\partial}{\partial z} \operatorname{erf}[\eta(z, t)] = \frac{\partial \eta}{\partial z} \frac{d}{d\eta} \operatorname{erf}(\eta)$,

$$\begin{aligned} j(0) &= \frac{k(T_s - T_0)}{2\sqrt{Dt}} \left. \frac{d}{d\eta} \operatorname{erf}(\eta) \right|_{\eta=0} \\ &= \frac{k(T_s - T_0)}{2\sqrt{Dt}} \left. \frac{2}{\sqrt{\pi}} e^{-\eta^2} \right|_{\eta=0} \\ &= \frac{k(T_s - T_0)}{\sqrt{D\pi}} t^{-1/2}. \end{aligned}$$

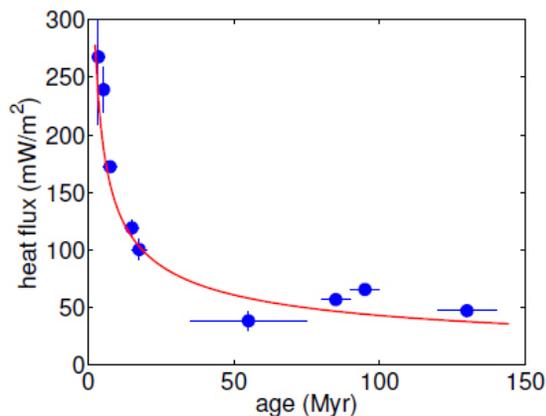
The heat flux thus decreases like $t^{-1/2}$. (The singularity at $t = 0$ is merely the consequence of the instantaneous imposition of a point source.)

We now return to the problem of the generation of new seafloor at mid-ocean ridges.

New seafloor is created at a high temperature and is cooled from above as it ages. Typical dimensional quantities are

$$\begin{aligned} T_0 - T_s &= 1300 \text{ }^\circ\text{K} \\ D &= 1 \text{ mm}^2 \text{ s}^{-1} \\ k &= 3.3 \text{ W m}^{-1} \text{ }^\circ\text{K}^{-1} \end{aligned}$$

Measurements of heat flux at the seafloor compare reasonably well with predictions based on its age t (Pacific data from Ref. [5]):



1.6.4 Seafloor topography

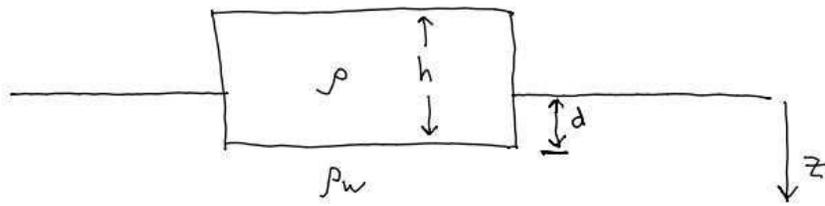
As the new seafloor cools, it becomes more dense and therefore less buoyant with respect to the more dense, underlying mantle. From this observation we can predict the depth of the seafloor as a function of its age.

The key point is that the mantle acts like a viscous fluid and the seafloor (i.e., the oceanic lithosphere) floats on it, but at a height determined by its density.

To quantify this, recall *Archimedes principle*:

Any floating object displaces its own weight of fluid.

Consider, for example, a wooden block of vertical extent h in a bathtub. The block is partially immersed to a depth $d < h$:



Take ρ_w to be the density of the water and ρ the density of the block. From Archimedes' principle, we have the *hydrostatic equilibrium* in which

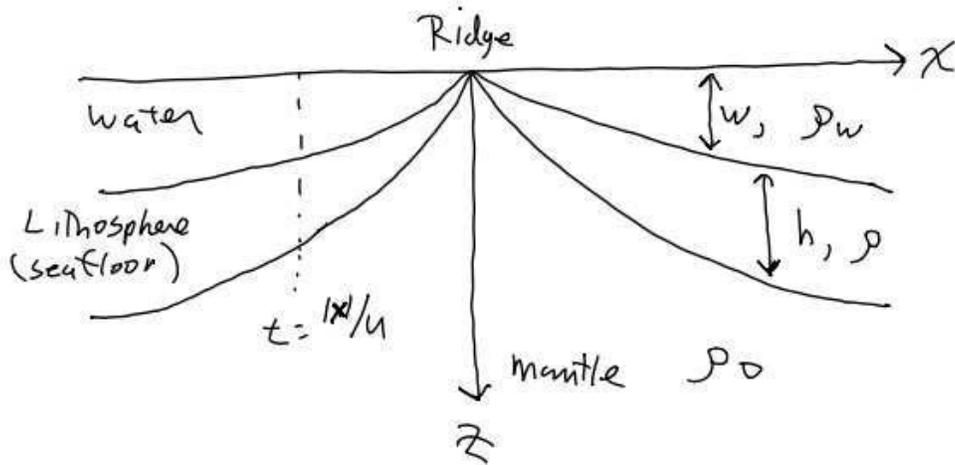
$$\rho h = \rho_w d.$$

Thus the height $h - d$ of the block relative to the water level is

$$\begin{aligned} h - d &= h - \frac{\rho}{\rho_w} h \\ &= h \left(1 - \frac{\rho}{\rho_w} \right). \end{aligned}$$

To apply this reasoning to the seafloor, we note that the combined weight of ocean + seafloor must remain the same as the seafloor cools and becomes more dense.

We consider depth $z(x)$ relative to the height of the mid-ocean ridge, and seek the specific depth $w(x)$ of the seafloor. (x is the distance from the ridge, and w is analogous to $h - d$ above.)



The seafloor (i.e., oceanic lithosphere) thickens as it cools, because underlying hot mantle material is converted to lithosphere.

Define

$$h(x) = \text{seafloor thickness a distance } x \text{ from the ridge.}$$

Then the weight of any column (of ocean + lithosphere) a depth h beneath the seafloor is simply the weight of the water + the weight of the lithospheric part of the column.

Let

$$\rho(x, z) = \text{density of lithosphere a distance } x \text{ from the ridge at depth } z.$$

From Archimedes, we know that the integrated mass in any column to a depth $w + h$ must equal the mass computed using the density ρ_0 of the mantle over the same vertical extent. Thus

$$\rho_w w + \int_0^h \rho dz = \rho_0(w + h)$$

or

$$w(\rho_w - \rho_0) + \int_0^h (\rho - \rho_0) dz = 0 \quad (8)$$

This is known as *isostatic equilibrium*. The first term is negative (mantle rock is denser than water). The second term is positive, since, as it cools, the density of the lithosphere becomes heavier than the mantle density.

Specifically, assume that the density ρ of the seafloor increases with respect to the mantle density ρ_0 according to

$$\rho - \rho_0 = -\rho_0 \alpha (T - T_0),$$

where T_0 is the temperature of new seafloor before it starts to cool, and α is the coefficient of thermal expansion.

Inserting equation (7) for $T - T_0$ we obtain

$$\rho - \rho_0 = -\rho_0 \alpha (T_s - T_0) \operatorname{erfc} \left(\frac{z}{2\sqrt{Dt}} \right) \quad (9)$$

where T_s is the temperature of the seawater above the seafloor and we take

$$\rho(x, z) \rightarrow \rho(t, z), \quad t = |x|/u$$

where t is the age of the seafloor and u is the spreading rate.

We insert (9) into the isostatic equilibrium (8) to obtain

$$w(t)(\rho_0 - \rho_w) = \rho_0\alpha(T_0 - T_s) \int_0^\infty \operatorname{erfc}\left(\frac{z}{2\sqrt{Dt}}\right) dz \quad (10)$$

where $w(t)$ is now the depth of the seafloor with age t and the integral now extends to $z = \infty$.

Substituting the similarity variable $\eta = z/(2\sqrt{Dt})$ into the integral, we have

$$\begin{aligned} \int_0^\infty \operatorname{erfc}\left(\frac{z}{2\sqrt{Dt}}\right) dz &= \int_0^\infty \operatorname{erfc}(\eta) \left|\frac{dz}{d\eta}\right| d\eta \\ &= 2\sqrt{Dt} \int_0^\infty \operatorname{erfc}(\eta) d\eta \\ &= \frac{2\sqrt{Dt}}{\sqrt{\pi}}. \end{aligned}$$

where we have used $\int_0^\infty \operatorname{erfc}(\eta) d\eta = 1/\sqrt{\pi}$.

Finally, we substitute this expression into (10) to obtain

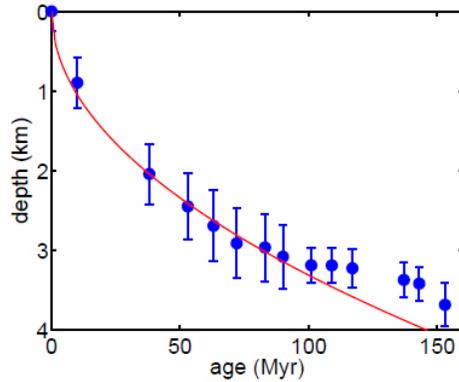
$$w(t) = \frac{2D^{1/2}\rho_0\alpha(T_0 - T_s)}{\pi^{1/2}(\rho_0 - \rho_w)} t^{1/2}.$$

Note that we have once again found a diffusive scaling in which a length varies like $t^{1/2}$.

We test this prediction by comparison with seafloor bathymetry from the North Atlantic, using data from Ref. [6]. We use the same values of $T_s - T_0$ and D we used for the heat flux in Section 1.6.3, supplemented by

$$\begin{aligned} \rho_0 &= 3300 \text{ kg m}^{-3} \\ \rho_w &= 1000 \text{ kg m}^{-3} \\ \alpha &= 2.8 \times 10^{-5} \text{ }^\circ\text{K}^{-1} \end{aligned}$$

The agreement for ages up to about 80 Myr is excellent:



Possibly the departure from theory at old ages is due to some secondary heat transfer from the mantle due, e.g., to convection.

1.6.5 Spreading rate and CO₂ production

The foregoing analysis amounts to an excellent confirmation of seafloor spreading.

We then ask: what controls the rate at which CO₂ is “produced” by degassing of magma at mid-ocean ridges?

A reasonable assumption is that the CO₂ production rate is proportional to the rate of seafloor production, which we express in terms of the spreading rate u :

$$\begin{aligned} \text{CO}_2 \text{ production rate} &\propto u = \text{spreading rate} \\ &\simeq 2\text{--}10 \text{ cm yr}^{-1}. \end{aligned}$$

We found above, in Section 1.6.3, that the heat flux j from the seafloor decreases with its age t like

$$j \propto t^{-1/2}.$$

Assuming that seafloor age is uniformly distributed, the mean (spatially averaged) heat flux \bar{j} then scales like

$$\begin{aligned} \bar{j} &\propto \frac{1}{t_{\max}} \int_0^{t_{\max}} t^{-1/2} dt \\ &\propto t_{\max}^{-1/2}, \end{aligned}$$

where t_{\max} is the typical age at which seafloor is consumed (i.e., subducted).

Taking L to be a characteristic distance between ridges and subduction zones, the age of subducting material then scales with the spreading rate like

$$t_{\max} \sim L/u.$$

The mean heat flux therefore scales like

$$\bar{j} \sim \frac{1}{(L/u)^{1/2}} \propto u^{1/2}. \quad (11)$$

Invoking the assumption that CO_2 production scales like the spreading rate u , we then have that the

$$\text{CO}_2 \text{ production rate} \propto \bar{j}^2.$$

The quadratic dependence of CO_2 production and spreading rate on the heat flux results from diffusive scaling.

This nonlinear amplification informs our knowledge of past environments: because the heat flux from the early Earth would have been about 2–3 times greater than today, CO_2 production from “volcanism” would have been about 4–9 times greater [7].

1.6.6 Kelvin’s estimate of the age of the Earth

Reference: [8].

The foregoing amounts to the “new view of the Earth” that first developed during the 1960s. One way to appreciate how huge such changes were is to consider Kelvin’s famous 19th century estimate of the age of the Earth.

Kelvin’s reasoning was delightfully simple, making the following assumptions:

- The Earth began its geological history at a uniform temperature of about $T = 4000$ °C.
- The earth then cooled, due to a constant $T \simeq 0$ °C boundary condition at the surface.

Reasoning as in Section 1.6.3, he found that

$$\text{heat flux} \propto \frac{1}{\sqrt{\text{age}}} \quad (12)$$

and used estimates of the heat flux in Edinburgh and appropriate dimensional constants to find that the Earth's age was about 20 Myr.

This not only threatened the biblical version of Earth history, but also the view of the geologist Lyell, for whom the Earth was unchanging throughout essentially infinite time (a concept known as “uniformitarianism”).

The eventual knowledge of radioactivity and the advent of age dating proved Kelvin wrong: the Earth is about 4.5 billion years old, about 100 times older than Kelvin's estimate.

What was wrong with Kelvin's theory?

There are two problems:

- Kelvin was unaware of the heating due to radioactive decay, which provides a prolonged source.
- He ignored thermal convection.

The latter problem turns out to be the more serious deficiency, because convection greatly increases the observed heat flow (see equation (11)), and therefore, via (12), it can easily lead to a serious underestimate of age.

1.7 Summary

Thermal convection is the “engine” that drives plate tectonics and volcanism.

It turns out that volcanism is, over the long-term, responsible for the CO₂ in the atmosphere, and thus the source of carbon that is fixed by plants.

Thus volcanism may be said to sustain life.

Without it, there probably would be no CO₂ in the atmosphere, and therefore we wouldn't be around to discuss it...

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