12.520 Lecture Notes 19

Plates (continued)

Flexural equation: $D\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} + \Delta\rho g w = q(x)$

where $D = \frac{Eh^3}{12(1-\nu^2)}$.

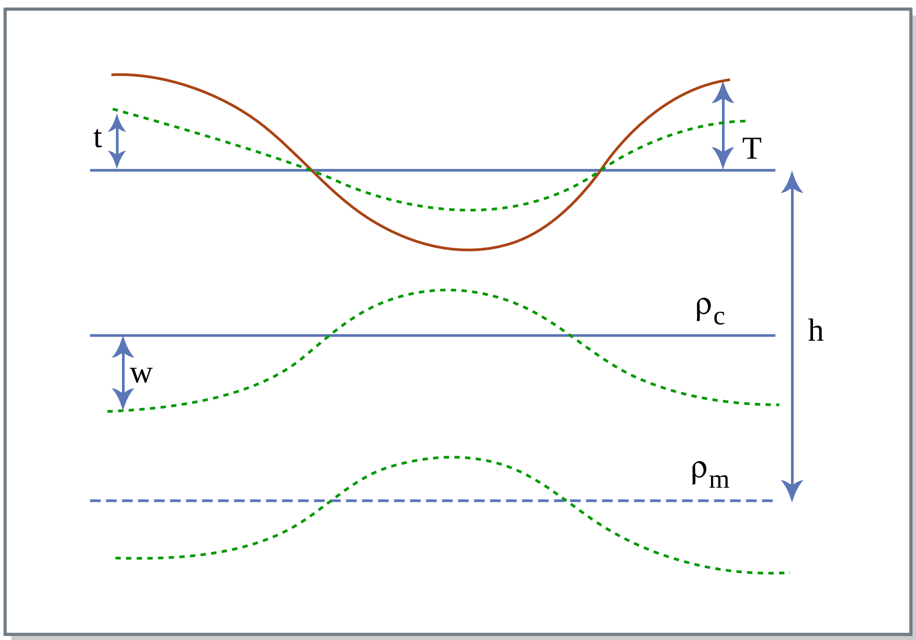

Figure 19.1
Figure by MIT OCW.

$T = T_0 \cos kx = T_0 \cos\frac{2\pi x}{\lambda}$

"Harmonic" load

$t = t_0 \cos kx\,, \quad w = w_0 \cos kx\,, \quad t_0 = T_0 - w_0$

$$D\frac{d^4 w}{dx^4} + \rho_m g w = t_0 \rho_c g \cos kx \text{ when } P = 0$$

$$k^4 D w_0 \cos kx + \rho_m g w_0 \cos kx = t_0 \rho_c g \cos kx$$

$$w_0 = \frac{t_0}{\frac{\rho_m}{\rho_c} - 1 + \frac{Dk^4}{\rho_c g}}$$

Call $2\pi\left(\frac{D}{\rho_c g}\right)^{1/4} \equiv \lambda_l$ flexural wavelength

For λ ? λ_l, $(\lambda_l k)^4 = 1$, w_0 ; $\frac{t_0}{\frac{\rho_m}{\rho_c} - 1}$; isostacy

For $\lambda = \lambda_l$, w_0 ; $0 \Rightarrow$ uncompensated

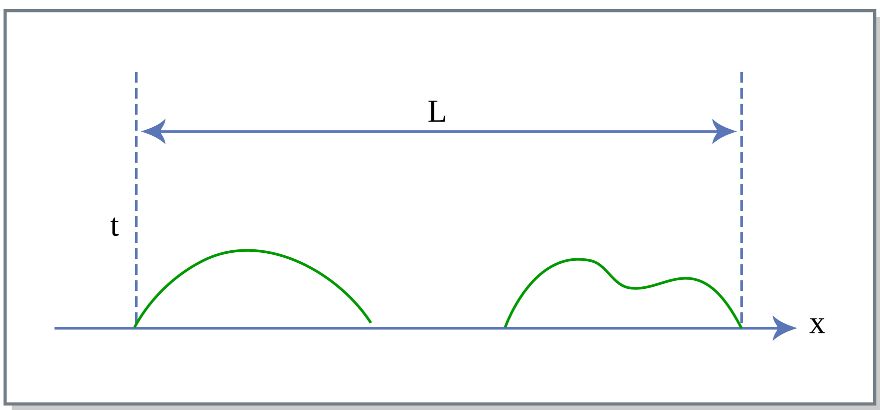

Figure 19.2
Figure by MIT OCW.

$$t(x) = \sum_{n=0}^{\infty}\left[t_n^c \cos\frac{2\pi n x}{L} + t_n^s \sin\frac{2\pi n x}{L}\right]$$

Find t_n^c, t_n^s Assume D, calculate w_n^s, w_n^c

Synthesize $w(x)$, compare to observations.

Plate subject to an end load

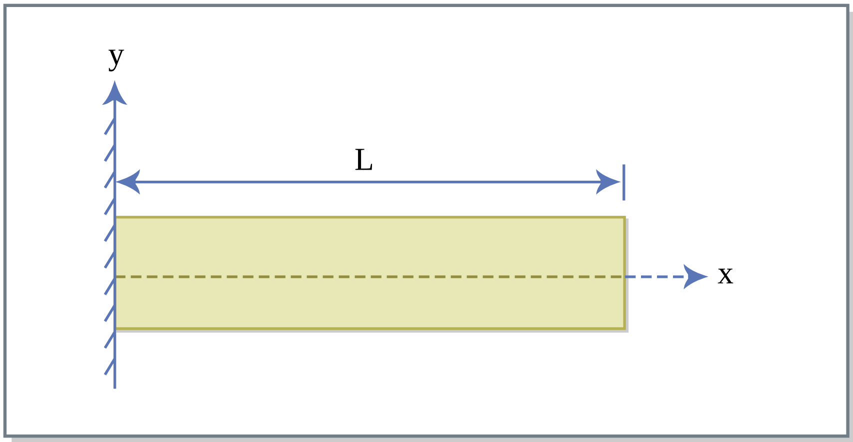

Figure 19.3
Figure by MIT OCW.

Shear force: $\frac{dV}{dx} = -q$

Since $q = 0$, V = const. = V_a

Bending moment:

$$\frac{dM}{dx} = V + P\frac{dw}{dx}$$

Since $P = 0$, $\frac{dM}{dx} = V \Rightarrow M = V_a x + \text{const} \Rightarrow 0$ at $x = L$

$$M = V_a(x - L)$$

Displacement:

$$\frac{d^4w}{dx^4} = 0 \quad \Rightarrow \quad \frac{d^3w}{dx^3} = \text{const}$$

But $M = -D\dfrac{d^2w}{dx^2} = 0$, $\quad \dfrac{dM}{dx} = -D\dfrac{d^3w}{dx^3} = V_a$

$$\frac{d^3w}{dx^3} = -\frac{V_a}{D}$$

$$\frac{d^2w}{dx^2} = -\frac{V_a}{D}(x - L)$$

Subject to $w, \dfrac{dw}{dx} = 0$ at $x = 0$

$w = \dfrac{V_a x^2}{2D}(L - \dfrac{x}{3}) \quad \Rightarrow \quad$ cubic displacement

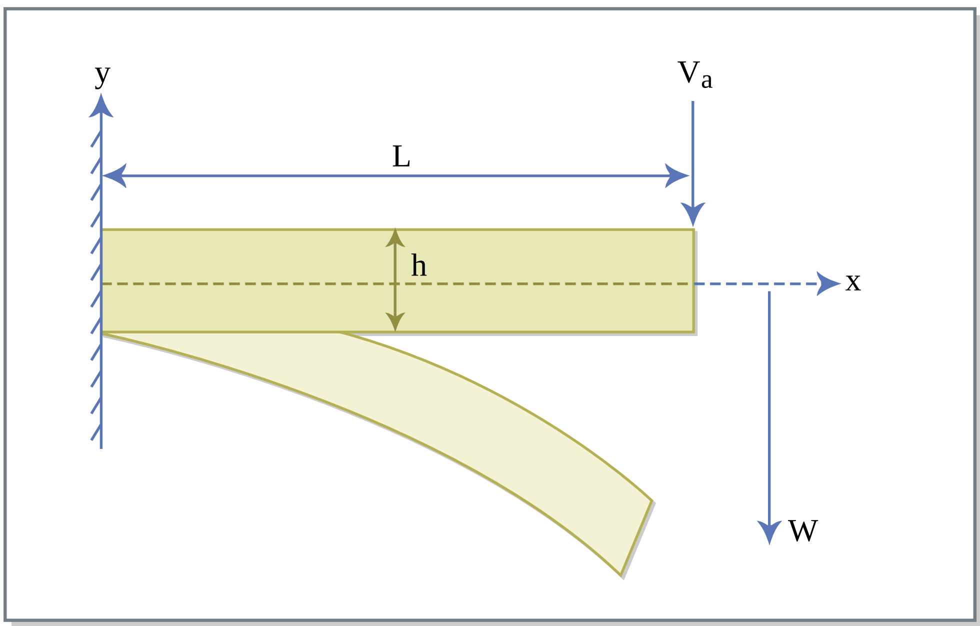

Figure 19.4

Figure by MIT OCW.

$$w = \frac{V_a}{2D}x^2(L - \frac{x}{3})$$

Assumption $|\sigma_{xy}| = |\sigma_{xx}|$

$$\sigma_{xx} = \frac{E}{1-\nu^2}\varepsilon_{xx}$$

$$\varepsilon_{xx} = -y\frac{d^2w}{dx^2}$$

$$M = -D\frac{d^2w}{dx^2}$$

$$\varepsilon_{xx} = \frac{y}{D}M$$

$$\sigma_{xx}^{\max} = \frac{E}{1-\nu^2}\frac{h}{2}\frac{1}{D}V_a L = \frac{6V_a L}{h^2} = \frac{6V_a}{h}\left(\frac{L}{h}\right)$$

$$\langle\sigma_{xy}\rangle = \frac{V_a}{h} = \frac{1}{6}\frac{h}{L}\sigma_{xx}^{\max}$$