Lab 1: Stress, Mohr's circles
EXERCISES

Fall 2005

1 Mohr circles for stress

For the problems that you use a Mohr circle construction, please show the circles you draw, labeling the points on the construction corresponding to the relevant planes.

1. A plane has an area of 10 square metres. Across this plane a force of 10^7 newtons is applied, the direction of which is oriented at 30 degrees from the normal to the plane. What is the normal stress on the plane? What is the maximum shear stress on the plane?

2. Across a particular plane, the state of stress is measured as σ =1.2 kbar and τ =-0.6 kbar. On a plane at right angles to the first, the state of stress is σ =0.6 kbar and τ =0.6 kbar.
Assuming that one principal stress lies parallel to the intersection of the planes, (i) What are the values of σ_1 and σ_3?; (ii) What is the angle between σ_1 and the normal to the first plane?
What basic assumption do you make in order to arrive at these answers?

3. Maximum and least principal stresses in a homogeneous rock body are measured as 1.5 kbar and 0.8 kbar respectively. What are the normal and maximum shear stresses on the following planes, parallel to the intermediate axis of stress and whose normals to σ_1 are inclined at
(i) 25°
(ii) 45°
(iii) 60°
(iv) 65°
(v) 80°

4. Assume that in both problems 2 and 3 the stress states are two-dimensional.
(i) What is the value of the mean stress in each case?
(ii) What is the value of the deviatoric normal and shear stress in each case?

5. Draw examples of Mohr's circles (labelling $\sigma_1, \sigma_2, \sigma_3$ on the diagram) for the following states of stress:
(i) General compression.
(ii) General tension.
(iii) Lithostatic stress.
(iv) Pure shear.
(v) Uniaxial tension
(vi) Uniaxial compression.

2 Some simple geodynamic applications of stress

Isostasy

For a column of rock in equilibrium, the normal force on horizontal planes is given by $\sigma_{zz} = \rho g z$. That is, the normal stress (per unit area) on horizontal planes increases linearly with depth. The normal stress due to the weight of the overlying rock or overburden is known as the lithostatic stress. If hydrostatic equilibrium holds, then the lithostatic stress beneath two columns of rock has to be equal. In geodynamics, this is known as isostasy.

1. A mountain range has an elevation of 5km. Assuming that $\rho_m = 3300$ kg m^{-3} and $\rho_c = 2750$ kg m^{-3}, and that the mountain range is in isostatic equilibrium with 35 km thick continental crust whose top surface is at sea-level, determine the thickness of the continental crust beneath the mountain range.

2. The Tibetan plateau has an average elevation of about 5km above sea-level. At some distant point in the future, this elevation will eventually be reduced to zero. Suppose that the only process for decreasing elevation is erosion (i.e. you take material from the top). How much material do you have to erode in order to bring Tibet back down to sea-level? If this occurs, you will expose rocks that were once at great depths. What metamorphic grade (in kilobars) will mineral assemblages found in these rocks record? Strangely enough, there are no areas of the Earth's surface that expose an equivalent area of such high grade rocks. This implies either that something like the Tibetan plateau is unprecedented in the Earth's history, or there processes other than erosion for reducing the thickness of continental crust. What other process might do this? Expand on your answer with reference to another example of a continent-continent collision and its fate (You could try looking out the window).

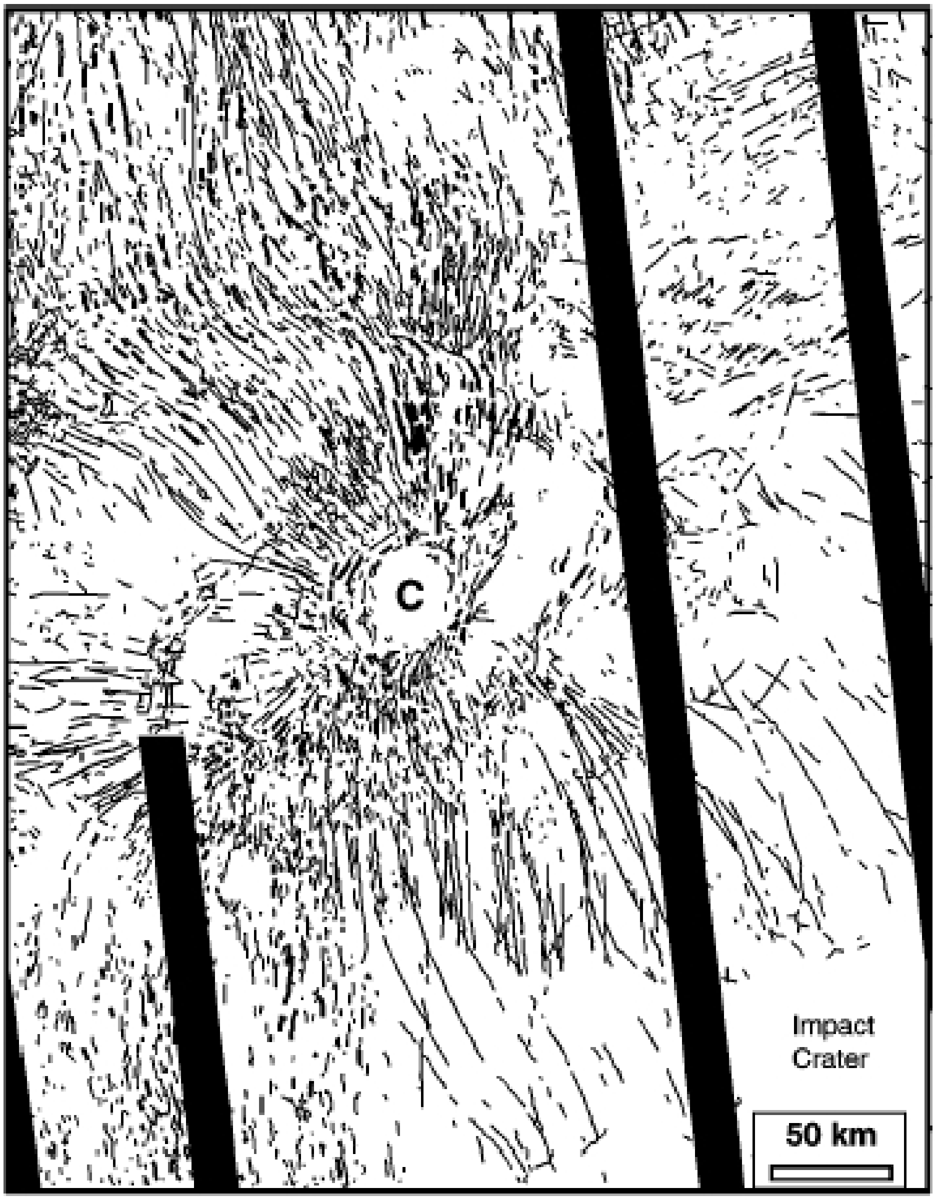
The orientation of principal stresses

At the scale of mountain belts, we can make some "broad side of the barn" generalisations about the orientations of principal stresses. Suppose that faults form at about 30°to σ_1 and contain σ_2 (this is a coarse version of Anderson's theory of faulting, which we will return to later). To a first order, one of the principal stresses is usually oriented vertically and is due to the weight of the rocks and the horizontal stresses are at least partly due to "far-field" tectonic forces.

1. Draw cartoon maps for the following three basic kinds of plate tectonic boundaries:
(i) A convergent boundary, faults are thrust faults;
(ii) A divergent boundary, faults are normal faults;
(iii) A transcurrent boundary, faults are strike-slip.

2. What happens to the thickness of the crust in each of these situations? What happens to magnitude of the principal stresses as a consequence? Can mountain belts grow (in height) forever?

Of course, these are broad generalisations. Stress variations are typically quite pronounced at a variety of scales. Stress trajectory maps are used to illustrate the variation of the orientation of principal strains within a map area. The attached figure shows a volcanic intrusion and associated dyke swarms. Dykes typically fill extensional fractures, which you can assume form perpendicular to the smallest principal stress.


C
Impact
Crater
50 km