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### 12.510 Introduction to Seismology

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## Today's lecture (Part 1)

1. Fault geometry
2. First motions
3. Stereographic fault plane representation

## 1 Fault geometry

There are three main types of faults:

1. Normal faults
2. Reverse faults
3. Stike-slip faults

We can think of these three types of faults as forming a set of basis functions. All faults can be described as a combination of these 3 basis faults.

The fault geometry is described in terms of the orientation of the fault plane and the direction of slip along the plane. The geometry of this model is shown in figure 2.

The dip angle $\delta$ is the angle between the fault plane and the horizontal.
The slip angle, $\lambda$ is the angle between the slip-vector and the horizontal.
The stike angle $\phi$ is used to orientate this system relative to the geographic one. It is defined as the angle in the plane of the earth's surface measured clockwise from north to the $x_{1}$ axis.

We can use the slip angle, $\lambda$ to specify the type of motion on the fault.
$\lambda=0$ implies left-lateral (sinstral) fault motion
$\lambda=180$ implies right-lateral (dextral) fault motion
$\lambda=270$ implies normal faulting (extension)
$\lambda=90$ implies reverse faulting
Most earthquakes consist of a combination of these motions and have a slip angle between these values.

Seismologists refer to the direction of slip in an earthquake and the orientation on the fault on which it occurs as the 'focal mechanism'. They typically display the focal mechanisms on maps as a 'beach-ball' symbol. We will talk more about this


Figure by MIT OpenCourseWare.
Figure 1: Basic types of faulting. Strike-slip motion can be right- or left-lateral. Dip slip-faulting can occur as either reverse or normal faulting (1)


Figure by MIT OCW.
Figure 1. Fault geometry used in earthquake studies. [Adapted from Stein and Wysession, 2003]

representation later in the lecture

## 2 First Motions

The focal mechanism uses the fact that the pattern of radiated seismic waves depends on the fault geometry. The simplest motion is the first motion, or polarity, of body waves.
The first motion is compression if the station is located so that material near the fault moves 'towards' the station, or 'dilation' where the motion is 'away from' the station. A vertical-component seismogram will record either an upward or downward first motion, corresponding to either compression or dilation. Figure 2 illustrates the first motion concept for a strike-slip earthquake on a vertical fault. Two vertical planes are shown in the diagram, which are related to the geometry of the fault
-One of these is the fault plane
-One of these runs perpendicular to the fault plane and is known as the auxiliary plane.
These planes define four quadrants. Two of these are compressional and two of these are dilational. If these perpendicular planes can be found the fault geometry will be known. One difficulty is that the first motion on the actual fault plane is the same as that on the auxiliary plane. The first motions alone are therefore insufficient to resolve which plane is the actual fault plane. This is a fundamental ambiguity

in inverting seismic observations for fault models. Additional geologic or geodetic information is needed to identify which is the actual fault plane.

## 3 Stereographic projections

The fault geometry can be found from the distribution of data on a sphere around the focus. We can trace rays from the earthquake onto a hemisphere using the eikonal equation. We can then use a stereographic projection to transform the hemisphere to a plane. The graphic construction that allows us to do this is called a stereonet (figure 3)

Consider how planes will appear on this net.
-A vertically dipping, N-S striking plane will plot as a straight line.
-A N-S striking plane with a different type will appear as a curve going from top to bottom.
-A horizontal plane will appear as a perimeter.
-Different type of fault will appear differently on a stereonet (see figure 5). For example, a four-quadrant 'checkerboard' indicates pure strike-slip motion.


Figure by MIT OCW.


Figure by MIT OCW.

## Schematic diagram of a focal mechanism



## 4 'Beach Balls’

We established earlier that you can represent focal mechanisms using 'beach ball' diagrams. Using a simple model, faulting will occur on planes, $45^{\circ}$ from the maximum and minimum compressive stresses. It follows that the maximum compressive ( P ) and minimum compressive stress ( T ) axis are found by bisecting the compressional quadrants respectively.

Note: The 'tension' (T) axis is actually the minimum compressive stress, because compression occurs at the source. The 'upward' and 'downward' first motions are from the observers point of view, but the ' P ' and ' T ' axes refer to what is happening at the source.

The 'beach-ball' representation of focal mechanisms is important in tectonic studies.

## 200701010105A NORTHERN MID-ATLANTIC RI

Date: 2007/ $1 / 1$ Centroid Time: 1: 5:16.1 GMT
Lat $=32.75 \quad$ Lon $=-39.78$
Depth $=12.0 \quad$ Half duration $=0.7$
Centroid time minus hypocenter time: 2.7
Moment Tensor: Expo=23 $-2.7900 .458 \quad 2.330-0.701-1.8901 .200$
$\mathrm{Mw}=5.0 \quad \mathrm{mb}=4.8 \quad \mathrm{Ms}=0.0 \quad$ Scalar Moment $=3.49 \mathrm{e}+23$
Fault plane: $\quad$ strike $=210 \quad$ dip $=28 \quad$ slip $=-84$
Fault plane: $\quad$ strike $=23 \quad \operatorname{dip}=63 \quad$ slip $=-93$

Figure by MIT OpenCourseWare.

## 5 The Harvard-CMT catalogue

Seismologists collect data in real-time when there is an earthquake and produce a representation of the focal-mechanism of an earthquake as a 'beach-ball' diagram. These are listed as entries in the Harvard-CMT catalogue. Each entry includes the following data:
-the location of the earthquake
-the time at which it occurred
-the depth of the earthquake
-the half-duration of the earthquake
Note that there are four different measurements of the magnitude of the earthquake listed in the Harvard-CMT catalogue. Note also that 2 sets of values for the strike, slip and dip directions of the fault plane are listed. This is because we are unable to distinguish between the fault plane and the auxiliary plane when these entries are produced.

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## MOMENT TENSOR

To know the source properties from the observed seismic displacements, the solution of the equation of motion can be separated as below.

$$
\begin{equation*}
u_{i}(\vec{x}, t)=G_{i j}\left(\vec{x}, t ; \vec{x}_{0}, t_{0}\right) f_{j}\left(\vec{x}_{0}, t_{0}\right) \tag{1}
\end{equation*}
$$

Where $u_{i}$ is the displacement, $f_{j}$ is the force vector. The Green's function $G_{i j}$ gives the displacement at point $X$ that results from a unit force function applied at point $X_{o}$. Internal forces, $f$, must act in opposing directions, - $f$, at a distance $d$ so as to conserve momentum (force couple). For angular momentum conservation, there also exists a complementary couple that balances the forces (double couple). There are nine different force couples as shown in Figure 1.


Figure by MIT OCW.
Figure 1. The nine different force couples for the components of the moment tensor. [Adapted from Shearer 1999]

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We define the moment tensor M as:

$$
M=\left[\begin{array}{lll}
M_{11} & M_{12} & M_{13}  \tag{2}\\
M_{21} & M_{22} & M_{23} \\
M_{31} & M_{32} & M_{33}
\end{array}\right]
$$

$M_{i j}$ represents a pair of opposing forces pointing in the $i$ direction, separated in the $j$ direction. Its magnitude is the product $f d$ [unit: $\mathrm{N}-\mathrm{m}$ ] which is called the seismic moment. For angular momentum conservation, the condition $M_{i j}=M_{j i}$ should be satisfied, so the moment tensor is symmetric. Therefore we have only six independent elements. This moment tensor represents the internally generated forces that can act at a point in an elastic medium.

The displacement for a force couple with a distance d in the $\hat{X}_{k}$ direction is given by:

$$
\begin{equation*}
u_{i}(\vec{x}, t)=G_{i j}\left(\vec{x}, t ; \vec{x}_{0}, t_{0}\right) f_{j}\left(\vec{x}_{0}, t_{0}\right)-G_{i j}\left(\vec{x}, t ; \vec{x}_{0}-\hat{x}_{k} d, t_{0}\right) f_{j}\left(\vec{x}_{0}, t_{0}\right)=\frac{\partial G_{i j}\left(\vec{x}, t ; \vec{x}_{0}, t_{0}\right)}{\partial x_{k}} f_{j}\left(\vec{x}_{0}, t_{0}\right) d \tag{3}
\end{equation*}
$$

The last term can be replaced by the moment tensor and thus:

$$
\begin{equation*}
u_{i}(\vec{x}, t)=\frac{\partial G_{i j}\left(\vec{x}, t, \vec{x}_{0}, t_{0}\right)}{\partial x_{k}} M_{j k}\left(\vec{x}_{0}, t_{0}\right) \tag{4}
\end{equation*}
$$

There is a linear relationship between the displacement and the components of the moment tensor that involves the spatial derivatives of the Green's functions. We can see the internal force $f$ is proportional to the spatial derivative of moment tensor when equation (1) and (4) are compared.

$$
\begin{equation*}
f_{i} \sim \frac{\partial}{\partial x_{j}} M_{i j} \tag{5}
\end{equation*}
$$

Consider right-lateral movement on a vertical fault oriented in the $\mathrm{x}_{1}$ direction and the corresponding moment tensor is given by:

$$
M=\left[\begin{array}{ccc}
0 & M_{12} & 0  \tag{6}\\
M_{21} & 0 & 0 \\
0 & 0 & 0
\end{array}\right]=\left[\begin{array}{ccc}
0 & M_{0} & 0 \\
M_{0} & 0 & 0 \\
0 & 0 & 0
\end{array}\right]
$$

Where $M_{o}=\mu \bar{D} s$ is called the scalar seismic moment. This is a measure of earthquake size and energy release where: $\mu$ is shear modulus, $\bar{D}=D(x) / L$ is average displacement, and s is area of the fault. $M_{o}$ can

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be time dependent, so that $M_{o}(t)=\mu \bar{D}(t) s(t)$. The right-hand side time dependent terms become the source time function, $\mathrm{x}(\mathrm{t})$, thus the seismic moment function is given by

$$
\begin{equation*}
M(t)=M_{0} x(t)=\mu D(t) s(t) \tag{7}
\end{equation*}
$$

We can diagonalize the moment matrix (6) to find principal axes. In this case, the principal axes are at $45^{\circ}$ to the original $\mathrm{x}_{1}$ and $\mathrm{x}_{2}$ axes.

$$
M^{\prime}=\left[\begin{array}{ccc}
M_{0} & 0 & 0  \tag{8}\\
0 & -M_{0} & 0 \\
0 & 0 & 0
\end{array}\right]
$$

The principal axes become tension and pressure axis. The above matrix represents that $x_{1}{ }^{\prime}$ coordinate is the tension axis, T, and $\chi_{2}^{\prime}$ is the pressure axis, P. (Figure 2)


Figure by MIT OCW.
Figure 2. The double-coupled forces and their rotation along the principal axes. [Adapted from Shearer 1999]

