

12.520 Problem Set 6

Due 11/13/06

1) (20%) The relationship between stress and strain for a simple isotropic elastic material is:

$$\tau_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij}$$

(λ and μ are constants, the Lamé parameters, and are called “moduli”)

It is sometimes useful to present ε_{ij} rather than σ_{ij} , as the independent variable. One way of determining the equation for ε_{ij} in terms of σ_{ij} and σ_{kk} is to first solve the equation above for ε_{kk} in terms of σ_{kk} , then substitute this value for ε_{kk} and solve for ε_{ij} .

First, determine the expression for ε_{kk} in terms of σ_{kk} . Then, write an equation equivalent to the one above, but with ε_{ij} on the left side and stresses on the right hand side. (The constants multiplying the stresses are called “compliances”.)

2) (30%) For the small strains involved in most elastic problems, the mathematics describing stresses and strains are linear. This means that the solution to a more complicated problem can be found by breaking the problem into smaller, simpler pieces and adding the solutions together. As an example, you can determine the relationship between the bulk modulus and other elastic moduli such as Young's modulus E, the Lamé parameters λ and μ (where $\mu = G$, the shear modulus), and Poisson's ratio ν in the following way:

i) First consider the stresses and strains involved in uniaxial stress along the x_1 axis. That is, consider the experiment used to define Young's modulus E: for $\sigma_{11} = \sigma_0$ and all other $\sigma_{ij} = 0$, write each component of the strain tensor, first in terms of λ and μ , then in terms of E and ν .

ii) Next, consider uniaxial stress σ_0 along the x_2 direction. Write each component of the strain tensor, first in terms of λ and μ , then in terms of E and ν .

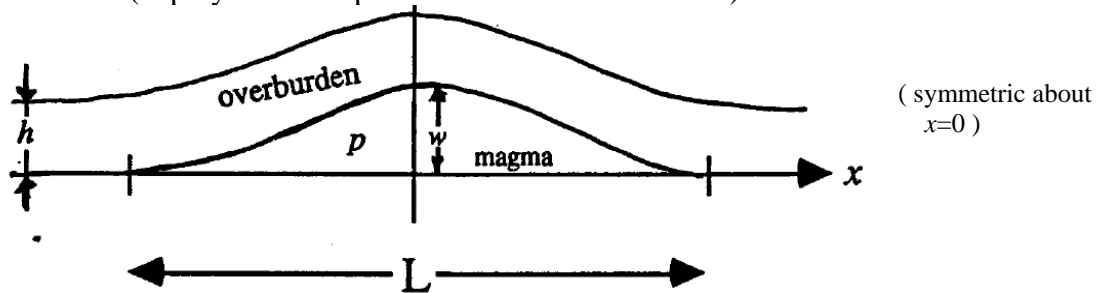
iii) Next, consider uniaxial stress σ_0 along the x_3 direction. Write each component of the strain tensor, first in terms of λ and μ , then in terms of E and ν .

iv) Finally, add these three solutions together to determine the total strains involved when a stress σ_0 is applied along each axis at the same time. From these relations between stress and strain, determine the bulk modulus K, first in terms of λ and μ , then in terms of E and ν .

3) (20%) Treat the lithosphere as an elastic plate of thickness h, Young's modulus E, subjected to a compressive force P. Assuming that buckling will occur in the form of a sine curve, find the minimum P needed to initiate "subduction" (i.e., buckling) by this mechanism. Estimate needed parameters. (See discussion in T&S).

4) (30%) Consider the (2-D) laccolith (flat bottomed igneous intrusion) shown below. The overburden is to be treated as a thin elastic plate loaded by the uniform pressure p of the magma. The lithostatic pressure can be ignored, since it is only the excess pressure that leads to deformation.

a) Calculate w , the deflection of the overburden, assuming that w and dw/dx are 0 at $x = \pm L/2$. (A polynomial expansion in x works well here.)



b) The figure below shows the results of a repeat leveling survey in the Owens Valley near Mammoth Lakes. The large uplift observed in 1980 is believed to be the result of the inflation of a magma chamber.

i) If the magma chamber can be approximated as a 2-D laccolith, find its depth and the overpressure. (The dots are the data - the dashed line is a different kind of theoretical model, assuming a spherical magma chamber. Fit the data, not the model. Note: your answer may surprise you!)

ii) Is the inferred normal faulting along the Hilton Creek fault consistent with the near-surface stress predicted for the laccolith model? What about if the fault nucleated at a depth of 10 km?

